Is it Possible to Gain from Brain Drain?

Annukka Hujanen

University of Helsinki

Faculty of Social Sciences

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HELSINGIN YLIOPISTO
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UNIVERSITY OF HELSINKI

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Tiivistelmä – Referat – Abstract

The number of international migrants has been steadily increasing from the 1960s onwards. Especially the developed countries have been net gainers of migration, whereas the developing countries have been net losers of emigrants. Because particularly the amount of skilled emigrants has been increasing, brain drain has become a concern. A common understanding of detrimental brain drain dominated the economic literature until the mid-1990s. However, during the 1990s it was noticed that migration into developed countries engenders incentives to invest in education and consequently increases the amount of skilled individuals in developing countries. The purpose of this thesis is therefore to study potential gains from brain drain.

The thesis first introduces a simple overview of brain drain. Because the level of schooling determines to a great extent the state of the economic development, the survey pays attention to the effects of emigration on educational decisions. By allowing a proportion of the skilled workers of a developing country to migrate into a developed country, the latent ability of the last individual to acquire education decreases. Consequently, brain drain induces an incentive effect.

Secondly, a more detailed human capital accumulation model is introduced to find support for the simple overview of brain drain. By utilizing a wage-setting equation determined by the production sector and a skill-setting equation defined by the utility maximizing individuals, an equilibrium analysis of brain drain is performed. It is assumed that the level of economic development, and consequently the wage-setting equation, depends on human capital through imitation and innovation activities. By examining the impacts of human capital on economic growth, it is shown that an increase in the stock of skilled human capital speeds up the growth the more the closer the economy is to the world´s technological frontier. As the country still approaches the technological frontier, the accelerating effect weakens.

Both approaches find certain circumstances which lead to economic gains as a result of brain drain. Particularly, it is detected that the wage rate of the developed country relative to the wage rate of the developing country needs to be sufficiently high in order to engender an incentive effect. Furthermore, the emigration possibility of the skilled individuals must be low enough. The simple overview of brain drain also shows that the amount of highly skilled individuals cannot be previously high. Otherwise the incentive effect does not operate. At the same time, the human capital accumulation model emphasizes that individuals face a minimum subsistence level which creates a liquidity constraint. If the standard of living of the developing country falls below the standard of living of the developed country, the liquidity constraint restricts the educational decisions. Therefore, poverty trap is still a possibility for extremely poor countries. According to the analysis, the best potential for brain drain to result in positive effects on the source country of emigration is for middle-income countries.

Avainsanat – Nyckelord – Keywords
Brain Drain, Economic Growth, Migration, Technological Development

HELSINGIN YLIOPISTO
HELSINGFORS UNIVERSITET
UNIVERSITY OF HELSINKI

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Tiivistelmä – Referat – Abstract

Kansainvälisten siirtolaisten määrä on kasvanut tasaista vauhtia 60-luvulta lähtien. Erityisesti kehittyneet maat ovat olleet siirtolaisten vastaanottajia, kun taas kehittymättömät maat ovat menettäneet asukkaitaan muuttoliikkeen seurauksena. Koska varsinkin koulutettujen maastamuuttajien määrä on lisääntynyt, huoli aivovuodosta on ollut esillä. Yleinen käsitys aivovuodon haitallisista seurauksista hallitsi talouskirjallisuutta aina 90-luvun puoliväliin asti. 90-luvun aikana kuitenkin huomattiin, että muuttomahdollisuus kehittyneisiin maihin kannustaa yksilöitä investoimaan koulutukseen ja täten lisää koulutettujen määrää kehitysmaissa. Tämän tutkielman tarkoituksena onkin tarkastella aivovuodon mahdollisia hyötyjä.

Tutkielma esittelee ensin taloustieteellisen yleiskatsauksen aivovuotoon. Huomiota kiinnitetään erityisesti maastamuuton vaikutuksiin koulutuspäätöksiin, sillä taloudellinen kehitys riippuu pitkälti kouluttautuneiden määrästä. Kun osalla kehitysmaan koulutetusta väestöstä on mahdollisuus muuttaa kehittyneempään maahan, myös osa niistä yksilöistä, jotka eivät aikaisemmin investoineet koulutukseen, hankkivat koulutuksen. Näin ollen aivovuoto saa aikaan kannustinvaikutuksen.

Seuraavaksi esitellään yksityiskohtaisempi inhimillisen pääoman malli. Tarkastelemalla tuotantosektorin vaikutuksia palkan määräytymiseen ja hyötyä maksimoivien yksilöiden vaikutuksia inhimillisen pääoman kertymiseen, talouden tasapainot voidaan määritellä. Inhimillisen pääoman oletetaan vaikuttavan talouskehitykseen, ja täten myös palkkaan, jäljittelyn ja innovaatioiden kautta. Kouluttautuneiden työntekijöiden määrän lisääntymisen voidaan näyttää kiihdyttävän talouskasvua sitä enemmän mitä lähempänä kehitysmaan teknologinen kehitys on kehittyneen maan teknologian tasoa. Kun kehitysmaan teknologia ylittää tietyn tason, vaikutus kuitenkin hidastuu.

Molempien lähestymistapojen mukaan on olemassa tietyt ehdot, joiden täytyttyä aivovuoto johtaa hyödyllisiin seurauksiin. Ensinnäkin palkkaeron on oltava riittävän suuri kehitysmaan ja kehittyneen maan välillä, jotta kannustinvaikutus syntyy. Lisäksi kouluttautuneiden yksilöiden muuttomahdollisuuden on oltava tarpeeksi pieni. Yleiskatsauksen mukaan kouluttautuneiden työntekijöiden määrä ei myöskään saa jo olla korkea, koska silloin kannustinvaikutus ei toimi. Toisaalta inhimillisen pääoman malli painottaa, että on olemassa vähimmäistoimeentulo, joka on täytettävä. Jos kehitysmaan elintaso on todella alhainen, likviditeettirajoite rajaa koulutusmahdollisuuksia. Tämän vuoksi köyhyysloukkuun jääminen on yhä mahdollista kaikkein köyhimmille maille. Näin ollen keskituloisilla mailla on paras mahdollisuus hyötyä aivovuodosta.

Avainsanat – Nyckelord – Keywords
Aivovuoto, Muuttoliike, Talouskasvu, Teknologinen kehitys

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List of symbols

α	The share of physical capital in national income
γ	The relative efficiency of innovation compared to imitation in generating productivity growth
η	The efficiency of the process of technological improvements
θ	The number of efficiency units offered by a high-skilled worker additional to those offered by a low-skilled worker
κ	The elasticity of unskilled labor in innovation
λ	A parameter describing time preference and relative length of the retirement period
$\widehat{\mu_t}$	The minimum subsistence level in period t
π_t	The proportion of educated of an economy in period t
σ	The elasticity of unskilled labor in imitation
ϕ_t	The risk premium factor in period t
ω_t	The wage rate of a developing country relative to the wage rate of a developed country in period t
A_t	Total factor productivity in period t
$A_{i,t}$	Total factor productivity of sector i in period t
b_i	The latent ability of an individual i to learn new skills
c	The cost of education
E	The amount of educated/skilled workers in an economy
$e_{m,i,t}$	The amount of educated/skilled labor used in imitation in sector i in period t
$e_{n,i,t}$	The amount of educated/skilled labor used in innovation in sector i in period t
H_t	Labor in efficiency units used in production in period t

h_t The average number of efficiency units of labor in period t

K_t Physical capital used in production in period t

k_t The stock of capital per worker in period t

l_t The amount of land used in production in period t

m The average number of children

p The probability of migration for educated workers

$p_{i,t}$ The price of an intermediate input i in period t

$q_{i,t}$ The flow of intermediate good i in period t

R_t^* The international interest factor in period t

r_t The net return to physical capital in period t

s_t Savings in period t

U The amount of uneducated/unskilled workers in an economy

$u_{m,i,t}$ The amount of unskilled labor used in imitation in sector i in period t

$u_{n,i,t}$ The amount of unskilled labor used in innovation in sector i in period t

w_t The wage rate per efficiency units of labor in period t

$w_{e,t}$ The wage rate of an educated worker in period t

$w_{u,t}$ The wage rate of an uneducated worker in period t

X_t The vector of country characteristics determining the wage rate in period t

x_t Education decision in period t (a dichotomous variable)

Y_t Gross domestic product in period t/ a composite good produced in period t

y_t Gross domestic product per capita in period t

1 Introduction

During the last five decades the number of international migrants has been steadily increasing. Comparing the different development groups reveals that the developed countries have been net gainers of international migration, whereas the developing countries have been main sending countries of international emigrants. According to the United Nations (2007) between the years 1990 and 2000 the developed countries were gaining 2,5 million migrants annually. Hereby, there are more people living outside their home countries than never before and it seems like the increase in the amount of international migrants is not going to die down in the near future. Especially environmental disasters, demographic disparities, and deterioration of employment opportunities ensure that emigration out of the developing countries continues to dominate most likely. The report of International Organization for Migration (2010) predicts that if the number of international migrants continues to increase as quickly as during the last 20 years, the migrant population could reach the amount of 405 million by 2050.

The increase of migration causes challenges not only for the receiving developed countries but also for the developing countries. There is a lot of evidence that the share of high-skilled emigration has been increasing relative to the share of low-skilled emigration[1]. Particularly, the amount of educated emigrants has grown in absolute terms. The departure of a nation´s most educated individuals is commonly referred to as *brain drain*. This term has been used especially when considering emigration flows from developing to developed countries[2].

The brain drain literature originated in the late 1960s. The first contributions did not find that brain drain would have any great influences on the welfare of those remaining in the source country. For example Grubel and Scott (1966) discovered that there are only some

[1] See for example Defoort (2008).
[2] See for example Gibson and McKenzie (2011).

short run losses resulting from the emigration of the highly skilled. In the short run there are especially some adjustment costs caused by inefficient employment of new factors of production. Thus, the production losses only last until new replacements for inputs have been introduced and disappear in the long run. Of a greater importance are the long run effects on the welfare which are inflicted by failures to allocate resources efficiently on the free market. Grubel and Scott reported that also the losses of the long run tend to be negligible because governments are able to redistribute tax incomes and expenditures quite effortlessly. Furthermore, emigration of highly skilled individuals causes positive externalities through remittances and new knowledge produced by scientists in the destination countries. Hereby, Grubel and Scott actually supported the free movement of international migrants.

A new understanding of brain drain was formed in the 1970s. For example Bhagwati and Hamada (1974) challenged the previous studies by pointing out that because of externalities social marginal product normally exceeds private marginal product. They took into consideration three economic inefficiencies: two sticky wages for educated and uneducated workers and free education. Paying attention to the well observed fact that integrated markets for educated workers tend to create an increment in the wages of highly skilled individuals in the source country of emigration, they also took into account the leap-frogging process. The leap-frogging process signifies that as a consequence of the increasing wages of educated individuals, the wages of uneducated workers also tend to be pulled up. Carrying out a welfare analysis, Bhagwati and Hamada concluded that brain drain causes unfavorable effects on the national income and on the employment of both educated and uneducated individuals. Along with Bhagwati and Hamada, many other authors of the 1970s found negative impacts of highly skilled emigration on economic development. For example McCulloch and Yellen (1977) examined the effects of policy-induced changes in factor earnings and discovered that policies aimed to assist less developed regions may actually result in undesirable influences such as overinvestment in education.

Even in the 1990s there was still a lot of support given for the stance suggesting that brain drain leads to harmful effects on national income and growth[3]. However, the understanding about the impacts of brain drain was slowly started to be questioned during the decade. The more recent analyses have seen brain drain to increase incentives to acquire education in addition to the new employment opportunities[4]. Thus, a possibility of emigration into a more advanced country with a higher wage rate produces an incentive effect. Models with this assumption have found certain circumstances which lead to brain gain instead of brain drain. An example of such a model was developed by Vidal in 1998. According to Vidal, emigration of educated individuals may result in bifurcation in the dynamics of an economy and hereby free some of the developing countries from poverty trap. Therefore, brain drain should not be condemned detrimental without a closer examination.

The purpose of this thesis is to find out whether there actually exists a possibility of beneficial brain drain. And if so, what are the conditions under which high-skilled emigration leads to brain gain instead of brain drain. The thesis will at first introduce a simple overlapping generations model of brain drain from the 1990s. The model will be derived in three stages in section 2. The production sector and the decision making of individuals will first be presented separately before concentrating on the effects of high-skilled emigration on the productivity of an economy. Because the amount of human capital of the economy is considered as an essential determinant of economic growth[5], the main focus is on how individuals decide whether to invest in education, and how the possibility of emigration affects the amount of educated individuals of the economy.

The second overlapping generations model introduced in section 3 takes into consideration the possible equilibria determined by the production sector and decisions made by individuals of the economy. The production sector of the economy determines a *wage-setting equation* which can have either increasing or diminishing marginal returns

[3] See for example Miyagiwa (1991), Haque and Kim (1995), and Wong and Yip (1999).
[4] See for example Stark et al. (1997), and Stark et al. (1998).
[5] See for example Romer (1990).

with respect to the average level of human capital of the economy. Therefore, the properties of the wage-setting curve will be studied more closely in section 3.2 assuming that human capital affects the productivity growth through industries benefiting imitations and innovations. The behavior of utility maximizing individuals, on the other hand, determines a *skill-setting equation* which defines how the average level of human capital develops in an economy. By combining the wage-setting and the skill-setting equations in section 4, the equilibria of the model will be utilized to examine the dynamics caused by emigration of some of the highly skilled individuals of the economy and, hereby, to figure out the possible conditions for beneficial brain drain. Section 5 concludes.

2 Overview of Brain Drain

In order to examine the effects of brain drain on the growth of the source country of emigration, the first objective of the thesis is to analyze the impacts of emigration on educational decisions based on the paper written by Andrew Mountford in 1997. As shown by many empirical and theoretical studies[6], the level of schooling is a significant explanatory variable for differing growth rates across countries. An essential assumption for the overview is that educational decisions are made endogenously, and thus individuals of the economy tend to acquire different levels of education. As a consequence, individuals are categorized into two different educational classes: the educated and the uneducated individuals, or alternatively the highly skilled and the low skilled individuals. Educated and highly skilled refer to individuals with a tertiary schooling. In addition, there is no certainty for the possibility of emigration. This illustrates the fact that there are some restrictions for emigration.

[6] See for example Mankiw et al. (1992), or Cohen and Soto (2007) for an empirical research, and Galor and Zeira (1993) for a theoretical approach.

We will first take a look at an economy where there does not exist a possibility of emigration. Firstly, the production side of the economy will be specified, and as a result the factor prices will be defined. Secondly, it will be examined how individuals make decisions concerning their level of education. It is assumed that abilities of the decision makers are distributed according to a certain distribution function. After the educational decisions have been studied in a closed economy, the possibility of emigration will be included into the analysis. The details of the calculations of the section are given in Appendix A.

2.1 Production and Factor Prices

In an economy of overlapping generations, one good is produced according to a constant returns to scale production function. Two factors of production, physical capital and efficiency units of labor, are utilized in the production. The total amount of physical capital available in period t is denoted by K_t, while the total amount of efficiency units of labor in period t is denoted by H_t. It is assumed that there exists a continuum of individuals in each period of time, but for simplicity the amount of individuals born in every period is normalized to unity. The state of the technology is depicted by A_t and the output produced in period t is thus described by the production function written as

$$Y_t = F(K_t, A_t H_t).$$

Because of the assumption of constant returns, the production function can be modified into intensive form[7]. Consequently, the production function can be rewritten as

$$Y_t = A_t H_t F\left(\frac{K_t}{A_t H_t}, 1\right)$$

and by denoting $f(k_t) \equiv F\left(\frac{K_t}{A_t H_t}, 1\right)$, the production function becomes

$$Y_t = f(k_t) A_t H_t, \tag{2.1}$$

where $k_t = \frac{K_t}{A_t H_t}$ is the capital stock per efficiency unit of labor in period t. The following

[7] See for example Romer (2001), pp. 10.

assumptions can be assumed to apply for the function f with all the values of k:

$$f(k) > 0,$$

$$f'(k) > 0,$$

$$f''(k) < 0,$$

$$\lim_{k \to 0} f(k) = 0,$$

$$\lim_{k \to 0} f'(k) = \infty$$

and

$$\lim_{k \to \infty} f'(k) = 0,$$

where the last three assumptions are the Inada conditions.

Supposing that perfect competition is prevailing in the economy, factor prices are determined by the marginal products of the factors. As a consequence, the net return to physical capital is given by

$$r_t = \frac{dY_t}{dK_t} = f'(k_t). \tag{2.2}$$

However, because of perfect mobility of capital the world net rate of return is a constant r^*, presupposing that the world is in a steady state equilibrium. As a result, also r_t is equal to r^* and the amount of capital in efficiency units is thus a constant k in every period t. Consequently, for a given level of technology the wage rate per efficiency unit of labor is determined by k as

$$w_t = \frac{dY_t}{dH_t} = A_t[f(k) - kf'(k)] = A_t w(k), \tag{2.3}$$

where $w(k) \equiv f(k) - kf'(k)$.

The production side of the economy and the factor prices have now been determined. Next it will be considered how individuals make educational decisions taking into account a cost of education and a latent ability typical for the individual.

2.2 Education Decision

It can be quite naturally assumed that individuals are not equally able to learn new skills. Due to these differences in latent abilities, each individual i of the economy possesses his own level of latent ability denoted by b_i. The latent abilities of all the individuals are distributed over a closed interval $[0, B]$ and follow a density function $D(b_i)$ which has the following properties:

$$\int_0^B D(b_i)db_i = 1$$

and

$$D(b_i) > 0, \forall\ b_i \in [0, B].$$

It is assumed that the latent abilities of all the generations receive values from the interval $[0, B]$ and, in addition, the latent abilities of children do not depend on the abilities of their parents[8].

Individuals of the economy are assumed to live for three periods and they derive utility only from their third period consumption. The utility function is chosen in this way in order to make the analysis as simple as possible. However, the same results can be obtained even though utility would depend on consumption of all the three periods[9]. During the first period individuals have a possibility to invest in education. The assumption of three periods is necessary because individuals borrow to fund their

[8] Mountford takes into account the dependence of a latent ability on the average level of "parental" human capital in his previous model derived in 1995. This assumption does not, however, affect the final conclusions.

[9] This has been done in the paper of Mountford in 1995. In addition, the utility of individuals of the model derived in section 3 will depend on consumption of all the three periods of their lives.

education. In order to finance the education they borrow assets from the capital market at the world´s interest rate r^*. In the second period of their life, individuals work and repay the debt borrowed for education. The three period structure of the economy is required because the individuals who are repaid must be still alive.

The cost of education, which is a fixed cost, is assumed to be c units of output. Those who invest in education in the first period possess b_i efficiency units of labor in the second period. At the same time, uneducated individuals are assumed to possess only one efficiency unit of labor. In addition to working and repaying the debt from the first period, individuals must also save funds in the second period in order to consume in the third period of their life. The third period is namely a retirement period and individuals work no longer. It is assumed that individuals have similar preferences.

An educational decision is made by comparing utility with and without an investment in education. It is optimal for an individual i to invest in education if his income in the second period will be higher with the educational investment than without the investment. That is when

$$A_t w(k) b_i - c(1 + r^*) > A_t w(k). \tag{2.4}$$

A critical value for the latent ability can be found out by solving the inequality. As can be easily seen, the critical value for the latent ability is given by

$$\hat{b}_t = \frac{A_t w(k) + c(1+r^*)}{A_t w(k)}. \tag{2.5}$$

As a consequence, it is profitable for those individuals with a latent ability higher than $\hat{b}_t$ to invest in education, while for those who possess a latent ability equal to or lower than $\hat{b}_t$ it is not profitable to acquire education. It is assumed that $\hat{b}_t \in [0 + \varepsilon, B - \varepsilon]$, where $0 < \varepsilon < \frac{B}{2}$.

The proportion of educated workers in the previous period is assumed to cause an economy wide growth externality. This illustrates the fact that knowledge and other intangible assets have imperfect property rights. Once a new innovation has been created it diffuses throughout the society and throughout the generations[10]. As a result, A_t positively depends on the proportion of educated individuals in the previous period, denoted by π_{t-1}, and can thus be written as

$$A_t = A(\pi_{t-1}), \tag{2.6}$$

where

$$\pi_{t-1} = \int_{\hat{b}_{t-1}}^{B} D(b_i) db_i,$$

and

$$A'(\pi_{t-1}) > 0.$$

Because

$$\frac{d\hat{b}_t}{d\pi_{t-1}} = -\frac{c(1+r^*)A'(\pi_{t-1})}{A^2(\pi_{t-1})w(k)} < 0 \tag{2.7}$$

and as a consequence

$$\frac{d\pi_t}{d\pi_{t-1}} = D(\hat{b}_t)\frac{c(1+r^*)A\prime(\pi_{t-1})}{A^2(\pi_{t-1})w(k)} > 0, \tag{2.8}$$

it can be easily proven that the proportion of educated individuals in period t is an increasing function of the proportion of educated workers in period $t-1$. This leads to an even greater externality in period $t+1$. The state of the technology also has the following properties: $A(0) = 1$, standing for the fact that there exists no externality if there were no educated individuals in the previous period and, on the other hand, $A(1)$ is finite.

[10] See for example Tamura (1991).

It can be assumed that it is profitable for the most talented individual to acquire education even though there were no educated individuals in the previous period, that is B is high enough. On the other hand, there is always an individual i with his latent ability equaling zero and thus there must exists at least one steady state equilibrium $\bar{\pi}$ for the amount of educated individuals. Whether there are more equilibria depends on the properties of the $A(\pi_{t-1})$ function. The possibility of multiple equilibria will be analyzed more closely later on in the thesis. But before that it will be considered how emigration of some of the educated individuals affects the amount of highly skilled individuals in the economy and thus also the economic growth of the source country.

2.3 Highly Skilled Emigration

As already mentioned earlier, it is reasonable to assume that only a proportion of the educated individuals face an opportunity to emigrate from their home country. An exogenous emigration probability for the highly skilled individuals is thought to be given by p. The uneducated individuals do not have the chance to emigrate and thus spend their whole life in the source country of emigration. This assumption is justifiable because there is a lot of evidence for positive selection when considering emigration from developing countries into developed countries. For example Grogger and Hanson (2011) have demonstrated that migrants tend to be more educated relative to non-migrants, especially the larger the skill-related difference in wage rates between the source and the destination countries. The highly skilled individuals are encouraged to emigrate because the wage rate per efficiency unit of labor in the receiving country, w^*, exceeds the local wage rate per efficiency unit, $A_t w$. This results from the assumption that the level of technology in the more advanced country A_t^* is larger than $A(1)$, the highest possible level of technology in the less developed country.

The emigration possibility now modifies the education decision into an expected utility problem. In order to keep the analysis simple, it is assumed that individuals are risk neutral. An individual deciding whether to invest in education once again compares the

income of the second period with an educational investment and without the investment. The education decision problem can now be written as

$$(pw^* + (1-p)A_t w)b_i - c(1+r^*) > A_t w. \quad (2.9)$$

The critical value for the latent ability, by solving the inequality (2.9), now becomes

$$\hat{b}_t = \frac{A_t w + c(1+r^*)}{pw^* + (1-p)A_t w}, \quad (2.10)$$

and all the individuals possessing a latent ability greater than $\hat{b}_t$ find it profitable to acquire education. The average proportion of the educated individuals staying in the source country of emigration is given by

$$\pi_t = \frac{(1-p)\int_{\hat{b}_t}^{B} D(b_i)db_i}{1-p\left(\int_{\hat{b}_t}^{B} D(b_i)db_i\right)}, \quad (2.11)$$

where the numerator represents the amount of educated workers staying in the home country and the denominator stands for the amount of the whole population after emigration has taken place. It can now be shown that there exists a possibility of beneficial brain drain if a sufficient number of individuals is motivated to invest in education with a sufficiently low possibility of emigration.

An economy is in autarky when $p = 0$. On the other hand, $p = 1$ indicates that $\pi_t = 0$ and brain drain causes only negative effects on the source economy as there are no highly skilled individuals staying in the developing country. As a result, the emigration possibility needs to be low enough in order to lead to beneficial brain drain. The sufficient condition for the brain drain to result in productivity improvements can thus be estimated at $p = 0$. Brain drain causes positive effects on the productivity growth whenever $\frac{d\pi_t}{dp} > 0$, that is when the proportion of the educated individuals increases as a result of the highly skilled emigration.

The differentiation of the equation (2.11) with respect to the emigration possibility p gives

$$\frac{d\pi_t}{dp} = \frac{\partial \pi_t}{\partial p} + \frac{\partial \pi_t}{\partial \hat{b}_t}\frac{\partial \hat{b}_t}{\partial p}, \tag{2.12}$$

where

$$\frac{\partial \pi_t}{\partial p} = -\frac{\int_{\hat{b}_t}^{B} D(b_i)db_i\left[1-\int_{\hat{b}_t}^{B} D(b_i)db_i\right]}{\left[1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right]^2}, \tag{2.13}$$

$$\frac{\partial \pi_t}{\partial \hat{b}_t} = -\frac{(1-p)D(\hat{b}_t)}{\left[1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right]^2} \tag{2.14}$$

and

$$\frac{\partial \hat{b}_t}{\partial p} = -\frac{\left(A_t w + c(1+r^*)\right)(w^* - A_t w)}{[pw^* + (1-p)A_t w]^2}. \tag{2.15}$$

Clearly, it can be seen that $\frac{\partial \pi_t}{\partial p} < 0$, $\frac{\partial \pi_t}{\partial \hat{b}_t} < 0$ and $\frac{\partial \hat{b}_t}{\partial p} < 0$.

The estimation of the optimal level of brain drain at $p = 0$ results in

$$\frac{d\pi_t}{dp}\,|_{p=0} = -\int_{\hat{b}_t}^{B} D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B} D(b_i)db_i\right) + D(\hat{b}_t)\frac{\left(A_t w + c(1+r^*)\right)(w^* - A_t w)}{(A_t w)^2} > 0$$

$$\Leftrightarrow \frac{D(\hat{b}_t)\left(A_t w + c(1+r^*)\right)(w^* - A_t w)}{(A_t w)^2} > \int_{\hat{b}_t}^{B} D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B} D(b_i)db_i\right). \tag{2.16}$$

Because $\int_{\hat{b}_t}^{B} D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B} D(b_i)db_i\right)$ cannot exceed the value of $\frac{1}{4}$, the condition for beneficial brain drain can be summarized as stated in Proposition 1.

Proposition 1 Whenever it is valid for $D(\hat{b}_t)$, A_t, w, w^*, c and r^* that

$$\frac{D(\hat{b}_{NM})\left(A_t w + c(1+r^*)\right)(w^* - A_t w)}{(A_t w)^2} > \frac{1}{4},$$

brain drain induces positive effects on the productivity of the source economy. In Proposition 1, $\hat{b}_{NM}$ depicts the lowest level of ability of an individual choosing to acquire education when the economy is in autarky.

The intuition behind the Proposition 1 can be presented more simply by assuming that the latent abilities are uniformly distributed. The uniform distribution leads to certain properties for the density function $D(b_i)$. First of all $D(b_i) = \frac{1}{B}$ [11], and consequently $\int_{\hat{b}_t}^{B} D(b_i)db_i = \int_{\hat{b}_t}^{B} \frac{1}{B} db_i = 1 - \frac{\hat{b}_t}{B}$. Using these properties, the condition for beneficial brain drain can be redefined as

$$\frac{d\pi_t}{dp} > 0 \Leftrightarrow 1 - \frac{\hat{b}_t}{B} < (1-p)\frac{w^* - A_t w}{pw^* + (1-p)A_t w}. \qquad (2.17)$$

From the equation (2.17) it can be seen that emigration of highly skilled individuals, that is brain drain, increases the amount of educated individuals of the developing country if the emigration possibility p is low and if the wage rate per efficiency unit of labor of the developed country, w^*, is high relative to the wage rate per efficiency unit of labor of the developing country, $A_t w$. On the other hand, also the proportion of educated individuals of the previous period affects the outcome of brain drain through A_t. If the number of educated individuals in the previous period is low, there are positive effects on the growth of the source country. This can be seen in Figure 1 by illustrating the influences of brain drain.

[11] See for example Hastings and Peacock (1974), pp. 116.

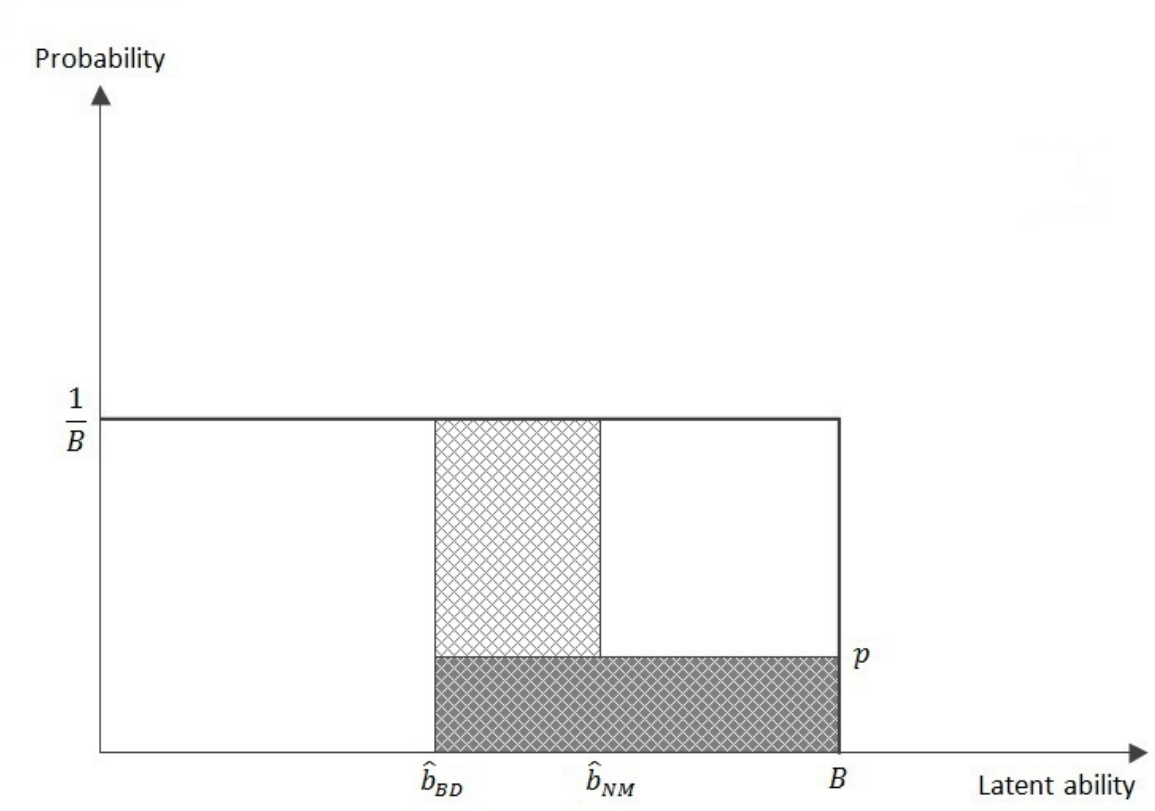


Figure 1. Influences of brain drain on the proportion of highly skilled individuals staying in the source country of emigration.

As shown in Figure 1, the threshold level of the latent ability reduces from $\hat{b}_{NM}$ to $\hat{b}_{BD}$ as a result of brain drain. This can be seen by comparing the equations (2.5) and (2.10). Because the denominator of the equation (2.10) is larger than the denominator of the equation (2.5), the emigration possibility decreases the critical level of latent ability required for a profitable educational investment. The amount of educated individuals thus increases by $\frac{\hat{b}_{NM}-\hat{b}_{BD}}{B}$ when a proportion p of the educated individuals of the developing country emigrate. However, the amount of educated individuals staying in their home country decreases by $p\frac{B-\hat{b}_{BD}}{B}$. Whether the effect of brain drain on the source country is positive or negative can be found out by comparing the lined rectangle in Figure 1, representing the individuals encouraged to acquire education as a result of the emigration possibility, with the darker rectangle in Figure 1, representing the educated individuals emigrating from the source country as a result of the emigration possibility. As

can be seen from the Figure 1, brain drain results in positive effects if the possibility of emigration is sufficiently low and if the proportion of educated individuals was previously low. From the equation (2.17) it can also be seen that if the wage rate per efficiency unit of labor of the receiving country is high enough compared to the wage rate of the source country there will exist a positive level of emigration possibility p such that results in positive productivity effects in the less developed country. This represents the incentive effect produced by higher possible earnings in the developed country. The results of this section are summarized in Proposition 2.

Proposition 2 Brain drain results in positive effects on the productivity of the source country if

1. the emigration possibility is low enough,
2. the proportion of educated individuals in the previous period was low, and
3. the wage rate per efficiency unit of labor in the developed country is sufficiently high.

In this section no strict assumptions about the properties of the production function were made. It was only assumed that the production function has constant returns to scale. On the other hand, it was also ignored that the uneducated individuals have a possibility to earn income during the first period while the others are acquiring education. Therefore, an approach with more comprehensive assumptions about the production possibilities and the lifetime utilities of educated and uneducated workers will be derived in the next section. In order to examine whether there exists a possibility of multiple equilibria, the production function and the utility function of an individual will be determined more accurately. When there is more information about the existing equilibria, the dynamics caused by highly skilled emigration can be examined.

3 Role of Human Capital Accumulation

In this section, an overlapping generations model of human capital accumulation and endogenous growth will be introduced based on a paper written by Frédéric Docquier and Hillel Rapoport in 2012. As in the previous section, we will first examine an economy without migration possibilities and will later include the emigration of highly skilled workers into the analysis.

The first step is to build a *wage-setting equation* which is determined by the production sector of the economy. When constructing the wage-setting equation, it is again assumed that the economic performance of a country depends on the average level of human capital of the country. The properties of the wage-setting curve will be studied more closely in section 3.2 where human capital affects technological evolution through imitation and innovation industries. After the impacts of the amount of skilled labor on the equilibrium growth rate have been examined, a *skill-setting equation* will be derived to illustrate the accumulation of human capital in the economy. These two equations will then determine the possible equilibria of the economy. The mathematical results of this section are derived in Appendix B.

3.1 Wage-Setting

Similarly to the previous section, the economy is assumed to consist of firms and individuals who live for three periods. During the first two periods, youth and adulthood, individuals work and decide on education and the third period, on the other hand, is a retirement period. Workers are assumed to be either low skilled or high skilled depending on their education decision made in the first period. On the contrary to the previous section, the production function is determined more accurately. It is assumed that a composite good in the economy is produced according to a Cobb-Douglas production function

$$Y_t = A_t {K_t}^{\alpha} {H_t}^{1-\alpha}, \tag{3.1}$$

where A_t describes total factor productivity, K_t represents physical capital, and H_t labor in efficiency units used in the production in period t. H_t includes both highly skilled and low skilled labor which are considered as perfect substitutes for the time being. The number of efficiency units offered by a low skilled worker is normalized to one and a highly skilled worker can then be assumed to offer $1 + \theta$ efficiency units of labor, where $\theta > 0$. In addition, α depicts the share of physical capital in national income and $\alpha \in [0, 1]$. Writing the production function in the per worker form it becomes

$$y_t = A_t {k_t}^{\alpha} {h_t}^{1-\alpha}, \tag{3.2}$$

where y_t is the GDP per worker, k_t denotes the stock of capital per worker, and h_t describes the average number of efficiency units of labor.

The returns to physical capital are assumed to be equalized at the international level because of perfect mobility of capital. There is thus one prevalent interest rate across nations, and R_t^* is used to denote the risk-free, international interest factor which is defined as $R_t^* = 1 + r_t^*$. However, in contrast to section 2 each country is characterized by a certain risk premium typical for the country, and $\phi_t \geq 1$ is equal to one plus the risk premium for the given country. The existing risk premia illustrate the fact that in spite of the integration of financial markets the real interest rates still tend to differ across countries[12]. Given these conditions the returns to physical capital are determined by the marginal product of capital as

$$R_t^* \phi_t = \frac{dy_t}{dk_t} = \alpha A_t {k_t}^{\alpha-1} {h_t}^{1-\alpha}. \tag{3.3}$$

The wage rate per efficiency unit of labor is, on the other hand, given by the marginal product of labor as

$$w_t = \frac{dy_t}{dh_t} = (1-\alpha) A_t {k_t}^{\alpha} {h_t}^{-\alpha}. \tag{3.4}$$

[12] See for example Mishkin (1984).

From the equation (3.3), the value of k_t can be solved to be

$$k_t = \left(\frac{R_t^* \phi_t}{A_t \alpha}\right)^{\frac{1}{\alpha-1}} h_t. \tag{3.5}$$

By inserting the equation (3.5) into the equation (3.2), the GDP per worker of the economy becomes

$$y_t = \left(\frac{A_t \alpha^\alpha}{R_t^{*\alpha} {\phi_t}^\alpha}\right)^{\frac{1}{1-\alpha}} h_t. \tag{3.6}$$

The same presumptions about the production function can be assumed to apply for a technologically more advanced country. By denoting the parameters of a leading economy by a superscript *, the GDP per capita of the developed country can be written as

$$y_t^* = \left(\frac{A_t^* \alpha^\alpha}{R_t^{*\alpha} \phi_t^{*\alpha}}\right)^{\frac{1}{1-\alpha}} h_t^*. \tag{3.7}$$

Now, the GDP of the developing country can be expressed relative to that of the developed country in the following way:

$$\frac{y_t}{y_t^*} = \left(\frac{A_t}{A_t^*}\right)^{\frac{1}{1-\alpha}} \left(\frac{\phi_t}{\phi_t^*}\right)^{\frac{-\alpha}{1-\alpha}} \left(\frac{h_t}{h_t^*}\right). \tag{3.8}$$

Therefore, as can be seen from the equation (3.8), the gap in the economic performance between the countries depends on the total factor productivities of the countries, the risk premiums, and the average numbers of efficiency units of labor.

The solved k_t in the equation (3.5) can also be inserted into the equation (3.4) in order to solve the relative wage ratio between the countries. For the developing country the wage rate per efficiency unit of labor can be written as

$$w_t = (1-\alpha)\left(\frac{A_t \alpha^\alpha}{R_t^{*\alpha} {\phi_t}^\alpha}\right)^{\frac{1}{1-\alpha}}, \tag{3.9}$$

and thus for the developed country it applies that

$$w_t^* = (1-\alpha)\left(\frac{A_t^*\alpha^\alpha}{R_t^{*\alpha}\phi_t^{*\alpha}}\right)^{\frac{1}{1-\alpha}}. \tag{3.10}$$

Hereby, when comparing the wage rate per efficiency unit of labor in the developing country relative to that of the developed country, the ratio of the wage rates becomes

$$\frac{w_t}{w_t^*} = \left(\frac{A_t}{A_t^*}\right)^{\frac{1}{1-\alpha}}\left(\frac{\phi_t}{\phi_t^*}\right)^{\frac{-\alpha}{1-\alpha}}. \tag{3.11}$$

The relation of the wage rates per efficiency units of labor will be denoted by ω_t in what follows.

From the equation (3.11) it can be seen that the ratio of the wage rates depends on the total factor productivities of the countries and also on the risk premiums. Thus, ω_t is not directly relative to the human capital endowments. The level of technology and the risk premiums can nevertheless be considered to depend on the quality of workers[13]. As a consequence, the *wage-setting equation* can be determined to depend on the average level of domestic human capital, the average stock of human capital of the developed country, and other characteristics according to a function W as

$$\omega_t = W(h_t, h_t^*, X_t), \tag{3.12}$$

where X_t represents a vector of country characteristics affecting the wage level. It is assumed that $W(0,\ h_t^*,\ X_t) > 0$, $W_h' > 0$, and $W_{hh}'' \lesseqgtr 0$. The last assumption implies that the economy might follow local increasing returns or threshold externalities. Both of these concepts extend the traditional growth models which expect the per capita growth rate to be a decreasing function of the level of per capita capital stock. We will shortly take a look at the concepts.

[13] As stated by Docquier and Rapoport (2012), migration affects international transaction costs and thus facilitates the movements of goods and capital between countries. These network externalities, on the other hand, impact on international trade and FDI (foreign direct investment). The effects can be included into the risk premium factor ϕ by having an impact on international transaction costs and on institutional risk. In particular, an increase in the average level of human capital decreases the risk premium of the country. For more information see Docquier and Rapoport (2012). The effect of average level of human capital on the total factor productivity will be paid attention to in the following subsection 3.2.

Local increasing returns refer to a phenomenon where knowledge, or alternatively human capital, is considered as an input that exhibits increasing marginal productivity. By assuming endogenous technological change a model of long-run growth with increasing returns can be introduced with this presumption[14]. The accumulation of knowledge is considered as the main factor driving the long-run growth. New knowledge is produced by a research technology that exhibits diminishing returns. However, creation of new knowledge causes a natural externality. If a firm invests in the production of knowledge, a positive external effect is caused because the production possibilities of other firms also expand. As a consequence, knowledge has an increasing marginal product and it is always beneficial to invest in the creation of new knowledge. Nevertheless, the properties of the production of output and knowledge guarantee an existence of a competitive equilibrium. Because the research technology exhibits diminishing returns, there exists a maximum rate of growth for knowledge and this leads to an existence of a maximum feasible rate of growth for the output.

Technological externalities with a threshold property have, on the other hand, been utilized to explore the fact that per capita growth rates tend to differ sustainably between countries, even though the countries are identically structured. An externality can arise from spillovers of capital stocks or alternatively from the process of creating human capital. The main idea behind the threshold externalities is that with a certain technology, differences in the stocks of inputs may lead to different growth rates. Once physical capital or the level of human capital exceeds a certain critical value, production possibilities may expand and the expansion might be rapid[15]. This means that as the per capita production surpasses a critical value, the inputs become more favorable to the economic expansion. Because different initial values for the inputs result in different dynamics, an economy may generate multiple equilibria. These differences in dynamic behavior are called *threshold effects* and they arise from variations in social returns to

[14] See for example a model derived by Romer in 1986.

[15] See for example Azariadis and Drazen (1990) for a growth model of technological externalities with threshold properties. They have also tested empirically whether economic growth is correlated with investments in human capital relative to per capita GDP by using literacy as a measure of the quality of human capital. As a result, they have found out that none of the countries of their data sets was able to grow quickly without a highly qualified labor force.

scale. In order to examine the possible impacts of increasing returns and threshold externalities on the wage-setting curve, the effects of human capital on the technological development will be examined more closely in the following subsection. The properties of the wage-setting curve will be specified utilizing a paper written by Jérôme Vandenbussche et al. in 2006. Similarly to the local increasing returns, it will be assumed that human capital is the main factor inducing economic growth.

3.2 Properties of the Wage-Setting Curve

Human capital can affect technological progress through two different sources. Following Vandenbussche et al. (2006), these sources are adoption or imitation of existing technologies and innovation of new technologies. It can be assumed that imitation and innovation activities require different types of human capital. Especially, unskilled human capital is more suitable in the process of imitation than in innovation. On the other hand, skilled human capital is more efficient in order to engender innovations. In addition to the allocation of human capital across imitation and innovation, the country's level of technological development affects the impact of human capital on growth. Consequently, both the composition of human capital and the distance to the technological frontier must be paid attention to when considering the effects of human capital on technological development.

The economy includes i sectors which carry out technological improvements by combining innovations and imitations. The two activities exploit both high skilled and low skilled labor. The endowments of educated and uneducated workers are denoted by E and U respectively. Final output of the sectors producing technological improvements in terms of efficiency units of labor, y_t, is also produced competitively according to the Cobb-Douglas production function using a continuum of mass 1 of intermediate inputs and land. The production function can thus be written as

$$y_t = l_t^{1-\alpha} \int_0^1 A_{i,t}^{1-\alpha} q_{i,t}^{\alpha} di, \tag{3.13}$$

where $A_{i,t}$ is the productivity of sector i of the economy, $q_{i,t}$ is the flow of intermediate good i, and l_t is the amount of land used in the production in period t. The total supply of land can be normalized to one and $\alpha \in [0,1]$ as before. Because of the competitive nature of the production, the price of each intermediate good is determined by its marginal product in the following way:

$$p_{i,t} = \frac{dy_t}{dq_{i,t}} = \alpha A_{i,t}^{1-\alpha} q_{i,t}^{\alpha-1}. \tag{3.14}$$

It can be thought that there is one producer in every intermediate good sector i. The producer produces with the productivity $A_{i,t}$, which will be converted endogenous in what follows, and uses final good as capital according to a one-for-one technology. The producer desires to maximize his profits and thus chooses the flow of intermediate good $q_{i,t}$ in order to solve his maximization problem

$$\max_{q_{i,t}} (p_{i,t} q_{i,t} - q_{i,t}).$$

Inserting the price of an intermediate good i in the equation (3.14) into the maximization problem and solving it gives the equilibrium demand for each intermediate input as

$$q_{i,t} = \alpha^{\frac{2}{1-\alpha}} A_{i,t}. \tag{3.15}$$

As a result, the profit of a producer producing in sector i can be written as

$$\Pi_{i,t} = (p_{i,t} - 1) q_{i,t} = (\alpha A_{i,t}^{1-\alpha} q_{i,t}^{\alpha-1} - 1) q_{i,t} = \left(\alpha A_{i,t}^{1-\alpha} \left(\alpha^{\frac{2}{1-\alpha}} A_{i,t} \right)^{\alpha-1} - 1 \right) \alpha^{\frac{2}{1-\alpha}} A_{i,t}$$

$$= \left(\alpha A_{i,t}^{1-\alpha} \alpha^{\frac{2(\alpha-1)}{1-\alpha}} A_{i,t}^{\alpha-1} - 1 \right) \alpha^{\frac{2}{1-\alpha}} A_{i,t} = \left(\frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}} A_{i,t}.$$

By denoting $\delta \equiv \left(\frac{1}{\alpha} - 1 \right) \alpha^{\frac{2}{1-\alpha}}$, the profit function becomes

$$\Pi_{i,t} = \delta A_{i,t}. \tag{3.16}$$

At the beginning of each period, producers decide how to combine imitation and innovation activities in their production. Hereby, they also choose the amounts of high skilled and low skilled workers demanded. A function G can be used to describe the dynamics of the technology according to the following equation:

$$A_{i,t} = A_{i,t-1} + G\big(A^*_{t-1} - A_{t-1}, A_{t-1}, u_{m,i,t}, e_{m,i,t}, u_{n,i,t}, e_{n,i,t}\big), \qquad (3.17)$$

where A^*_{t-1} is the productivity at the world's technological frontier in period $t-1$ and A_{t-1} is the productivity at the country's technological frontier at the end of the period $t-1$. $u_{m,i,t}$ and $e_{m,i,t}$ are the amounts of unskilled and skilled workers utilized by sector i in imitation in period t. On the other hand, $u_{n,i,t}$ and $e_{n,i,t}$ are the amounts of unskilled and skilled labor inputs used in innovation during the same period. Function G is increasing in all of its arguments. If technological progress is assumed to be a linear function of imitation and innovation, the equation (3.17) takes the form

$$A_{i,t} = A_{i,t-1} + \eta\big[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A^*_{t-1} - A_{t-1}) + \gamma {u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}A_{t-1}\big]. \qquad (3.18)$$

In the equation (3.18), σ represents the elasticity of unskilled labor in the activity of imitation and ,similarly, κ is the elasticity of unskilled labor in innovation. $\gamma > 0$ describes the fact that innovations are more productive in the generation of technological growth than adopting of the existing technologies. The efficiency of the whole process of technological improvements is depicted by $\eta > 0$. Because skilled labor is utilized more intensively in the activity of innovation, the elasticity of skilled labor must be larger in innovation than in imitation. On the contrary, the elasticity of low skilled labor must be higher in imitation than in innovation. These properties of the elasticities result in the condition $\kappa < \sigma$. The condition will be made use of later on.

The wage rates of the uneducated and educated workers working for technological improvements are now denoted by $w_{u,t}A^*_{t-1}$ and $w_{e,t}A^*_{t-1}$ respectively. As a consequence, an improvement in the productivity requires a labor cost

$$C_{i,t} = \big[w_{u,t}\big(u_{m,i,t} + u_{n,i,t}\big) + w_{e,t}\big(e_{m,i,t} + e_{n,i,t}\big)\big]A^*_{t-1} \qquad (3.19)$$

by the producer of sector i. Producers choose their labor inputs in order to maximize their current profits net of labor costs. The maximization problem of the producer i is thus given by

$$\max_{u_{m,i,t},u_{n,i,t},e_{m,i,t},e_{n,i,t}} \delta\left[A_{i,t-1} + \eta\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A^*_{t-1} - A_{t-1}) + \gamma{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}A_{t-1}\right]\right] - C_{i,t}. \quad (3.20)$$

However, as $\delta A_{i,t-1}$ does not depend on the values of labor inputs of the period t, the producer's maximization problem can be simplified into the form

$$\max_{u_{m,i,t},u_{n,i,t},e_{m,i,t},e_{n,i,t}} \delta\eta\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(1 - a_{t-1}) + \gamma{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}a_{t-1}\right]A^*_{t-1} - C_{i,t}, \quad (3.21)$$

where $a_{t-1} \equiv A_{t-1}/A^*_{t-1}$ represents country's inverse distance to the technological frontier in period $t-1$. On the other hand, because all the producers face the same maximization problem, the i subscripts can be ignored and as a consequence $E = e_{m,t} + e_{n,t}$ and $U = u_{m,t} + u_{n,t}$ in equilibrium. The maximization problem can again be rewritten as

$$\max_{u_{m,t},u_{n,t},e_{m,t},e_{n,t}} \delta\eta\left[{u_{m,t}}^{\sigma}{e_{m,t}}^{1-\sigma}(1 - a_{t-1}) + \gamma{u_{n,t}}^{\kappa}{e_{n,t}}^{1-\kappa}a_{t-1}\right]A^*_{t-1} - C_t. \quad (3.22)$$

Solving the producer's maximization problem gives two equalities for an interior solution:

$$\sigma{u_m}^{\sigma-1}{e_m}^{1-\sigma}(1-a) = \gamma\kappa(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa}a \quad (3.23)$$

and

$$(1-\sigma){u_m}^{\sigma}{e_m}^{-\sigma}(1-a) = \gamma(1-\kappa)(U-u_m)^{\kappa}(E-e_m)^{-\kappa}a. \quad (3.24)$$

By dividing both equations by their right-hand sides and by then combining the equations, the following result can be received:

$$\sigma(1-\kappa)e_m(U-u_m) = (1-\sigma)\kappa u_m(E-e_m). \quad (3.25)$$

For simplicity, the time indices are ignored. By rewriting the equation (3.25) as

$$\left(\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}\right)\left(\frac{U-u_m}{E-e_m}\right)=\frac{u_m}{e_m}$$

$$\Leftrightarrow\ \psi\left(\frac{u_n}{e_n}\right)=\frac{u_m}{e_m}, \tag{3.26}$$

where $\psi \equiv \frac{\sigma(1-\kappa)}{(1-\sigma)\kappa} > 1$, it can be seen that between innovation and imitation the ratios of unskilled to skilled labor are proportional to each other.

The amount of unskilled workers demanded for imitation as a function of skilled workers in imitation can be solved from the equation (3.25) as

$$u_m = \frac{\psi U e_m}{E+(\psi-1)e_m}. \tag{3.27}$$

Now, substituting the equation (3.27) into the equation (3.24) results in the following result:

$$(\psi-1)e_m = z(a)U - E, \tag{3.28}$$

where $z(a) \equiv \left(\frac{(1-\sigma)\psi^{\sigma}(1-a)}{(1-\kappa)\gamma a}\right)^{\frac{1}{\sigma-\kappa}}$ is a decreasing function of a.

Because in equilibrium it must be that $e_m \leq E$ and $e_n \leq E$, an interior solution only exists if it holds true that

$$\frac{z(a)}{\psi} \leq \frac{E}{U} \leq z(a). \tag{3.29}$$

This can be realized by using the equation (3.28). Equation (3.29) determines certain conditions for relative endowments of educated and uneducated labor. When a country is relatively abundant in unskilled labor, $\frac{E}{U}$ is small and the country approaches the lower boundary of the condition. At the lower boundary, when $\frac{E}{U} = \frac{z(a)}{\psi}$, $e_m = E$ and the country specializes in imitation. On the other hand, when a country is relatively more abundant in skilled labor it comes closer to the upper boundary. At the upper boundary

$e_n = E$ and the country invests only in innovation in order to accomplish technological improvements.

Using the equations (3.27) and (3.28), the relative factor intensities can be written as

$$\frac{u_m}{e_m} = \frac{\psi}{z(a)} \tag{3.30}$$

and

$$\frac{u_n}{e_n} = \frac{1}{z(a)}. \tag{3.31}$$

From the equations (3.30) and (3.31) it can be noted that the factor intensities do not depend on the total factor endowments. Thus, Figure 2 can be utilized to analyze the properties of the sectors implementing technological improvements.

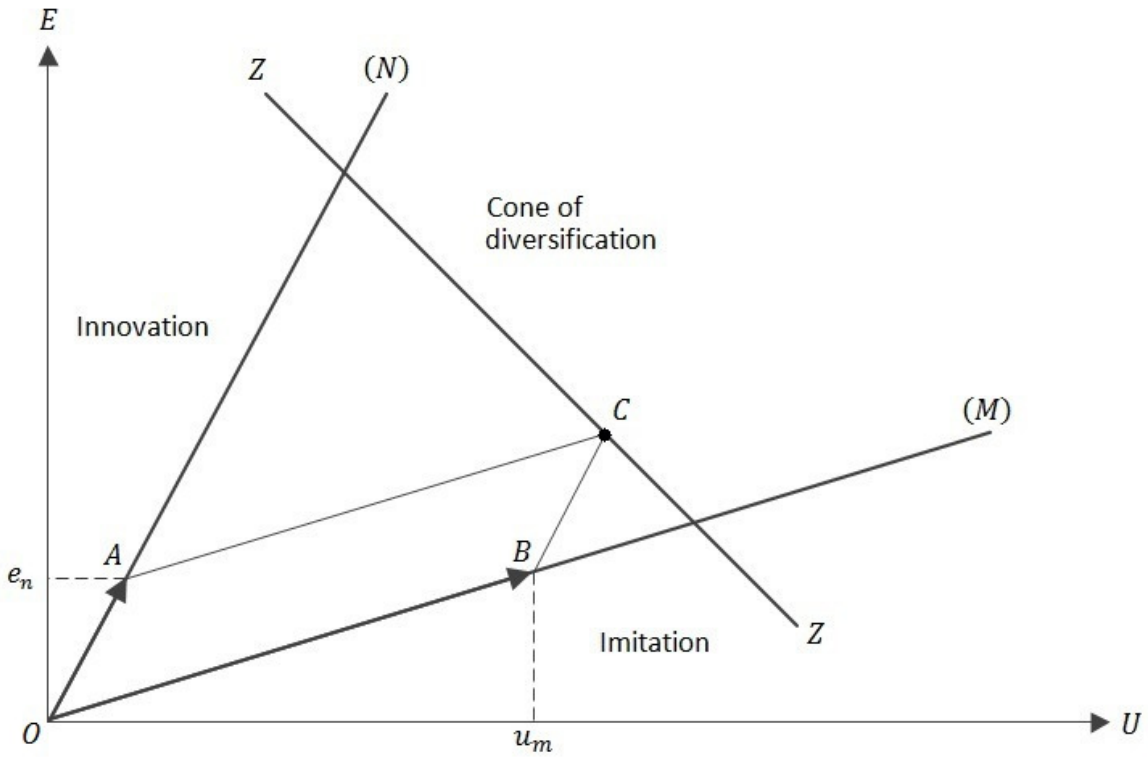


Figure 2. Relative factor intensities and the allocation of human capital across imitation and innovation.

In Figure 2, the amount of uneducated labor is given on the horizontal axis, while the amount of educated labor is given on the vertical axis. (M) and (N) lines are depicted using the condition (3.29). (N) line represents the upper boundary of the condition. Above the (N) line $e_n = E$ and, therefore, the country is fully specialized in innovation. On the other hand, below the (M) line the whole labor force is dedicated to imitation. If the country is located between these two lines, for example at point C, there are investments both in innovation and imitation. In Figure 2, vector OA represents the share of the labor force used in innovation and vector OB the share of labor force used in imitation. Along the (ZZ) line the aggregate level of labor is kept constant but the composition of unskilled and skilled labor between the activities changes. If the (ZZ) line, however, moves further away from the origin, both of the labor inputs increase in equal proportion and there is also a proportionally equal increase in skilled and unskilled labor in both innovation and imitation.

Due to an increase in the amount of skilled labor, an economy moves closer to the (N) line, as shown by the equation (3.29), and both skilled and unskilled labor are used more intensively in innovation and less intensively in imitation. This is a consequence of the condition $\kappa < \sigma$. Because the elasticity of skilled labor in innovation is larger than the elasticity of skilled labor in imitation, an increase in E leads to a proportionally greater increase in the amount of skilled labor utilized in innovation than in the amount of skilled labor utilized in imitation. As a consequence, the marginal productivity of unskilled labor increases more in innovation and a part of the unskilled labor force is attracted away from imitation. As the amount of unskilled labor employed in imitation decreases, also the productivity of skilled labor decreases. As a result, even more of the skilled workers shift from imitation to innovation and eventually the amount of both types of workers increases in innovation and decreases in imitation. On the contrary, if the amount of unskilled labor increases, more of both of the labor inputs are utilized in imitation.

In addition to the absolute quantities of the labor inputs, the value of a needs to be taken into consideration because it also affects the allocation of labor endowments between innovation and imitation. If the value of a increases the economy approaches the world´s technological frontier. In that case $z(a)$ diminishes and both the lower and the upper boundary of the condition (3.29) decline. In Figure 2, (N) and (M) lines shift downwards into the direction of the horizontal axis and, as a consequence, skilled and unskilled labor transfer from imitation to innovation. On the contrary, further away from the technological frontier more of the labor force is distributed into the activity of imitation. Hereby, it can be summarized that when both imitation and innovation are made use of in order to accomplish technological improvements, the optimal amount of skilled and unskilled labor utilized in imitation is increasing (decreasing) in the total number of unskilled (skilled) labor, and decreasing in the distance to the technological frontier. In reality it can be hypothesized that imitation and innovation contain complementarity, at least to some extent. Consequently, corner solutions can be ruled out and from now on the analysis concentrates only on the interior solutions.

Next, it will be considered how changes in the supply of skilled labor affect the equilibrium growth rate of the economy. The growth rate in period t is given by

$$g_t = \int_0^1 \frac{A_{i,t} - A_{t-1}}{A_{t-1}} di.$$

By using the equations (3.18), (3.27) and (3.28), the equilibrium growth rate can be defined as

$$\frac{g}{\eta\gamma} = \frac{1}{\sigma-\kappa}[\sigma(1-\kappa)z(a)^{-\kappa}E - (1-\sigma)\kappa z(a)^{1-\kappa}U]. \qquad (3.32)$$

As can been seen from the equation (3.32), as a function of distance to the technological frontier the two types of human capital have bidirectional effect on the growth rate. Because

$$\left(\frac{1}{\eta\gamma}\right)\frac{dg}{dE} = \frac{\sigma(1-\kappa)}{(\sigma-\kappa)} z(a)^{-\kappa} > 0$$

and

$$\left(\frac{1}{\eta\gamma}\right)\frac{dg}{dU} = -\frac{(1-\sigma)\kappa}{(\sigma-\kappa)} z(a)^{1-\kappa} < 0,$$

the growth rate is increasing in the amount of skilled labor[16], while on the other hand the productivity growth is decreasing in the amount of unskilled labor. The positive effect of an increase in the stock of skilled human capital on the productivity growth can be seen to dominate the negative effect of unskilled human capital the more the closer the economy is situated to the technological frontier.

As the country approaches the technological frontier, the first term of the equation (3.32) becomes larger and larger. On the contrary, the second term in the square brackets becomes smaller and smaller as the value of a comes closer to one. As a result, a marginal increase in the stock of skilled human capital speeds up the growth rate all the more the closer the economy is situated to the technological frontier. This phenomenon

[16] This is in line with Turcotte and Rennison (2004) who have proved empirically, by using statistics of Canada, that the productivity level depends positively on the share of workers who have completed a university degree, and on the share of workers who have taken part in formal trainings.

results from the fact that $z(a)$ is a decreasing function of a and on the other hand from the observation already stated previously that an increase in the stock of skilled labor induces both educated and uneducated workers to move over from imitation to innovation sector. Consequently, the innovation term of a technological improvement increases in the equation (3.18), while the imitation term decreases. The positive impact on innovation can be seen to dominate the negative impact on imitation because according to the equations (3.30) and (3.31) the factor intensities are proportional to each other and moreover increasing in a. Therefore, as the country approaches the technological frontier the amount of unskilled labor utilized in innovation must be increasing more rapidly than the amount of skilled labor utilized in innovation. On the other hand, the amount of unskilled labor employed in imitation must be decreasing slower than the amount of skilled labor in imitation. As a consequence, an increase in the stock of skilled labor speeds up technological progress. Because the wage-setting equation depends positively on the total factor productivity of the country, as shown by the equation (3.11), also the wage-setting curve is increasing in the average quality of human capital and in the proximity to the technological frontier.

The accelerating effect of an increase of the stock of skilled labor on the productivity growth does not however last endlessly but deteriorates gradually as the country comes closer to the technological frontier. This can be seen by differentiating the derivative of the growth rate with respect to the skilled human capital for the second time with respect to the proximity to the technological frontier. As a result we have

$$\frac{1}{\eta\gamma}\frac{d^2g}{dEda} = \left[-\frac{\kappa\sigma(1-\kappa)}{(\sigma-\kappa)}z(a)^{-\kappa-1}\right]z'(a). \qquad (3.33)$$

Because $z(a) \equiv \left(\frac{(1-\sigma)\psi^{\sigma}(1-a)}{(1-\kappa)\gamma a}\right)^{\frac{1}{\sigma-\kappa}}$, the derivative of $z(a)$ with respect to a is

$$z'(a) = \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}}\left(\frac{1}{a}-1\right)^{\frac{1-\sigma+\kappa}{\sigma-\kappa}}. \qquad (3.34)$$

As the country approaches the technological frontier, the limit of $z'(a)$ becomes zero because

$$\lim_{a \to 1} \left(\frac{1}{a} - 1 \right) = 0.$$

Consequently,

$$\lim_{a \to 1} \frac{d^2 g}{dE da} = 0$$

and also the limit of the growth rate approaches zero the closer the country is situated to the technological frontier. Therefore, the growth rate slows down as the technological development proceeds. This follows from the fact that if the creation of innovations exploits skilled labor more intensively than the imitation activities, even countries situating far from the technological frontier utilize most of the skilled labor in innovation. As a result, the amount of skilled workers transferring form imitation to innovation must decrease little by little as a country approaches the frontier. The results of this section are summarized in Proposition 3.

Proposition 3

1. A marginal increase in the stock of skilled human capital speeds up growth as the country approaches the technological frontier.
2. The accelerating effect becomes weaker the closer the country is situated to the technological frontier because $\kappa < \sigma$.

3.3 Skill-Setting

Before proceeding into the equilibrium analysis, we will still take a look at the decision making of individuals based on the paper by Docquier and Rapoport (2012). The utility of individuals is now assumed to depend on their consumption during all the three periods of their lives. In period t young individuals maximize their lifetime utility and can choose to spend their time working or, alternatively, investing in education. If they decide to work they will receive a wage w_t. Education x_t is again an either-or decision. Hereby, if an individual invests in education $x_t = 1$, and otherwise $x_t = 0$. Education entails a cost

cw_t, where c captures individual's ability to learn and is assumed to be uniformly distributed on $[0, 1]$. It can be thought that the cost represents the wages lost during acquiring education. The adulthood wage depends on whether an individual acquired education during the youth or not. Uneducated individuals will receive a wage w_{t+1}, while those who invested in education will receive a wage $(1 + \theta)w_{t+1}$. The wage in the second period is used for consumption and savings. The savings s_{t+1} alone determine the consumption during the third period.

The utility function is now assumed to be logarithmic and is written as

$$U(x_t, s_{t+1}) = \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1 - \lambda)\ln(w_{t+1}(1 + x_t\theta) - s_{t+1}) + \lambda \ln(s_{t+1}R^*_{t+2}), \quad (3.35)$$

where $\widehat{\mu_t} > 0$ signifies a minimum level of subsistence in the first period[17]. It is assumed that there is no such minimum subsistence level during the other two periods. λ, on the other hand, represents time preference and the relative length of the retirement period. By maximizing the utility function with respect to the savings, the fraction of income that individuals save during the second period of their lives becomes

$$\frac{dU(x_t,s_{t+1})}{ds_{t+1}} = -\frac{(1-\lambda)}{w_{t+1}(1+x_t\theta)-s_{t+1}} + R^*_{t+2}\frac{\lambda}{s_{t+1}R^*_{t+2}} = 0$$

$$\Leftrightarrow \qquad s_{t+1} = \lambda\big(w_{t+1}(1 + x_t\theta)\big). \quad (3.36)$$

From the equation (3.36) it can thus be seen that savings equal a fraction λ of the second period income. As a result, the utility function can be rewritten as

$$U\left(x_t, \lambda\big(w_{t+1}(1 + x_t\theta)\big)\right) = \ln(w_t - \widehat{\mu_t} - x_t c w_t) + \ln\big(w_{t+1}(1 + x_t\theta)\big) + \lambda \ln(R^*_{t+2})$$

$$+\lambda \ln(\lambda) + (1 - \lambda)\ln(1 - \lambda). \quad (3.37)$$

Because $\lambda \ln(\lambda) + (1 - \lambda)\ln(1 - \lambda)$ is a constant, a new function I can be defined as

$$I \equiv \lambda \ln(\lambda) + (1 - \lambda)\ln(1 - \lambda) \quad (3.38)$$

[17] See for example Belot and Hatton (2012) for the importance of the liquidity constraint on educational selectivity.

and, consequently, the utility function becomes

$$V(x_t) = \ln(w_t - \widehat{\mu_t} - x_t c w_t) + \ln\big(w_{t+1}(1 + x_t\theta)\big) + \lambda \ln(R^*_{t+2}) + I. \qquad (3.39)$$

When considering whether to acquire education, an individual compares his discounted lifetime incomes with an educational investment and without the investment. It is profitable for an individual to invest in education if $V(1) > V(0)$, that is if his expected lifetime utility with education is greater than the utility without education. Solving the inequality gives:

$$V(1) > V(0) \;\Leftrightarrow\; \left(\frac{\theta}{1+\theta}\right)\left(\frac{w_t - \widehat{\mu_t}}{w_t}\right) > c. \qquad (3.40)$$

As a consequence, there exists a critical value for the cost of education defined as

$$\widehat{c_t} = \left(\frac{w_t - \widehat{\mu_t}}{w_t}\right)\left(\frac{\theta}{1+\theta}\right). \qquad (3.41)$$

The minimum level of consumption during the first period can be assumed to be a constant fraction μ of the wage rate of the developed country, that is $\widehat{\mu_t} = \mu w^*_t$. This is a reasonable assumption because the liquidity constraints are more binding in the low income countries than in the technologically more advanced countries. In this case the critical value for the cost of education can be rewritten as

$$\widehat{c_t} = \left(\frac{w_t - \mu w^*_t}{w_t}\right)\left(\frac{\theta}{1+\theta}\right) = \left(\frac{w_t}{w_t} - \mu\frac{w^*_t}{w_t}\right)\left(\frac{\theta}{1+\theta}\right) = \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right). \qquad (3.42)$$

From the equation (3.40) it can be seen that if an individual faces a cost $c < \widehat{c_t}$ he will acquire education and if $c \geq \widehat{c_t}$ there is no incentive to invest in education. For the critical value, given in the equation (3.41), it holds true that

$$\frac{d\widehat{c_t}}{dw_t} = \left(\frac{w_t - (w_t - \widehat{\mu_t})}{w_t^2}\right)\left(\frac{\theta}{1+\theta}\right) = \left(\frac{\widehat{\mu_t}}{w_t^2}\right)\left(\frac{\theta}{1+\theta}\right) > 0 \qquad (3.43)$$

and

$$\frac{d\widehat{c_t}}{d\theta} = \left(\frac{w_t - \widehat{\mu_t}}{w_t}\right)\left(\frac{(1+\theta) - \theta}{(1+\theta)^2}\right) = \left(\frac{w_t - \widehat{\mu_t}}{w_t}\right)\left(\frac{1}{(1+\theta)^2}\right) > 0, \qquad (3.44)$$

because the minimum subsistence level cannot exceed the income earned in period t.

Because c is assumed to be uniformly distributed, $\widehat{c_t}$ also determines the proportion of the young who educate themselves. When there is no possibility of emigration, as is assumed for the time being, $\widehat{c_t}$ also specifies the proportion of the educated in the second period. If the proportion of skilled workers is denoted by π_t, it follows that $\pi_{t+1} = \widehat{c_t}$ as long as the country is in autarky. The proportion of the educated individuals in the second period positively depends on the wage rate of the developing country w_t and on the skill premium θ as shown by the equations (3.43) and (3.44). Because $\pi_{t+1} = \widehat{c_t}$, the average level of human capital of an economy where each adult has m children can be written as

$$h_{t+1} = \frac{(1-\pi_{t+1})+m+(1+\theta)\pi_{t+1}}{1+m} = 1 + \frac{\pi_{t+1}\theta}{1+m} = 1 + \frac{\widehat{c_t}\theta}{1+m} \qquad (3.45)$$

for the second period. This follows from the assumption that uneducated individuals offer one efficiency unit of labor while the educated individuals offer $1+\theta$ efficiency units of labor. As can be seen from the equation (3.45), the average level of human capital increases with the critical value for the cost of education.

If the critical value for the cost of education in the equation (3.42) is inserted into the equation (3.45), the average level of human capital in the second period becomes

$$h_{t+1} = 1 + \frac{1}{1+m}\left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\theta = 1 + \frac{1}{1+m}\left(\frac{\theta^2}{1+\theta}\right)\left(1 - \frac{\mu}{\omega_t}\right) = 1 + \frac{\theta^2}{(1+m)(1+\theta)}\left(1 - \frac{\mu}{\omega_t}\right)$$

$$\equiv H(\omega_t). \qquad (3.46)$$

The equation (3.46) determines how the average level of human capital develops in an economy and is thus called the *skill-setting equation*. For the $H(\omega_t)$ function it is valid that

$$\frac{dH(\omega_t)}{d\omega_t} = \frac{\mu\theta^2}{(1+m)(1+\theta)\omega_t^2} > 0 \qquad (3.47)$$

and

$$\frac{d^2H(\omega_t)}{d\omega_t^2} = -\frac{2\mu\theta^2}{(1+m)(1+\theta)\omega_t^3} < 0. \quad (3.48)$$

These results assume that $\omega_t \geq \mu$ because otherwise the minimum subsistence level would exceed the income earned during the first period. Therefore, if $\omega_t < \mu$ it must be that $H(\omega_t) = 0$.

Now there is enough information in order to determine the possible equilibria of the economies. When specifying the equilibria, the properties of the skill-setting curve derived in equations (3.47) and (3.48) can be utilized. In addition, the features of the wage-setting curve described in section 3.2 will be needed. By combining the skill-setting and the wage-setting curves the dynamic effects of brain drain can finally be examined.

4 Effects of Brain Drain

In this section, we will finally pay our attention to the equilibria of the model of human capital accumulation. In the possible balanced growth equilibria both the *wage-setting* and the *skill-setting equations*, that is equations (3.12) and (3.46), are satisfied. It will be shown by using the properties of the equations that there exists a possibility of multiple equilibria. Thereafter, the existing equilibria can be utilized to examine the effects of different policies on the productivity growth of the economy. We are particularly interested in the consequences of brain drain. The equations of this section are derived in Appendix C.

4.1 Steady States of the Economy

In section 3 a wage-setting equation set by the production sector of the economy and a skill-setting equation designated by the individuals maximizing their lifetime utility were defined. From now on we will focus on the steady state values of the intensive variables

denoted by a subscript ss. As a consequence, the wage-setting equation can be redetermined to be

$$\omega_{ss} = \mathcal{W}(h_{ss}),$$

and the skill-setting equation can be rewritten as

$$H(\omega_{ss}) = 1 + \frac{\theta^2}{(1+m)(1+\theta)}\left(1 - \frac{\mu}{\omega_{ss}}\right).$$

The shape of the skill-setting curve can be easily determined by taking advantage of the equations (3.47) and (3.48) which show that the skill-setting curve is increasing but has diminishing marginal returns with respect to ω_{ss}. Thus, on a (h_{ss}, ω_{ss}) plane the skill-setting curve is upward sloping and converges to the vertical axis. In addition, ω_{ss} must exceed the assumed minimum subsistence level μ as already mentioned earlier. The shape of the wage-setting curve is more complex because its second derivative can be either positive or negative as pointed out in section 3.1. However, as was shown in the following section 3.2, an increase in the stock of highly skilled human capital enhances technological progress and the productivity growth. When the economy approaches the world´s technological frontier and begins replacing imitation with innovation, the growth rate tends to even speed up. Because the wage-setting equation positively depends on the level of the technology, as indicated by the equation (3.11), it also increases as a function of the average level of human capital. However, another significant observation made in section 3.2 was that as the country comes still closer to the technological frontier the productivity growth begins diminishing. This is because as the economy verges on the technological frontier, the amount of skilled labor shifting from imitation to innovation decreases little by little because it is assumed that innovation sector benefits highly skilled labor even far from the frontier. Consequently, the effect of an increase in the amount of highly skilled labor on the productivity growth decreases once the country has reached a certain technological stage. These properties of the productivity growth with respect to the average level of human capital lead to an S-shaped wage-setting curve. The skill-setting and the wage-setting curves are depicted in Figure 3.

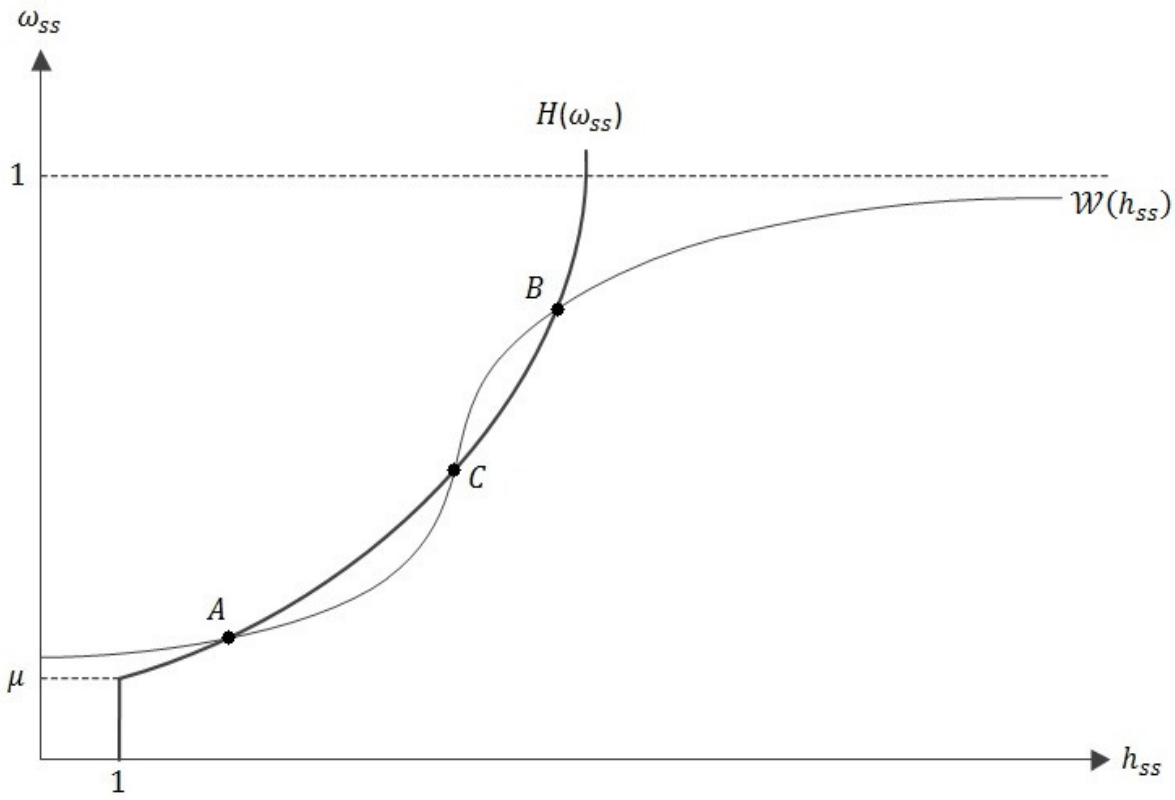


Figure 3. Long-run equilibria of a closed economy.

As can be seen from the Figure 3, the intersections of $H(\omega_{ss})$ and $\mathcal{W}(h_{ss})$ curves determine three different equilibrium points A, B and C. Point A and point B are dynamically stable equilibria and point C, on the other hand, is a dynamically unstable equilibrium. It can be thought that point A represents a developing country with low average level of human capital situating far from the technological frontier while, at the same time, point B depicts a developed country with high average level of human capital. The existence of multiple equilibria indicates that variations in the amount of educated workers may have long run effects on the productivity of a country. This is in line with threshold externalities mentioned earlier in section 3.1. From the Figure 3, it can be seen that if an economy starts out with a human capital stock below a critical value, it will converge to the steady state A with low level of per capita income. On the contrary, if the economy starts out with a human capital stock above the critical value, it will converge to the steady state B with high level of per capita income. Thus, if there are no outside

incentives or government interventions, the initial level of the inputs is the only factor affecting the final equilibrium that will be reached.

One possible cause of a threshold externality is the migration of some of the highly skilled individuals into a more advanced country. As already noticed in section 2, the emigration possibility of educated workers tends to increase the amount of individuals investing in education. The emigration possibility could thus operate as an incentive effect and enable the source country of emigration to shift from a low level of human capital equilibrium into a higher level of human capital equilibrium. The prospects for such shifts will be examined for the human capital accumulation approach in what follows. The curves in Figure 3 will be used to analyze the consequences of highly skilled emigration.

4.2 Brain Drain

Individuals living in the developing country are now permitted to migrate into a developed country. As in section 2, it is assumed that only the educated workers have the possibility to migrate and low skilled workers stay in their home country. Moreover, only a fraction p of the highly skilled individuals actually emigrate. The proportion of the highly skilled individuals staying in the developing country in the second period is thus given by

$$\pi_{t+1} = \frac{(1-p)\hat{c_t}}{1-p\hat{c_t}}, \tag{4.1}$$

where the numerator depicts the amount of the highly skilled individuals staying in the source country of emigration and the denominator represents the whole population after brain drain has taken place.

The utility function (3.39) must now be modified to include the migration prospect for the educated individuals. When $x_t = 1$, the utility function can be rewritten as

$$V(1) = \ln(w_t - \widehat{\mu_t} - cw_t) + (1-p)\ln\big(w_{t+1}(1+\theta)\big) + p\ln\big(w^*_{t+1}(1+\theta)\big)$$
$$+\lambda\ln(R^*_{t+2}) + I. \qquad (4.2)$$

By comparing the utility of an educated worker to the utility of an uneducated worker, $V(0)$ as in the equation (3.39), a new critical value for the cost of education can be calculated to be

$$V(1) > V(0) \Leftrightarrow c < \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{{\omega_{t+1}}^p}{1+\theta}\right). \qquad (4.3)$$

As a consequence, the critical value of the cost of education is given by

$$\widehat{c_t} = \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{{\omega_{t+1}}^p}{1+\theta}\right), \qquad (4.4)$$

when there exists a possibility of emigration for the educated individuals. Now, if $p = 0$ then $\omega^p_{t+1} = 1$ and

$$\widehat{c_t} = \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{1}{1+\theta}\right) = \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{1+\theta}{1+\theta} - \frac{1}{1+\theta}\right) = \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)$$

as in the equation (3.42) for an autarkic economy. If however $0 < p < 1$, $\omega^p_{t+1} < 1$ because it is assumed that the wage rate of the leading economy exceeds the wage rate of the local economy. The proportion of individuals acquiring education is now higher than without the possibility of emigration because

$$\left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{{\omega_{t+1}}^p}{1+\theta}\right) > \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right) \qquad (4.5)$$

applies with all $\omega_{t+1} < 1$ and $0 < p < 1$. This result supports the occurrence of the incentive effect.

Since we are interested in how the possibility of emigration affects the average level of human capital in the developing country, and as a result the productivity growth rate, the

effects of the value of p on the value of π_{t+1} must be studied. The differentiation of π_{t+1} in the equation (4.1) with respect to p results in

$$\frac{d\pi_{t+1}}{dp} = \frac{(1-p)\frac{d\widehat{c_t}}{dp} - \widehat{c_t}(1-\widehat{c_t})}{(1-p\widehat{c_t})^2}. \tag{4.6}$$

Because the denominator of the equation (4.6) is always positive, beneficial brain drain is possible only if the numerator is also positive. It can be easily seen that if p is close to one there is no possibility of beneficial brain drain because then $(1-p)\frac{d\widehat{c_t}}{dp}$ becomes small. As a consequence, the probability of highly skilled emigration needs to be low enough in order to yield possible gains. The numerator of the equation (4.6) can thus be estimated to be positive at $p = 0$. The derivative of the average level of human capital with respect to the emigration possibility at $p = 0$ is given by

$$\frac{d\pi_{t+1}}{dp}|_{p=0} = \frac{d\widehat{c_t}}{dp} - \widehat{c_t}(1-\widehat{c_t}). \tag{4.7}$$

The condition for beneficial brain drain at $p = 0$ now becomes

$$\frac{d\widehat{c_t}}{dp} - \widehat{c_t}(1-\widehat{c_t}) > 0$$

$$\Leftrightarrow \quad \ln\left(\frac{w_{t+1}^*}{w_{t+1}}\right) > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]. \tag{4.8}$$

From the equation (4.8) it can be interpreted that the ratio of the wage rates across the countries, that is ω_{ss}, needs to be low enough in order to enable beneficial brain drain. A reasonably large difference in skill prices generates a strong incentive to acquire education. If w_{ss} and w_{ss}^* are close to each other, the incentive effect vanishes. However, it must be noted that if the ratio of the skill prices is too low the liquidity constraint becomes strongly binding and it might be impossible to invest in education even though one would be willing to. A particularly large difference in wages increases the value of the right-hand side of the equation (4.8) and thus deteriorates the possibility of beneficial brain drain. The conditions for beneficial brain drain are summarized in Proposition 4.

Proposition 4 Conditions for beneficial brain drain:

1. The differential in skill prices (ω_{ss}) needs to be low enough to generate a strong incentive effect, but not so low that the liquidity constraint on education investment becomes strongly binding.
2. The probability of high-skilled emigration needs to be sufficiently low.

As a result of the equation (4.8), brain drain can lead to ambiguous effects on the skill-setting curve in Figure 3. The $H(\omega_{ss})$ curve might shift to the left for extremely poor countries because of the binding liquidity constraint. If individuals struggle to fulfill their minimum subsistence level, they certainly do not have the means to invest in education even though they would be motivated to. Similarly, the $H(\omega_{ss})$ curve might shift to the left for rich countries as well because the incentive effect does not work anymore. The best potential for the $H(\omega_{ss})$ curve to move to the right is for middle-income countries. In the case of middle-income countries, the incentive effect operates but the liquidity constraint does not restrain options made by individuals. The two possible effects on the skill-setting curve are illustrated in Figure 4.

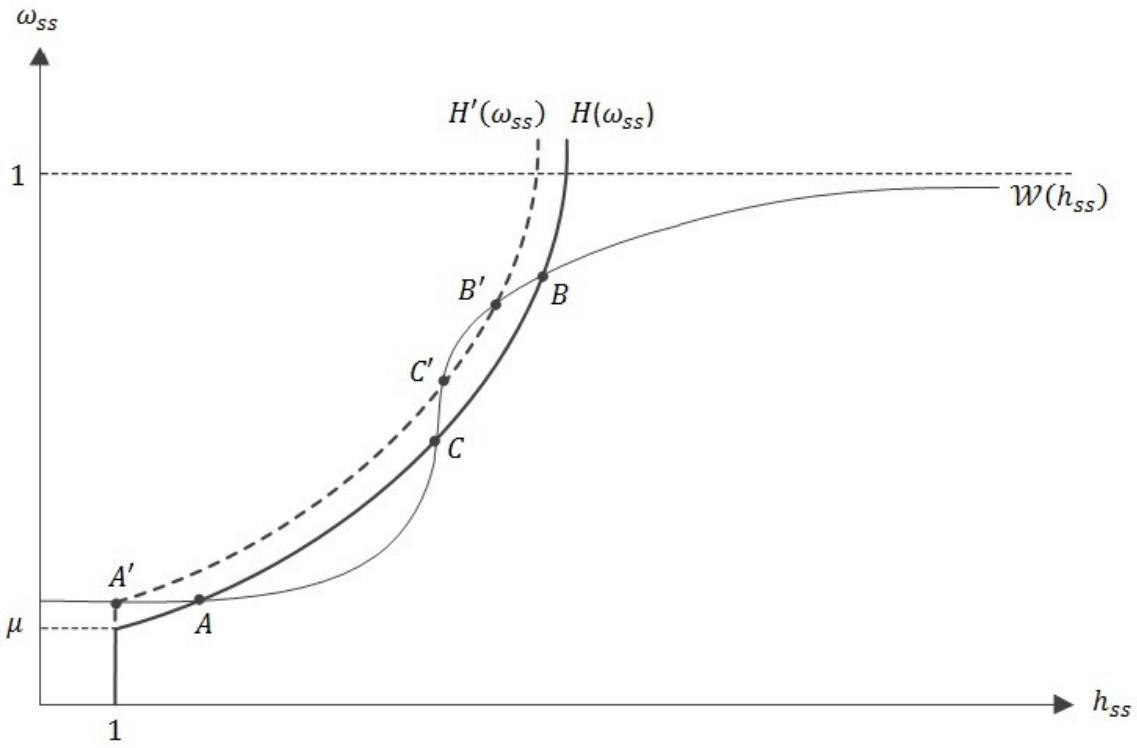


Figure 4a. The $H(\omega_{ss})$ curve might move to the left for rich countries and for extremely poor countries.

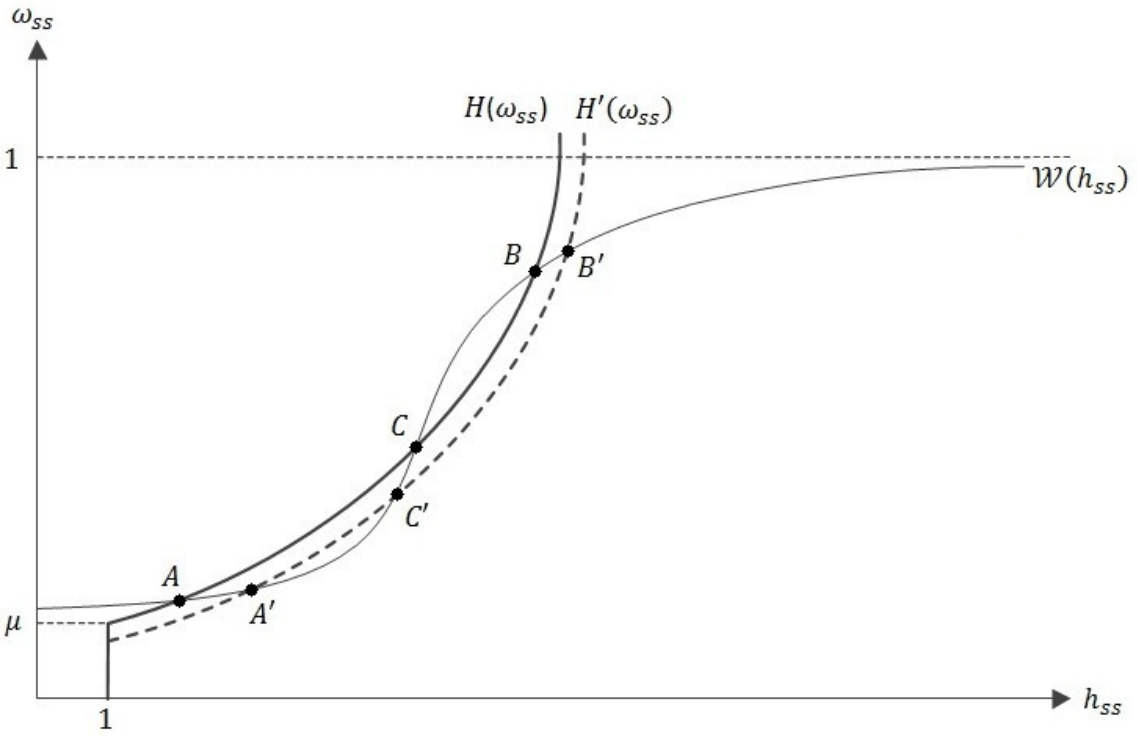


Figure 4b. The $H(\omega_{ss})$ curve might move to the right for middle-income countries.

As can be seen from the Figure 4a, the migration prospect might lower the average level of human capital for extremely poor countries. Simultaneously, they move further away from the technological frontier. This observation suggests that there still exists the possibility that some of the developing countries are trapped in a poverty trap. However, as depicted in the Figure 4b, the average level of human capital increases with the possibility of emigration for middle-income countries. Hereby, it is the country characteristics that mainly affect the final results of brain drain. To a large extent the initial conditions depend on the public policies adopted and can thus be influenced. Ultimately, based on the preceding analysis an important conclusion can be made that brain drain is a possible source for positive externalities and need not deplete the human capital stock of a country.

5 Concluding Remarks

The purpose of this thesis was to delve into the problem of brain drain from the perspective of an underdeveloped country. As stated at the beginning of the thesis, the conceptions about the effects of brain drain on economic development have contradicted each other since the outset of the brain drain literature. Therefore, the main objective of the thesis was to find out how high-skilled emigration affects the level of human capital and thus the productivity improvements in developing countries. The conditions leading to beneficial brain drain were also paid attention to.

At first, a simple economic overview of brain drain was introduced for a closed economy. No strict assumptions about the production function or the lifetime utilities of individuals were made. However, differences in innate abilities led to differences in educational decisions. It was shown that it is profitable only for individuals possessing a latent ability above a certain critical level to invest in education. By allowing a proportion of the educated workers to immigrate into a more advanced country, it was next pointed out that the critical value for the latent ability declines with the emigration possibility. As a

result, it was found out that brain drain results in positive effects on the productivity growth of the source country of emigration. This however requires that the emigration possibility is low enough, the proportion of the highly skilled was low in the previous period, and the wage rate per efficiency unit of labor in the developed country is high relative to the wage rate of the developing country.

After introducing the simple economic overview, an analysis with more accurate assumptions about the production function and the utility function was presented in order to examine the possibility of multiple equilibria. The model of human capital accumulation was derived in two stages. At first a *wage-setting equation* was developed and it was denoted that the wage rate of a developing country relative to the wage rate of a developed country depends on the states of the technologies of the countries. As a consequence, the ratio of the wage rates is also relative to the average quality of human capital. The effects of human capital on technological progress were taken into a closer consideration by assuming that technological improvements are created by imitation and innovation activities. An increase in the stock of skilled human capital was shown to attract both educated and uneducated workers from imitation to innovation, and contrariwise. In addition, the growth rate of the economy was proven to be increasing in the amount of skilled workers and decreasing in the amount of unskilled workers. However, the growth rate deteriorates gradually as the technological progress proceeds above a certain level. The second stage of the human capital accumulation model, on the other hand, derived a *skill-setting equation*. By exploiting the utility function of individuals it was found out that there exists a certain cost of education that determines the proportion of utility maximizing individuals who invest in education. According to that, the average level of human capital could be calculated. The development of the average level of human capital then specified the skill-setting equation.

Thereafter, the wage-setting and the skill-setting equations were combined in order to show that there exist multiple equilibria. A stable equilibrium with a low level of average human capital and a long distance to the technological frontier represented a developing

country, while on the other hand another stable equilibrium with a high level of average human capital represented the leading economy. The wage-setting and the skill-setting curves could then be used to study how high-skilled emigration affects the equilibrium outcomes. By allowing a part of the educated workers to leave the developing country it could be proven that the proportion of individuals acquiring education increases similarly as in section 2. However, two necessary conditions for beneficial brain drain were derived. First of all, the ratio of the wage rates between the countries should be low enough in order to encourage young people to invest in education. Nonetheless, the ratio of the skill prices should not be that low that the liquidity constraint becomes strongly binding. For the second, the probability of emigration for the highly skilled should be sufficiently low.

In summary, both of the models emphasize the significance of an incentive in the form of a wage differential between the countries. If the wage rates only differ slightly there is no additional incentive to invest in education, not to mention to go through the effort of migration. This supports the observations made by the United Nations. In addition, the incentive mechanism has also been supported by empirical evidence. For example Beine et al. (2008) have shown that emigration of some of the educated workers increases the level of human capital of a country. Furthermore, they give support to the other results of the simple overview of brain drain. Especially that those countries with low average levels of human capital and low high-skilled emigration rates have the best potential to benefit from brain drain. However, the existence of the minimum subsistence level also has to be taken into account as has been done in the model of human capital accumulation. The liquidity constraint limits the choices concerning education and as a consequence especially the extremely poor countries are affected by a large wage differential.

The analysis presented in this thesis can be utilized to study the consequences of other public policies as well. The observations of those studies could then be used to build ideal conditions for human capital formation together with high-skilled emigration. For example factors affecting the emigration possibility should be taken into consideration

when making policy decisions. Furthermore, another aspect that should also be paid attention to is the return migration of the educated individuals. For example Mayr and Peri (2009) have uncovered that 20 to 30 per cent of Eastern European and Asian emigrants return to their home countries while they still belong to the labor force. Thus, return migration increases the average level of human capital together with the initial incentive effect and is a significant determinant of the productivity growth. As a result of return migration, brain drain could have positive effects even on a larger group of countries than assessed based on this analysis. All in all, brain drain is a meaningful factor in inducing incentives to acquire education and should not be ignored as an education policy instrument in developing countries.

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Appendices

Appendix A

Derivation of equation (2.2)

The net return to physical capital is determined by the marginal product of capital, i.e. by the derivative of the production function with respect to the stock of physical capital. Because the production function can be written as

$$Y_t = F(K_t, A_t H_t) = f(k_t) A_t H_t = f\left(\frac{K_t}{A_t H_t}\right) A_t H_t, \qquad (A1)$$

the net return to physical capital is given by

$$r_t = \frac{dY_t}{dK_t} = f'\left(\frac{K_t}{A_t H_t}\right)\left(\frac{1}{A_t H_t}\right) A_t H_t = f'(k_t). \qquad (A2)$$

Derivation of equation (2.3)

The wage rate per efficiency unit of labor is given by the marginal product of labor. Thus, the wage rate can be defined by differentiating the production function with respect to the total amount of efficiency units of labor. Because the production function is determined as given in the equation $(A1)$, the wage rate per efficiency unit of labor is

$$w_t = \frac{dY_t}{dH_t} = A_t H_t f'(k_t)\left(-\frac{K_t}{A_t {H_t}^2}\right) + A_t f(k_t) = A_t\left[\left(-\frac{K_t}{A_t H_t}\right) f'(k_t) + f(k_t)\right]$$

$$= A_t[f(k_t) - k_t f'(k_t)]. \qquad (A3)$$

By defining $w(k_t) \equiv f(k_t) - k_t f'(k_t)$, the wage rate per efficiency unit of labor can be written as $w_t = A_t w(k_t)$.

Derivation of equations (2.7) and (2.8)

Using the fact that $A_t = A(\pi_{t-1})$, $\hat{b}_t$ can be written as

$$\hat{b}_t = \frac{A_t w(k) + c(1+r^*)}{A_t w(k)} = 1 + \frac{c(1+r^*)}{A(\pi_{t-1}) w(k)}. \qquad (A4)$$

Differentiation of $\hat{b}_t$ with respect to π_{t-1}thus results in

$$\frac{d\hat{b}_t}{d\pi_{t-1}} = -\frac{c(1+r^*)A'(\pi_{t-1})}{A^2(\pi_{t-1}) w(k)}. \qquad (A5)$$

Because $\pi_t = \int_{\hat{b}_t}^{B} D(b_i) db_i = \mathcal{D}(B) - \mathcal{D}(\hat{b}_t)$, where $\mathcal{D}$ is an integrated function of D, it follows that

$$\frac{d\pi_t}{d\pi_{t-1}} = -D(\hat{b}_t)\frac{d\hat{b}_t}{d\pi_{t-1}} = D(\hat{b}_t)\frac{c(1+r^*)A'(\pi_{t-1})}{A^2(\pi_{t-1}) w(k)}. \qquad (A6)$$

Derivation of equation (2.12)

The proportion of educated individuals is given by the equation (2.11) as

$$\pi_t = \frac{(1-p)\int_{\hat{b}_t}^{B} D(b_i) db_i}{1-p\left(\int_{\hat{b}_t}^{B} D(b_i) db_i\right)} = \frac{\int_{\hat{b}_t}^{B} D(b_i) db_i - p\left(\int_{\hat{b}_t}^{B} D(b_i) db_i\right)}{1-p\left(\int_{\hat{b}_t}^{B} D(b_i) db_i\right)}. \qquad (A7)$$

The differentiation of the average proportion of educated individuals with respect to the emigration possibility p thus gives

$$\frac{d\pi_t}{dp}=\frac{\left(-D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}-\left(\int_{\hat{b}_t}^{B}D(b_i)db_i-pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\right)\right)\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$-\frac{\left(\int_{\hat{b}_t}^{B}D(b_i)db_i-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)\left(-\left(\int_{\hat{b}_t}^{B}D(b_i)db_i-pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\right)\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=\frac{-D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}+pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\int_{\hat{b}_t}^{B}D(b_i)db_i-\int_{\hat{b}_t}^{B}D(b_i)db_i+p\int_{\hat{b}_t}^{B}D(b_i)db_i\int_{\hat{b}_t}^{B}D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{E}D(b_i)db_i\right)^2}$$

$$+\frac{pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}-p^2D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\int_{\hat{b}_t}^{B}D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{E}D(b_i)db_i\right)^2}$$

$$-\frac{-\int_{\hat{b}_t}^{B}D(b_i)db_i\int_{\hat{b}_t}^{B}D(b_i)db_i+pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\int_{\hat{b}_t}^{B}D(b_i)db_i+p\int_{\hat{b}_t}^{B}D(b_i)db_i\int_{\hat{b}_t}^{B}D(b_i)db_i-p^2D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}\int_{\hat{b}_t}^{B}D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=\frac{-D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}-\int_{\hat{b}_t}^{B}D(b_i)db_i+pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}+\int_{\hat{b}_t}^{B}D(b_i)db_i\int_{\hat{b}_t}^{B}D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=-\frac{\int_{\hat{b}_t}^{B}D(b_i)db_i-\int_{\hat{b}_t}^{B}D(b_i)db_i\int_{\hat{b}_t}^{B}D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}-\frac{D(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}-pD(\hat{b}_t)\frac{\partial \hat{b}_t}{\partial p}}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=-\frac{\int_{\hat{b}_t}^{B}D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B}D(b_i)db_i\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}-\frac{(1-p)D(\hat{b}_t)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}\frac{\partial \hat{b}_t}{\partial p}. \qquad (A8)$$

Because

$$\frac{\partial \pi_t}{\partial p}=\frac{-\int_{\hat{b}_t}^{B}D(b_i)db_i\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)-\left(\int_{\hat{b}_t}^{B}D(b_i)db_i-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)\left(-\int_{\hat{b}_t}^{B}D(b_i)db_i\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=-\frac{\int_{\hat{b}_t}^{B}D(b_i)db_i\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i-\int_{\hat{b}_t}^{B}D(b_i)db_i+p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2}$$

$$=-\frac{\int_{\hat{b}_t}^{B}D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B}D(b_i)db_i\right)}{\left(1-p\int_{\hat{b}_t}^{B}D(b_i)db_i\right)^2} \qquad (A9)$$

and

$$\frac{\partial \pi_t}{\partial \hat{b}_t} = \frac{(1-p)\left(-D(\hat{b}_t)\frac{d\hat{b}_t}{d\hat{b}_t}\right)\left(1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right)-(1-p)\int_{\hat{b}_t}^{B} D(b_i)db_i\left(-p\left(-D(\hat{b}_t)\frac{d\hat{b}_t}{d\hat{b}_t}\right)\right)}{\left(1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right)^2}$$

$$= \frac{(1-p)\left(-D(\hat{b}_t)+pD(\hat{b}_t)\int_{\hat{b}_t}^{B} D(b_i)db_i\right)-(1-p)\left(pD(\hat{b}_t)\right)\int_{\hat{b}_t}^{B} D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right)^2}$$

$$= \frac{-(1-p)D(\hat{b}_t)+(1-p)pD(\hat{b}_t)\int_{\hat{b}_t}^{B} D(b_i)db_i-(1-p)pD(\hat{b}_t)\int_{\hat{b}_t}^{B} D(b_i)db_i}{\left(1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right)^2}$$

$$= -\frac{(1-p)D(\hat{b}_t)}{\left(1-p\int_{\hat{b}_t}^{B} D(b_i)db_i\right)^2}, \qquad (A10)$$

$\frac{d\pi_t}{dp}$ can be written as

$$\frac{d\pi_t}{dp} = \frac{\partial \pi_t}{\partial p} + \frac{\partial \pi_t}{\partial \hat{b}_t}\frac{\partial \hat{b}_t}{\partial p}, \qquad (A11)$$

where $\hat{b}_t$ is given by the equation (2.10) and

$$\frac{\partial \hat{b}_t}{\partial p} = -\frac{\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(pw^*-(1-p)A_t w)^2}. \qquad (A12)$$

Determination of Proposition 1

According to the equation (2.16)

$$\frac{D(\hat{b}_t)\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(A_t w)^2} > \int_{\hat{b}_t}^{B} D(b_i)db_i\left(1-\int_{\hat{b}_t}^{B} D(b_i)db_i\right).$$

A function v can be determined as $v \equiv \int_{\hat{b}_t}^{B} D(b_i)db_i \left(1 - \int_{\hat{b}_t}^{B} D(b_i)db_i\right)$. By denoting $x \equiv \int_{\hat{b}_t}^{B} D(b_i)db_i$, the function v becomes $v = x(1-x) = x - x^2$. Now the maximization of the function v results in

$$\frac{dv}{dx} = 1 - 2x = 0 \Leftrightarrow x = \int_{\hat{b}_t}^{B} D(b_i)db_i = \frac{1}{2}.$$

Because

$$\frac{d^2v}{dx^2} = -2 < 0\,,$$

this is a maximum point. Thus, the maximum value of the function v is

$$v\left(\frac{1}{2}\right) = \frac{1}{2}\left(1 - \frac{1}{2}\right) = \frac{1}{4}$$

and consequently the condition of the equation (2.16) becomes

$$\frac{D(\hat{b}_t)(A_t w + c(1+r^*))(w^* - A_t w)}{(A_t w)^2} > \frac{1}{4}. \qquad (A13)$$

Derivation of equation (2.17)

Assuming that the density function $D(b_i)$ follows the uniform distribution, it has the following properties: $D(b_i) = \frac{1}{B}$ and $\int_{\hat{b}_t}^{B} D(b_i)db_i = 1 - \frac{\hat{b}_t}{B}$. Given these properties, the condition for beneficial brain drain now becomes:

$$\frac{d\pi_t}{dp} > 0$$

$$\Leftrightarrow \frac{\partial \pi_t}{\partial p} < \frac{\partial \pi_t}{\partial \hat{b}_t}\frac{\partial \hat{b}_t}{\partial p}$$

$$\Leftrightarrow \frac{\left(1-\frac{\hat{b}_t}{B}\right)\left(1-\left(1-\frac{\hat{b}_t}{B}\right)\right)}{\left(1-p\left(1-\frac{\hat{b}_t}{B}\right)\right)^2} < \frac{(1-p)\left(\frac{1}{B}\right)}{\left(1-p\left(1-\frac{\hat{b}_t}{B}\right)\right)^2}\frac{\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(pw^*+(1-p)A_t w)^2}$$

$$\Leftrightarrow \left(1-\frac{\hat{b}_t}{B}\right)\left(\frac{\hat{b}_t}{B}\right) < (1-p)\left(\frac{1}{B}\right)\frac{\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(pw^*+(1-p)A_t w)^2}$$

$$\Leftrightarrow 1-\frac{\hat{b}_t}{B} < (1-p)\left(\frac{1}{\hat{b}_t}\right)\frac{\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(pw^*+(1-p)A_t w)^2}$$

$$\Leftrightarrow 1-\frac{\hat{b}_t}{B} < (1-p)\left(\frac{pw^*+(1-p)A_t w}{A_t w+c(1+r^*)}\right)\frac{\left(A_t w+c(1+r^*)\right)(w^*-A_t w)}{(pw^*+(1-p)A_t w)^2}$$

$$\Leftrightarrow 1-\frac{\hat{b}_t}{B} < (1-p)\frac{w^*-A_t w}{pw^*+(1-p)A_t w}. \qquad (A14)$$

Appendix B

Derivation of equation (3.5)

Solving k_t from the equation (3.3) results in

$$R_t^*\phi_t = \alpha A_t k_t^{\alpha-1} h_t^{1-\alpha}$$

$$\Leftrightarrow\ k_t^{\alpha-1} = R_t^*\phi_t\alpha^{-1}A_t^{-1}h_t^{\alpha-1}$$

$$\Leftrightarrow\ k_t = R_t^{*\frac{1}{\alpha-1}}\phi_t^{\frac{1}{\alpha-1}}\alpha^{\frac{-1}{\alpha-1}}A_t^{\frac{-1}{\alpha-1}}h_t^{\frac{\alpha-1}{\alpha-1}}$$

$$\Leftrightarrow\ k_t = \left(\frac{R_t^*\phi_t}{A_t\alpha}\right)^{\frac{1}{\alpha-1}} h_t. \qquad (B1)$$

Derivation of equation (3.6)

By inserting the equation (3.5) into the equation (3.2), y_t becomes

$$y_t = A_t k_t^{\alpha} h_t^{1-\alpha} = A_t(R_t^{*\frac{1}{\alpha-1}}\phi_t^{\frac{1}{\alpha-1}}\alpha^{\frac{-1}{\alpha-1}}A_t^{\frac{-1}{\alpha-1}}h_t)^{\alpha}h_t^{1-\alpha} = A_t R_t^{*\frac{\alpha}{\alpha-1}}\phi_t^{\frac{\alpha}{\alpha-1}}\alpha^{\frac{-\alpha}{\alpha-1}}A_t^{\frac{-\alpha}{\alpha-1}}h_t^{\alpha}h_t^{1-\alpha}$$

$$= A_t^{\frac{-1}{\alpha-1}}R_t^{*\frac{\alpha}{\alpha-1}}\phi_t^{\frac{\alpha}{\alpha-1}}\alpha^{\frac{-\alpha}{\alpha-1}}h_t = A_t^{\frac{1}{1-\alpha}}R_t^{*\frac{-\alpha}{1-\alpha}}\phi_t^{\frac{-\alpha}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}h_t = \left(\frac{A_t\alpha^{\alpha}}{R_t^{*\alpha}\phi_t^{\alpha}}\right)^{\frac{1}{1-\alpha}} h_t. \qquad (B2)$$

Derivation of equation (3.9)

By inserting the equation (3.5) into the equation (3.4), w_t becomes

$$w_t = (1-\alpha)A_t k_t^{\alpha} h_t^{-\alpha} = (1-\alpha)A_t \left(R_t^{*\frac{1}{\alpha-1}} \phi_t^{\frac{1}{\alpha-1}} \alpha^{\frac{-1}{\alpha-1}} A_t^{\frac{-1}{\alpha-1}} h_t\right)^{\alpha} h_t^{-\alpha}$$

$$= (1-\alpha)A_t R_t^{*\frac{\alpha}{\alpha-1}} \phi_t^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{-\alpha}{\alpha-1}} A_t^{\frac{-\alpha}{\alpha-1}} h_t^{\alpha} h_t^{-\alpha} = (1-\alpha)A_t^{\frac{-1}{\alpha-1}} R_t^{*\frac{\alpha}{\alpha-1}} \phi_t^{\frac{\alpha}{\alpha-1}} \alpha^{\frac{-\alpha}{\alpha-1}}$$

$$= (1-\alpha)A_t^{\frac{1}{1-\alpha}} R_t^{*\frac{-\alpha}{1-\alpha}} \phi_t^{\frac{-\alpha}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} = (1-\alpha)\left(\frac{A_t \alpha^{\alpha}}{R_t^{*\alpha} \phi_t{}^{\alpha}}\right)^{\frac{1}{1-\alpha}}. \qquad (B3)$$

Derivation of equation (3.15)

By inserting the price of an intermediate input i given in equation (3.14) into the producer´s maximization problem, the maximization problem of a producer i becomes

$$\max_{q_{i,t}} \left(p_{i,t} q_{i,t} - q_{i,t}\right) = \max_{q_{i.t}} \left(\alpha A_{i,t}^{1-\alpha} q_{i,t}^{\alpha-1} q_{i,t} - q_{i,t}\right) = \max_{q_{i,t}} \left(\alpha A_{i,t}^{1-\alpha} q_{i,t}^{\alpha} - q_{i,t}\right).$$

The solution of the maximization problem, which determines the demanded amount of input i in equilibrium, is given as

$$\frac{d\left(\alpha A_{i,t}^{1-\alpha} q_{i,t}^{\alpha} - q_{i,t}\right)}{dq_{i,t}} = \alpha^2 A_{i,t}^{1-\alpha} q_{i,t}^{\alpha-1} - 1 = 0$$

$$\Leftrightarrow \ \alpha^2 A_{i,t}^{1-\alpha} q_{i,t}^{\alpha-1} = 1$$

$$\Leftrightarrow \ q_{i,t}^{\alpha-1} = \frac{1}{\alpha^2 A_{i,t}^{1-\alpha}}$$

$$\Leftrightarrow \ q_{i,t} = \left(\frac{1}{\alpha^2 A_{i,t}^{1-\alpha}}\right)^{\frac{1}{\alpha-1}} = \alpha^{\frac{2}{1-\alpha}} A_{i,t}. \qquad (B4)$$

Derivation of equations (3.23), (3.24) and (3.25)

The maximization problem of a producer is given by

$$\max_{u_{m,t},u_{n,t},e_{m,t},e_{n,t}} \delta\eta\left[{u_{m,t}}^{\sigma}{e_{m,t}}^{1-\sigma}(1-a_{t-1})+\gamma{u_{n,t}}^{\kappa}{e_{n,t}}^{1-\kappa}a_{t-1}\right]A_{t-1}^{*}-C_t.$$

From the maximization problem, a function P can be defined as

$$\begin{aligned}P\left(u_{m,t},u_{n,t},e_{m,t},e_{n,t}\right)&=\delta\eta\left[{u_{m,t}}^{\sigma}{e_{m,t}}^{1-\sigma}(1-a_{t-1})+\gamma{u_{n,t}}^{\kappa}{e_{n,t}}^{1-\kappa}a_{t-1}\right]A_{t-1}^{*}\\&-\left[w_{u,t}\left(u_{m,t}+u_{n,t}\right)+w_{e,t}\left(e_{m,t}+e_{n,t}\right)\right]A_{t-1}^{*},\end{aligned}\qquad (B5)$$

where $a_{t-1}\equiv A_{t-1}/A_{t-1}^{*}$. The differentiation of the function P with respect to $u_{m,t}$ and $u_{n,t}$, the amounts of unskilled labor demanded in imitation and innovation sectors, gives

$$\frac{dP}{du_{m,t}}=\sigma\delta\eta{u_{m,t}}^{\sigma-1}{e_{m,t}}^{1-\sigma}(1-a_{t-1})A_{t-1}^{*}-w_{u,t}A_{t-1}^{*}=0\qquad (B6)$$

and

$$\frac{dP}{du_{n,t}}=\kappa\delta\eta\gamma{u_{n,t}}^{\kappa-1}{e_{n,t}}^{1-\kappa}a_{t-1}A_{t-1}^{*}-w_{u,t}A_{t-1}^{*}=0.\qquad (B7)$$

From the equation $(B7)$ $w_{u,t}$ can be solved to be

$$\begin{aligned}&\kappa\delta\eta\gamma{u_{n,t}}^{\kappa-1}{e_{n,t}}^{1-\kappa}a_{t-1}A_{t-1}^{*}-w_{u,t}A_{t-1}^{*}=0\\&\Leftrightarrow\ w_{u,t}A_{t-1}^{*}=\kappa\delta\eta\gamma{u_{n,t}}^{\kappa-1}{e_{n,t}}^{1-\kappa}a_{t-1}A_{t-1}^{*}\\&\Leftrightarrow\ w_{u,t}=\kappa\delta\eta\gamma{u_{n,t}}^{\kappa-1}{e_{n,t}}^{1-\kappa}a_{t-1}.\end{aligned}\qquad (B8)$$

Inserting the value of $w_{u,t}$ in the equation $(B8)$ into the equation $(B6)$ results in

$$\sigma\delta\eta {u_{m,t}}^{\sigma-1} {e_{m,t}}^{1-\sigma}(1-a_{t-1})A_{t-1}^{*} = \kappa\delta\eta\gamma {u_{n,t}}^{\kappa-1} {e_{n,t}}^{1-\kappa} a_{t-1} A_{t-1}^{*}$$

$$\Leftrightarrow \sigma {u_{m,t}}^{\sigma-1} {e_{m,t}}^{1-\sigma}(1-a_{t-1}) = \kappa\gamma {u_{n,t}}^{\kappa-1} {e_{n,t}}^{1-\kappa} a_{t-1}. \quad (B9)$$

Because $u_{n,t} = U - u_{m,t}$ and $e_{n,t} = E - e_{m,t}$, equation $(B9)$ can be rewritten as

$$\sigma {u_{m,t}}^{\sigma-1} {e_{m,t}}^{1-\sigma}(1-a_{t-1}) = \kappa\gamma\left(U-u_{m,t}\right)^{\kappa-1}\left(E-e_{m,t}\right)^{1-\kappa} a_{t-1}. \quad (B10)$$

On the other hand, differentiating the function P with respect to $e_{m,t}$ and $e_{n,t}$, the amounts of educated workers demanded in imitation and innovation sectors, gives

$$\frac{dP}{de_{m,t}} = (1-\sigma)\delta\eta {u_{m,t}}^{\sigma} {e_{m,t}}^{-\sigma}(1-a_{t-1})A_{t-1}^{*} - w_{e,t}A_{t-1}^{*} = 0 \quad (B11)$$

and

$$\frac{dP}{de_{n,t}} = (1-\kappa)\delta\eta\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1} A_{t-1}^{*} - w_{e,t}A_{t-1}^{*} = 0. \quad (B12)$$

Again, $w_{e,t}$ can be solved from the equation $(B12)$ to be

$$(1-\kappa)\delta\eta\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1} A_{t-1}^{*} - w_{e,t}A_{t-1}^{*} = 0$$

$$\Leftrightarrow w_{e,t}A_{t-1}^{*} = (1-\kappa)\delta\eta\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1} A_{t-1}^{*}$$

$$\Leftrightarrow w_{e,t} = (1-\kappa)\delta\eta\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1}. \quad (B13)$$

Now, inserting the value of $w_{e,t}$ in the equation $(B13)$ into the equation $(B11)$, results in

$$(1-\sigma)\delta\eta {u_{m,t}}^{\sigma} {e_{m,t}}^{-\sigma}(1-a_{t-1})A_{t-1}^{*} = (1-\kappa)\delta\eta\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1} A_{t-1}^{*}$$

$$\Leftrightarrow\ (1-\sigma){u_{m,t}}^{\sigma} {e_{m,t}}^{-\sigma}(1-a_{t-1}) = (1-\kappa)\gamma {u_{n,t}}^{\kappa} {e_{n,t}}^{-\kappa} a_{t-1}. \qquad (B14)$$

As before, equation $(B14)$ can be rewritten as

$$(1-\sigma){u_{m,t}}^{\sigma} {e_{m,t}}^{-\sigma}(1-a_{t-1}) = (1-\kappa)\gamma\left(U-u_{m,t}\right)^{\kappa}\left(E-e_{m,t}\right)^{-\kappa} a_{t-1}. \qquad (B15)$$

Equations $(B10)$ and $(B15)$ are the first order conditions for an interior solution of the producer's maximization problem. For simplicity, the time indices can be ignored and, thus, the equations can be written as

$$\sigma {u_m}^{\sigma-1} {e_m}^{1-\sigma}(1-a) = \kappa\gamma(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa} a \qquad (B16)$$

and

$$(1-\sigma){u_m}^{\sigma} {e_m}^{-\sigma}(1-a) = (1-\kappa)\gamma(U-u_m)^{\kappa}(E-e_m)^{-\kappa} a. \qquad (B17)$$

If both of the equations are divided by their right hand sides, it follows that

$$\frac{\sigma {u_m}^{\sigma-1} {e_m}^{1-\sigma}(1-a)}{\kappa\gamma(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa} a} = 1$$

and

$$\frac{(1-\sigma)u_m{}^{\sigma}e_m{}^{-\sigma}(1-a)}{(1-\kappa)\gamma(U-u_m)^{\kappa}(E-e_m)^{-\kappa}a}=1.$$

By combining these equations, it can be found out that

$$\frac{\sigma u_m{}^{\sigma-1}e_m{}^{1-\sigma}(1-a)}{\kappa\gamma(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa}a}=\frac{(1-\sigma)u_m{}^{\sigma}e_m{}^{-\sigma}(1-a)}{(1-\kappa)\gamma(U-u_m)^{\kappa}(E-e_m)^{-\kappa}a}$$

$$\Leftrightarrow\ \sigma u_m{}^{\sigma-1}e_m{}^{1-\sigma}(1-a)(1-\kappa)\gamma(U-u_m)^{\kappa}(E-e_m)^{-\kappa}a$$

$$=(1-\sigma)u_m{}^{\sigma}e_m{}^{-\sigma}(1-a)\kappa\gamma(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa}a$$

$$\Leftrightarrow\ \sigma u_m{}^{\sigma-1}e_m{}^{1-\sigma}(1-\kappa)(U-u_m)^{\kappa}(E-e_m)^{-\kappa}$$

$$=(1-\sigma)u_m{}^{\sigma}e_m{}^{-\sigma}\kappa(U-u_m)^{\kappa-1}(E-e_m)^{1-\kappa}$$

$$\Leftrightarrow\ \sigma(1-\kappa)e_m(U-u_m)=(1-\sigma)\kappa u_m(E-e_m). \qquad (B18)$$

Derivation of equation (3.27)

From the equation (3.25) it can be solved that

$$\sigma(1-\kappa)e_m(U-u_m)=(1-\sigma)\kappa u_m(E-e_m)$$

$$\Leftrightarrow\ \sigma(1-\kappa)e_mU-\sigma(1-\kappa)e_mu_m=(1-\sigma)\kappa u_m(E-e_m)$$

$$\Leftrightarrow\ \sigma(1-\kappa)e_mU=(1-\sigma)\kappa(E-e_m)u_m+\sigma(1-\kappa)e_mu_m$$

$$\Leftrightarrow\ \frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}e_mU=\frac{(1-\sigma)\kappa}{(1-\sigma)\kappa}(E-e_m)u_m+\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}e_mu_m$$

$$\Leftrightarrow\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}\ e_mU=\left[\frac{(1-\sigma)\kappa}{(1-\sigma)\kappa}(E-e_m)+\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}e_m\right]u_m$$

$$\Leftrightarrow\ u_m = \frac{\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}Ue_m}{\frac{(1-\sigma)\kappa}{(1-\sigma)\kappa}(E-e_m)+\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}e_m} = \frac{\psi U e_m}{E-e_m+\psi e_m} = \frac{\psi U e_m}{E+(\psi-1)e_m}, \qquad (B19)$$

where $\psi \equiv \frac{\sigma(1-\kappa)}{(1-\sigma)\kappa} > 1$.

Derivation of equation (3.28)

Using the fact that $u_m = \frac{\psi U e_m}{E+(\psi-1)e_m}$, the equation (3.24) becomes

$$(1-\sigma){u_m}^{\sigma}{e_m}^{-\sigma}(1-a) = \gamma(1-\kappa)(U-u_m)^{\kappa}(E-e_m)^{-\kappa}a$$

$$\Leftrightarrow\ (1-\sigma){u_m}^{\sigma}{e_m}^{-\sigma}(1-a) = \gamma(1-\kappa){u_n}^{\kappa}{e_n}^{-\kappa}a$$

$$\Leftrightarrow\ (1-\sigma){u_m}^{\sigma}{e_m}^{-\sigma}(1-a) = \gamma(1-\kappa)\left(\frac{u_n}{e_n}\right)^{\kappa}a$$

$$\Leftrightarrow\ (1-\sigma){u_m}^{\sigma}{e_m}^{-\sigma}(1-a) = \gamma(1-\kappa)\left(\frac{u_m}{\psi e_m}\right)^{\kappa}a$$

$$\Leftrightarrow\ (1-\sigma)\left(\frac{\psi U e_m}{E+(\psi-1)e_m}\right)^{\sigma}\left(\frac{1}{e_m}\right)^{\sigma}(1-a) = \gamma(1-\kappa)\left(\frac{\psi U e_m}{E+(\psi-1)e_m}\right)^{\kappa}\left(\frac{1}{\psi e_m}\right)^{\kappa}a$$

$$\Leftrightarrow\ (1-\sigma)\left(\frac{\psi U}{E+(\psi-1)e_m}\right)^{\sigma}(1-a) = \gamma(1-\kappa)\left(\frac{U}{E+(\psi-1)e_m}\right)^{\kappa}a$$

$$\Leftrightarrow\ (1-\sigma)^{\frac{1}{\sigma}}\left(\frac{\psi U}{E+(\psi-1)e_m}\right)(1-a)^{\frac{1}{\sigma}} = \gamma^{\frac{1}{\sigma}}(1-\kappa)^{\frac{1}{\sigma}}\left(\frac{U}{E+(\psi-1)e_m}\right)^{\frac{\kappa}{\sigma}}a^{\frac{1}{\sigma}}$$

$$\Leftrightarrow\ \psi\left(\frac{U}{E+(\psi-1)e_m}\right)^{1-\frac{\kappa}{\sigma}}\left(\frac{(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma}} = 1$$

$$\Leftrightarrow\ \psi\left(\frac{U}{E+(\psi-1)e_m}\right)^{\frac{\sigma-\kappa}{\sigma}}\left(\frac{(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma}} = 1$$

$$\Leftrightarrow\ \psi^{\frac{\sigma}{\sigma-\kappa}}\left(\frac{U}{E+(\psi-1)e_m}\right)\left(\frac{(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma-\kappa}} = 1$$

$$\Leftrightarrow\ \left(\frac{U}{E+(\psi-1)e_m}\right)\left(\frac{\psi^{\sigma}(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma-\kappa}} = 1$$

$$\Leftrightarrow U\left(\frac{\psi^{\sigma}(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma-\kappa}} = E+(\psi-1)e_m$$

$$\Leftrightarrow\ (\psi-1)e_m = U\left(\frac{\psi^{\sigma}(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma-\kappa}} - E. \qquad (B20)$$

Denoting $z(a) \equiv \left(\frac{\psi^{\sigma}(1-\sigma)(1-a)}{\gamma(1-\kappa)a}\right)^{\frac{1}{\sigma-\kappa}}$, function $(B20)$ can be written as

$$(\psi-1)e_m = z(a)U - E. \qquad (B21)$$

By taking a logarithm of $z(a)$, it becomes

$$\ln\big(z(a)\big) = \frac{1}{\sigma-\kappa}\left[\ln\left(\frac{\psi^{\sigma}(1-\sigma)}{\gamma(1-\kappa)}\right) + \ln(1-a) - \ln(a)\right].$$

Now, the differentiation of $\ln\big(z(a)\big)$ with respect to a gives

$$\frac{d\ln\big(z(a)\big)}{da} = \frac{1}{\sigma-\kappa}\left(-\frac{1}{1-a} - \frac{1}{a}\right). \qquad (B22)$$

Because $\kappa < \sigma$ and $0 < a < 1$, the derivative of $\ln\big(z(a)\big)$ with respect to a is negative. Consequently, also the derivative of $z(a)$ with respect to a is negative.

Derivation of equation (3.29)

For an interior solution to exist, it must be that $e_m \leq E$ and $e_n \leq E$. From the equation (3.28) it can be obtained that $e_m = \frac{z(a)U-E}{\psi-1}$. As a consequence, $e_n = E - e_m = E - \frac{z(a)U-E}{\psi-1} = \frac{E\psi-E-z(a)U+E}{\psi-1} = \frac{E\psi-z(a)U}{\psi-1}$.

Now, the inequalities can be solved:

$$e_m \leq E$$
$$\Leftrightarrow \frac{z(a)U-E}{\psi-1} \leq E$$
$$\Leftrightarrow z(a)U - E \leq E\psi - E$$
$$\Leftrightarrow \frac{z(a)}{\psi} \leq \frac{E}{U} \qquad (B23)$$

and

$$e_n \leq E$$
$$\Leftrightarrow \frac{E\psi-z(a)U}{\psi-1} \leq E$$
$$\Leftrightarrow E\psi - z(a)U \leq E\psi - E$$
$$\Leftrightarrow \frac{E}{U} \leq z(a). \qquad (B24)$$

By combining the equations $(B23)$ and $(B24)$, the following condition for the relative factor endowments can be obtained:

$$\frac{z(a)}{\psi} \leq \frac{E}{U} \leq z(a). \qquad (B25)$$

Derivation of equations (3.30) and (3.31)

Inserting $e_m = \frac{z(a)U-E}{\psi-1}$ into the equation (3.27) results in

$$u_m = \frac{\psi U e_m}{E+(\psi-1)e_m} = \frac{\psi U\left(\frac{z(a)U-E}{\psi-1}\right)}{E+(\psi-1)\left(\frac{z(a)U-E}{\psi-1}\right)} = \left(\frac{\psi U^2 z(a)-\psi UE}{\psi-1}\right)\left(\frac{1}{z(a)U}\right) = \left(\frac{\psi(Uz(a)-E)}{\psi-1}\right)\left(\frac{1}{z(a)}\right)$$

$$= \left(\frac{\psi(\psi-1)e_m}{\psi-1}\right)\frac{1}{z(a)} = \frac{\psi e_m}{z(a)}$$

$$\Leftrightarrow \frac{u_m}{e_m} = \frac{\psi}{z(a)}. \qquad (B26)$$

Because $\psi\frac{u_n}{e_n} = \frac{u_m}{e_m}$, as stated in the equation (3.26),

$$\frac{u_n}{e_n} = \left(\frac{1}{\psi}\right)\left(\frac{u_m}{e_m}\right) = \left(\frac{1}{\psi}\right)\left(\frac{\psi}{z(a)}\right) = \frac{1}{z(a)}. \qquad (B27)$$

Derivation of equation (3.32)

The growth rate in period t is given by

$$g_t = \int_0^1 \frac{A_{i,t}-A_{t-1}}{A_{t-1}}di = \frac{1}{A_{t-1}}\int_0^1 A_{i,t}di - 1,$$

where $A_{i,t} = A_{i,t-1} + \eta\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A_{t-1}^* - A_{t-1}) + \gamma{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}A_{t-1}\right]$ according to equation (3.18). Because A_{t-1} represents the country´s productivity frontier at the end of period $t-1$, it is defined as $A_{t-1} = \int_0^1 A_{i,t-1}di$. The growth rate in period t can thus be written as

$$g_t = \frac{1}{A_{t-1}}\int_0^1 A_{i,t-1}di + \frac{\eta}{A_{t-1}}\int_0^1\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A_{t-1}^* - A_{t-1})\right]di$$

$$+\frac{\eta\gamma}{A_{t-1}}\int_0^1\left[{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}A_{t-1}\right]di - 1$$

$$= 1 + \frac{\eta}{A_{t-1}}\int_0^1\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A_{t-1}^* - A_{t-1})\right]di + \eta\gamma\int_0^1\left[{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}\right]di - 1$$

$$= \frac{\eta}{A_{t-1}}\int_0^1\left[{u_{m,i,t}}^{\sigma}{e_{m,i,t}}^{1-\sigma}(A_{t-1}^* - A_{t-1})\right]di + \eta\gamma\int_0^1\left[{u_{n,i,t}}^{\kappa}{e_{n,i,t}}^{1-\kappa}\right]di. \qquad (B28)$$

At the steady state $A_{t-1} = A_{t-1}^{*}$ and, therefore, the first term on the right-hand side of the equation $(B28)$ vanishes. On the other hand, because of the symmetry of the industries also the i and t subscripts can be ignored at the steady state and the growth rate becomes

$$g = \eta\gamma {u_n}^{\kappa} {e_n}^{1-\kappa}. \qquad (B29)$$

Because $u_m = \frac{\psi U e_m}{E+(\psi-1)e_m}$ and $e_m = \frac{z(a)U-E}{\psi-1}$ according to the equations (3.27) and (3.28) and, in addition, $E = e_m + e_n$ and $U = u_m + u_n$ in equilibrium, the values of e_n and u_n can be calculated to be

$$e_n = E - e_m = E - \frac{z(a)U-E}{\psi-1} = \frac{E(\psi-1)-z(a)U+E}{\psi-1} = \frac{\psi E - z(a)U}{\psi-1} \qquad (B30)$$

and

$$u_n = U - u_m = U - \frac{\psi U e_m}{E+(\psi-1)e_m} = \frac{UE+U(\psi-1)e_m-\psi U e_m}{E+(\psi-1)e_m} = \frac{UE-Ue_m}{E+(\psi-1)e_m} = \frac{Ue_n}{E+(\psi-1)e_m}. \qquad (B31)$$

Now, the growth rate of the economy can be calculated to be

$$g = \eta\gamma {u_n}^{\kappa} {e_n}^{1-\kappa}$$

$$= \eta\gamma \left(\frac{Ue_n}{E+(\psi-1)e_m}\right)^{\kappa} {e_n}^{1-\kappa}$$

$$= \eta\gamma \left(\frac{U}{E+(\psi-1)\left(\frac{z(a)U-E}{\psi-1}\right)}\right)^{\kappa} e_n$$

$$= \eta\gamma \left(\frac{U}{E+z(a)U-E}\right)^{\kappa} \left(\frac{\psi E - z(a)U}{\psi - 1}\right)$$

$$= \eta\gamma z(a)^{-\kappa} \left(\frac{\psi E - z(a)U}{\psi - 1}\right). \qquad (B32)$$

From the equation $(B32)$ it follows that

$$g = \eta\gamma z(a)^{-\kappa} \left(\frac{\psi E - z(a)U}{\psi - 1}\right)$$

$$\Leftrightarrow \frac{g}{\eta\gamma} = \frac{\psi}{\psi - 1} z(a)^{-\kappa} E - \frac{1}{\psi - 1} z(a)^{1-\kappa} U. \qquad (B33)$$

Because $\psi \equiv \frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}$ and as a consequence $\psi - 1 = \frac{\sigma(1-\kappa)}{(1-\sigma)\kappa} - \frac{(1-\sigma)\kappa}{(1-\sigma)\kappa} = \frac{\sigma - \sigma\kappa - \kappa + \sigma\kappa}{(1-\sigma)\kappa} = \frac{\sigma - \kappa}{(1-\sigma)\kappa}$, equation $(B33)$ becomes

$$\frac{g}{\eta\gamma} = \left(\frac{\sigma(1-\kappa)}{(1-\sigma)\kappa}\right)\left(\frac{(1-\sigma)\kappa}{\sigma - \kappa}\right) z(a)^{-\kappa} E - \left(\frac{(1-\sigma)\kappa}{\sigma - \kappa}\right) z(a)^{1-\kappa} U$$

$$\Leftrightarrow \frac{g}{\eta\gamma} = \left(\frac{\sigma(1-\kappa)}{\sigma - \kappa}\right) z(a)^{-\kappa} E - \left(\frac{(1-\sigma)\kappa}{\sigma - \kappa}\right) z(a)^{1-\kappa} U$$

$$\Leftrightarrow \frac{g}{\eta\gamma} = \frac{1}{\sigma - \kappa} [\sigma(1-\kappa) z(a)^{-\kappa} E - (1-\sigma)\kappa z(a)^{1-\kappa} U]. \qquad (B34)$$

Derivation of equation (3.34)

Because $z(a) = \left(\frac{(1-\sigma)\psi^{\sigma}(1-a)}{(1-\kappa)\gamma a}\right)^{\frac{1}{\sigma-\kappa}}$, the derivative of $z(a)$ with respect to a is given by

$$\frac{dz(a)}{da} = \left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \frac{d\left(\left(\frac{1-a}{a}\right)^{\frac{1}{\sigma-\kappa}}\right)}{da}$$

$$= \left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left[-\left(\frac{1}{\sigma-\kappa}\right)(1-a)^{\frac{1}{\sigma-\kappa}-1}a^{-\frac{1}{\sigma-\kappa}} + (1-a)^{\frac{1}{\sigma-\kappa}}\left(-\frac{1}{\sigma-\kappa}\right)a^{-\frac{1}{\sigma-\kappa}-1}\right]$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left[(1-a)^{\frac{1}{\sigma-\kappa}}a^{-\frac{1}{\sigma-\kappa}}((1-a)^{-1} + a^{-1})\right]$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left[(1-a)^{\frac{1}{\sigma-\kappa}}a^{-\frac{1}{\sigma-\kappa}}\left(\frac{a}{a(1-a)} + \frac{1-a}{a(1-a)}\right)\right]$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left[(1-a)^{\frac{1}{\sigma-\kappa}}a^{-\frac{1}{\sigma-\kappa}}\left(\frac{1}{a(1-a)}\right)\right]$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} (1-a)^{\frac{1-\sigma+\kappa}{\sigma-\kappa}}a^{-\frac{1-\sigma+\kappa}{\sigma-\kappa}}$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left(\frac{1-a}{a}\right)^{\frac{1-\sigma+\kappa}{\sigma-\kappa}}$$

$$= \left(-\frac{1}{\sigma-\kappa}\right)\left(\frac{(1-\sigma)\psi^{\sigma}}{(1-\kappa)\gamma}\right)^{\frac{1}{\sigma-\kappa}} \left(\frac{1}{a} - 1\right)^{\frac{1-\sigma+\kappa}{\sigma-\kappa}} \qquad (B35)$$

Derivation of equation (3.36)

An individual's utility function is given by

$$U(x_t,\, s_{t+1}) = \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1-\lambda)\ln(w_{t+1}(1 + x_t\theta) - s_{t+1}) + \lambda \ln(s_{t+1}R^*_{t+2}),$$

as in the equation (3.35). By maximizing the utility function with respect to the savings in period $t+1$, savings become a constant fraction λ of the second period income:

$$\frac{dU(x_t,s_{t+1})}{ds_{t+1}} = -\frac{1-\lambda}{w_{t+1}(1+x_t\theta)-s_{t+1}} + R^*_{t+2}\frac{\lambda}{s_{t+1}R^*_{t+2}} = 0$$

$$\Leftrightarrow \frac{1-\lambda}{w_{t+1}(1+x_t\theta)-s_{t+1}} = \frac{\lambda}{s_{t+1}}$$

$$\Leftrightarrow s_{t+1}(1-\lambda) = \lambda(w_{t+1}(1+x_t\theta) - s_{t+1})$$

$$\Leftrightarrow s_{t+1} - \lambda s_{t+1} = \lambda\big(w_{t+1}(1+x_t\theta)\big) - \lambda s_{t+1}$$

$$\Leftrightarrow s_{t+1} = \lambda\big(w_{t+1}(1+x_t\theta)\big). \qquad (B36)$$

Derivation of equation (3.37)

Because $s_{t+1} = \lambda\big(w_{t+1}(1+x_t\theta)\big)$, as shown by the equation $(B36)$, the utility function given in the equation (3.35) can be rewritten as

$$U(x_t, s_{t+1}) = U\left(x_t, \lambda\big(w_{t+1}(1+x_t\theta)\big)\right)$$

$$= \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1-\lambda)\ln\big(w_{t+1}(1+x_t\theta) - \lambda w_{t+1}(1+x_t\theta)\big)$$
$$+\lambda\ln(\lambda w_{t+1}(1+x_t\theta)R^*_{t+2})$$

$$= \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1-\lambda)\ln\big((1-\lambda)w_{t+1}(1+x_t\theta)\big)$$
$$+\lambda\ln(\lambda w_{t+1}(1+x_t\theta)R^*_{t+2})$$

$$= \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1-\lambda)\ln(1-\lambda) + (1-\lambda)\ln\big(w_{t+1}(1+x_t\theta)\big)$$
$$+\lambda\ln(\lambda) + \lambda\ln\big(w_{t+1}(1+x_t\theta)\big) + \lambda\ln(R^*_{t+2})$$

$$= \ln(w_t - \widehat{\mu_t} - x_t c w_t) + (1-\lambda)\ln(1-\lambda) + \ln\big(w_{t+1}(1+x_t\theta)\big)$$
$$-\lambda\ln\big(w_{t+1}(1+x_t\theta)\big) + \lambda\ln(\lambda) + \lambda\ln\big(w_{t+1}(1+x_t\theta)\big) + \lambda\ln(R^*_{t+2})$$

$$= \ln(w_t - \widehat{\mu_t} - x_t c w_t) + \ln\big(w_{t+1}(1+x_t\theta)\big) + \lambda\ln(R^*_{t+2}) + \lambda\ln(\lambda)$$
$$+(1-\lambda)\ln(1-\lambda). \qquad (B37)$$

Derivation of equation $(\mathbf{3.40})$

When deciding whether to invest in education, an individual compares his expected life time utility with and without an educational investment. If the expected utility with education exceeds the utility without education, that is $V(1) > V(0)$, it is profitable for an individual to invest in education. Solving the inequality gives the critical value for the cost of education as

$$
\begin{aligned}
& V(1) > V(0) \\
\Leftrightarrow\ & \ln(w_t - \widehat{\mu_t} - cw_t) + \ln\big(w_{t+1}(1+\theta)\big) + \lambda \ln(R^*_{t+2}) + I \\
& > \ln(w_t - \widehat{\mu_t}) + \ln(w_{t+1}) + \lambda \ln(R^*_{t+2}) + I \\
\Leftrightarrow\ & \ln(w_t - \widehat{\mu_t} - cw_t) + \ln(w_{t+1}) + \ln(1+\theta) > \ln(w_t - \widehat{\mu_t}) + \ln(w_{t+1}) \\
\Leftrightarrow\ & \ln\big((1+\theta)(w_t - \widehat{\mu_t} - cw_t)\big) > \ln(w_t - \widehat{\mu_t}) \\
\Leftrightarrow\ & w_t - \widehat{\mu_t} - cw_t + \theta w_t - \theta\widehat{\mu_t} - \theta c w_t > w_t - \widehat{\mu_t} \\
\Leftrightarrow\ & \theta(w_t - \widehat{\mu_t}) > (1+\theta) c w_t \\
\Leftrightarrow\ & \left(\frac{\theta}{1+\theta}\right)\left(\frac{w_t - \widehat{\mu_t}}{w_t}\right) > c. \qquad (B38)
\end{aligned}
$$

Appendix C

Derivation of equation $(\mathbf{4.3})$

The critical value for the cost of education can again be found out by solving the inequality $V(1) > V(0)$. However, a high skilled individual must now take into consideration that with a possibility p he will receive a wage w^*_{t+1} in the second period of his life. Hereby, the utility function of an educated individual is modified into the form

$$
\begin{aligned}
V(1) = \ln(w_t - \widehat{\mu_t} - cw_t) + (1-p)\ln\big(w_{t+1}(1+\theta)\big) + p\ln\big(w^*_{t+1}(1+\theta)\big) \\
+\lambda \ln(R^*_{t+2}) + I
\end{aligned}
$$

as in the equation (4.2). The utility function of an uneducated individual remains the same as before in section 3.3. The critical value for the cost of education with the emigration possibility for educated individuals now becomes

$$V(1) > V(0)$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) + (1-p)\ln\big(w_{t+1}(1+\theta)\big) + p\ln\big(w^*_{t+1}(1+\theta)\big) + \lambda\ln(R^*_{t+2}) + I$$
$$> \ln(w_t - \widehat{\mu_t}) + \ln(w_{t+1}) + \lambda\ln(R^*_{t+2}) + I$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) + (1-p)\ln(w_{t+1}) + (1-p)\ln(1+\theta) + p\ln(w^*_{t+1})$$
$$+p\ln(1+\theta) > \ln(w_t - \widehat{\mu_t}) + \ln(w_{t+1})$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) + \ln(w_{t+1}) - p\ln(w_{t+1}) + \ln(1+\theta) - p\ln(1+\theta) + p\ln(w^*_{t+1})$$
$$+p\ln(1+\theta) > \ln(w_t - \widehat{\mu_t}) + \ln(w_{t+1})$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) - p\ln(w_{t+1}) + \ln(1+\theta) + p\ln(w^*_{t+1}) > \ln(w_t - \widehat{\mu_t})$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) - \ln({w_{t+1}}^p) + \ln(1+\theta) + \ln((w^*_{t+1})^p) > \ln(w_t - \widehat{\mu_t})$$

$$\Leftrightarrow \ln(w_t - \widehat{\mu_t} - cw_t) - \ln(w_t - \widehat{\mu_t}) > \ln({w_{t+1}}^p) - \ln((w^*_{t+1})^p) - \ln(1+\theta)$$

$$\Leftrightarrow \ln\left(\frac{w_t - \widehat{\mu_t} - cw_t}{w_t - \widehat{\mu_t}}\right) > \ln\left(\frac{{w_{t+1}}^p}{(w^*_{t+1})^p}\right) - \ln(1+\theta)$$

$$\Leftrightarrow \ln\left(\left(\frac{w_t - \widehat{\mu_t}}{w_t - \widehat{\mu_t}}\right) - \frac{cw_t}{w_t - \widehat{\mu_t}}\right) > \ln({\omega_{t+1}}^p) - \ln(1+\theta)$$

$$\Leftrightarrow \ln\left(1 - \frac{cw_t}{w_t - \widehat{\mu_t}}\right) > \ln\left(\frac{{\omega_{t+1}}^p}{1+\theta}\right)$$

$$\Leftrightarrow 1 - \frac{cw_t}{w_t - \widehat{\mu_t}} > \frac{{\omega_{t+1}}^p}{1+\theta}$$

$$\Leftrightarrow -\frac{cw_t}{w_t - \widehat{\mu_t}} > \frac{{\omega_{t+1}}^p}{1+\theta} - 1$$

$$\Leftrightarrow -cw_t > \left(\frac{{\omega_{t+1}}^p}{1+\theta} - 1\right)(w_t - \widehat{\mu_t})$$

$$\Leftrightarrow -c > \left(\frac{{\omega_{t+1}}^p}{1+\theta} - 1\right)\left(\frac{w_t - \widehat{\mu_t}}{w_t}\right)$$

$$\Leftrightarrow c < \left(1 - \frac{{\omega_{t+1}}^p}{1+\theta}\right)\left(1 - \frac{\mu w^*_t}{w_t}\right)$$

$$\Leftrightarrow c < \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{{\omega_{t+1}}^p}{1+\theta}\right). \qquad (C1)$$

The proof of equation $(\mathbf{4.5})$

The proportion of individuals acquiring education is higher with the possibility of emigration for highly skilled individuals than without the possibility of emigration because comparing equations (3.42) and (4.4) gives

$$\left(1-\frac{\mu}{\omega_t}\right)\left(1-\frac{{\omega_{t+1}}^p}{1+\theta}\right) > \left(1-\frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)$$

$$\Leftrightarrow \frac{1+\theta}{1+\theta} - \frac{{\omega_{t+1}}^p}{1+\theta} > \frac{\theta}{1+\theta}$$

$$\Leftrightarrow 1+\theta-{\omega_{t+1}}^p > \theta$$

$$\Leftrightarrow 1-{\omega_{t+1}}^p > 0. \qquad (C2)$$

Equation $(C2)$ is valid with all $\omega_{t+1} < 1$ and $0 < p < 1$.

Derivation of equation $(\mathbf{4.6})$

By differentiating the proportion of highly skilled individuals staying in the source country of emigration with respect to the emigration possibility, it can be seen that the average level of human capital depends on p in the following way:

$$\frac{d\pi_{t+1}}{dp} = \frac{\left((-1)\hat{c_t}+(1-p)\frac{d\hat{c_t}}{dp}\right)(1-p\hat{c_t})-\left(\left((-1)\hat{c_t}-p\frac{d\hat{c_t}}{dp}\right)(1-p)\hat{c_t}\right)}{(1-p\hat{c_t})^2}$$

$$= \frac{\left(-\hat{c_t}+(1-p)\frac{d\hat{c_t}}{dp}\right)(1-p\hat{c_t})-\left(\left(-\hat{c_t}-p\frac{d\hat{c_t}}{dp}\right)(\hat{c_t}-p\hat{c_t})\right)}{(1-p\hat{c_t})^2}$$

$$= \frac{-\hat{c_t}(1-p\hat{c_t})+(1-p)\frac{d\hat{c_t}}{dp}(1-p\hat{c_t})-\left(-\hat{c_t}(\hat{c_t}-p\hat{c_t})-p\frac{d\hat{c_t}}{dp}(\hat{c_t}-p\hat{c_t})\right)}{(1-p\hat{c_t})^2}$$

$$= \frac{-\hat{c}_t + p\hat{c}_t^{\,2} + \left(\frac{d\hat{c}_t}{dp} - p\frac{d\hat{c}_t}{dp}\right)(1 - p\hat{c}_t) - \left(-\hat{c}_t^{\,2} + p\hat{c}_t^{\,2} - p\hat{c}_t\frac{d\hat{c}_t}{dp} + p^2\hat{c}_t\frac{d\hat{c}_t}{dp}\right)}{(1 - p\hat{c}_t)^2}$$

$$= \frac{-\hat{c}_t + p\hat{c}_t^{\,2} + \frac{d\hat{c}_t}{dp} - p\frac{d\hat{c}_t}{dp} - p\hat{c}_t\frac{d\hat{c}_t}{dp} + p^2\hat{c}_t\frac{d\hat{c}_t}{dp} + \hat{c}_t^{\,2} - p\hat{c}_t^{\,2} + p\hat{c}_t\frac{d\hat{c}_t}{dp} - p^2\hat{c}_t\frac{d\hat{c}_t}{dp}}{(1 - p\hat{c}_t)^2}$$

$$= \frac{\frac{d\hat{c}_t}{dp} - p\frac{d\hat{c}_t}{dp} - \hat{c}_t + \hat{c}_t^{\,2}}{(1 - p\hat{c}_t)^2} = \frac{(1-p)\frac{d\hat{c}_t}{dp} - \hat{c}_t(1 - \hat{c}_t)}{(1 - p\hat{c}_t)^2}. \qquad (C3)$$

Derivation of equation (4.8)

The derivative of the average level of human capital in the source country of emigration with respect to the emigration possibility at $p = 0$ is given by

$$\frac{d\pi_{t+1}}{dp}\Big|_{p=0} = \frac{d\hat{c}_t}{dp} - \hat{c}_t(1 - \hat{c}_t).$$

Because

$$\hat{c}_t = \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{\omega_{t+1}{}^{p}}{1+\theta}\right)$$

and

$$\frac{d\hat{c}_t}{dp} = \left(1 - \frac{\mu}{\omega_t}\right)\left(-\frac{1}{1+\theta}\right)\omega_{t+1}{}^{p} \ln \omega_{t+1},$$

the derivative of the average level of human capital with respect to the emigration possibility is positive at $p = 0$ if and only if

$$\left(1 - \frac{\mu}{\omega_t}\right)\left(-\ln \omega_{t+1} \frac{1}{1+\theta}\right) > \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{1}{1+\theta}\right)\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(1 - \frac{1}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; -\ln \omega_{t+1} \frac{1}{1+\theta} > \left(\frac{\theta}{1+\theta}\right)\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; -\ln \omega_{t+1} > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; -\ln \frac{w_{t+1}}{w_{t+1}^*} > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; -(\ln w_{t+1} - \ln w_{t+1}^*) > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; \ln w_{t+1}^* - \ln w_{t+1} > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]$$

$$\Leftrightarrow \; \ln\left(\frac{w_{t+1}^*}{w_{t+1}}\right) > \theta\left[1 - \left(1 - \frac{\mu}{\omega_t}\right)\left(\frac{\theta}{1+\theta}\right)\right]. \qquad (C4)$$