

SUOMEN GEODEETTISEN LAITOKSEN JULKAISUJA  
VERÖFFENTLICHUNGEN DES FINNISCHEN GEODÄTISCHEN INSTITUTES  
PUBLICATIONS OF THE FINNISH GEODETIC INSTITUTE

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**DIGITAL ELEVATION MODEL ERROR  
IN TERRAIN ANALYSIS**

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Academic Dissertation in Geography

Faculty of Science  
University of Helsinki

*To be presented, with the permission of the Faculty of Science of the University of Helsinki,  
for public criticism in the Auditorium XII of the Main Building (Unioninkatu 34) on  
November 21<sup>st</sup>, 2006, at 12 noon.*

KIRKKONUMMI 2006

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ISBN-13: 978-951-711-257-4 (paperback)

ISBN-10: 951-711-257-2 (paperback)

ISBN-13: 978-952-10-3350-6 (PDF, <http://ethesis.helsinki.fi/>)

ISBN-10: 952-10-3350-9 (PDF, <http://ethesis.helsinki.fi/>)

ISSN 0085-6932

Helsinki University Press  
Helsinki 2006

# ABSTRACT

Digital elevation models (DEMs) have been an important topic in geography and surveying sciences for decades due to their geomorphological importance as the reference surface for gravitation-driven material flow, as well as the wide range of uses and applications. When DEM is used in terrain analysis, for example in automatic drainage basin delineation, errors of the model collect in the analysis results. Investigation of this phenomenon is known as error propagation analysis, which has a direct influence on the decision-making process based on interpretations and applications of terrain analysis. Additionally, it may have an indirect influence on data acquisition and the DEM generation. The focus of the thesis was on the fine toposcale DEMs, which are typically represented in a 5–50m grid and used in the application scale 1:10 000–1:50 000.

The thesis presents a three-step framework for investigating error propagation in DEM-based terrain analysis. The framework includes methods for visualising the morphological gross errors of DEMs, exploring the statistical and spatial characteristics of the DEM error, making analytical and simulation-based error propagation analysis and interpreting the error propagation analysis results. The DEM error model was built using geostatistical methods.

The results show that appropriate and exhaustive reporting of various aspects of fine toposcale DEM error is a complex task. This is due to the high number of outliers in the error distribution and morphological gross errors, which are detectable with presented visualisation methods. In addition, the use of global characterisation of DEM error is a gross generalisation of reality due to the small extent of the areas in which the decision of stationarity is not violated. This was shown using exhaustive high-quality reference DEM based on airborne laser scanning and local semivariogram analysis. The error propagation analysis revealed that, as expected, an increase in the DEM vertical error will increase the error in surface derivatives. However, contrary to expectations, the spatial autocorrelation of the model appears to have varying effects on the error propagation analysis depending on the application. The use of a spatially uncorrelated DEM error model has been considered as a 'worst-case scenario', but this opinion is now challenged because none of the DEM derivatives investigated in the study had maximum variation with spatially uncorrelated random error. Significant performance improvement was achieved in simulation-based error propagation analysis by applying process convolution in generating realisations of the DEM error model. In addition, typology of uncertainty in drainage basin delineations is presented.

*Keywords:* digital terrain modelling, error propagation analysis, simulation, process convolution

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# ACKNOWLEDGEMENTS

The work has been carried out at the Finnish Geodetic Institute (FGI), Department of Geoinformatics and Cartography, which offered excellent facilities for the research and strong support for my postgraduate studies. I gratefully acknowledge my supervisor Professor Tapani Sarjakoski for introducing me to the world of digital terrain modelling and error propagation analysis. I also truly appreciate my supervisor's patience during the project, since we both know that not everything went as was originally planned. I express my gratitude to the FGI's director general, Professor Risto Kuittinen for his support and interest in my DEM research. Special thanks are reserved for Professor Markku Löytönen (Department of Geography, University of Helsinki), whose contribution during the final straight of the project has been really valuable.

The Ministry of Forestry and Agriculture and the Academy of Finland are acknowledged for their financial support.

Professor Risto Kalliola (Department of Geography, University of Turku) and Assistant Professor Ashton Shortridge (Department of Geography, Michigan State University) are warmly acknowledged for reviewing the thesis.

I thank Professor Juha Hyypä (Department of Remote Sensing and Photogrammetry, FGI) for organising the LIDAR campaign and Dr. Matti Ollikainen (Department of Geodesy and Geodynamics, FGI), Harri Kaartinen (Department of Remote Sensing and Photogrammetry, FGI) and Eero Ahokas (Department of Remote Sensing and Photogrammetry, FGI) for all their help during the data processing stage. The financial contribution to the LIDAR campaign by the National Land Survey of Finland is gratefully acknowledged.

I express my gratitude to Dr. Niina Käyhkö (Department of Geography, University of Turku) and the project 'Landscapes of the past, present

and future' (funded by the Academy of Finland, SA37121) for providing the raw data used in **II**.

Professor Matti Tikkanen (Department of Geography, University of Helsinki), Professor Jukka Käyhkö (Department of Geography, University of Turku), Dr. Olli Ruth (Department of Geography, University of Helsinki), Matti Ekholm (Finnish Environment Institute) and Jaakko Suikkanen (Finnish Environment Institute) are acknowledged for the manual drainage basin delineations in **III**.

I warmly thank all the past and present staff of the Department of Geoinformatics and Cartography, FGI. Special thanks go to Lic.Sc. (Tech.) Annu-Maaria Nivala, Dr. Tiina Sarjakoski, Lic.Sc. (Tech.) Lassi Lehto, Jaakko Kähkönen, Kirsti Filén, Dr. Antti Jakobsson (National Land Survey of Finland), Dr. Lars Harrie (University of Lund) and Dr. Karel Stanek (Masaryk University). In addition, without the help of our librarian, Kati Haanpää, things would have been much more complicated. Thanks are extended to all colleagues at the FGI, especially the coffee and tea club, whose contribution to this thesis cannot be emphasized too much.

I am deeply grateful for all the support provided by my parents, Ritva and Rauno Oksanen, brother Mikko Oksanen and his wife Minna Hokkanen. I would also like to express my appreciation to my in-laws Maarit and Timo Kivelä, Riikka Kivelä and Kimmo Kivelä. Thanks for all the encouragement!

My sincerest thanks go to my loving wife, Dr. Hanna Kivelä, with whom I have shared my highs and lows during this demanding process. I am also very grateful to our children, Atte and Lotta, who have given me new perspectives on life in general and reorganised my personal order of priorities completely.

Kirkkonummi, August 23rd, 2006  
Juha Oksanen

# ORIGINAL PUBLICATIONS

The thesis is based on the original articles, referred to in the text by their Roman numerals:

- I** OKSANEN J (2003). Tracing the Gross Errors of DEM - Visualisation Techniques for Preliminary Quality Analysis. *Proceedings of the 21st International Cartographic Conference, August 10-16, 2003, Durban, South Africa* (CD-ROM), 2410 - 2415.
- II** OKSANEN J and T SARJAKOSKI (2005). Error propagation of DEM-based surface derivatives. *Computers & Geosciences*, 31: 8, 1015-1027.
- III** OKSANEN J and T SARJAKOSKI (2005). Error propagation analysis of DEM-based drainage basin delineation. *International Journal of Remote Sensing*, 26: 14, 3085-3102.
- IV** OKSANEN J and T SARJAKOSKI (2006). Uncovering the statistical and spatial characteristics of fine toposcale DEM error. *International Journal of Geographical Information Science*, 20: 4, 345-369.

Previously unpublished results are presented.

In **II**, Oksanen contributed to the designing of the study and carried out the derivation of the analytical error propagation equations, the software tool development, data processing and analysis, interpreted the results and wrote the article.

In **III**, Oksanen contributed to the designing of the study and carried out the software tool development, data processing and analysis, interpreted the results and wrote the article.

In **IV**, Oksanen designed the study, carried out the data processing and analysis, interpreted the results and wrote the article.

In **II-IV**, the co-author acted as a supervisor.

The original publications have been reprinted with the kind permission of International Cartographic Association (**I**), Elsevier (**II**) and Taylor & Francis (**III**, **IV**).

# ABBREVIATIONS

AHN	Actual height model of the Netherlands (Actueel Hoogtebestand Nederland)
BB	Boolean drainage basin
BD	Boolean drainage divide
BKG	Federal Agency for Cartography and Geodesy, Germany (Bundesamt für Kartographie und Geodäsie)
CCDF	Conditional cumulative distribution function
DEM	Digital elevation model
DLG	Digital line graph
DOM	High precision digital surface model by the Swisstopo (digitales Oberflächenmodell)
DSM	Digital surface model
DTM	Digital terrain model
DTM-AV	Digital terrain model of the cadastral surveying by the Swisstopo (digitales Terrainmodell der Amtlichen Vermessung)
ETRS89	European terrestrial reference system 89
FGI	Finnish Geodetic Institute
GIS	Geographical information system
GISc	Geographical information science
GPS	Global positioning system
HIFI	Height interpolation by finite elements
InSAR	Interferometric synthetic aperture radar
IQR	Inter-quartile range
LIDAR	Light detection and ranging
LIDIC	Linear inverse distance interpolation from contours
LM	National Land Survey of Sweden (Lantmäteriet)
NED	National elevation dataset by the USGS
NLS	National Land Survey of Finland
NMA	National mapping agency
OS	Ordnance Survey
PB	Probable drainage basin
PD	Probable drainage divide
PZD	Probable zone of the drainage divide
RGB	Red-green-blue colour system
RMSE	Root mean squared error
RTK	Real time kinematic
SGS	Sequential Gaussian simulation
TDB	The topographic database by the NLS
TIN	Triangulated irregular network
USGS	United States Geological Survey

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# CHAPTER 1

## INTRODUCTION

### 1.1 Digital representation of terrain

Digital representation of terrain has been an important focus in geography and surveying sciences for many decades. Although there are numerous methods for representing terrain in digital form (Figure 1), the most frequently used methods in geographical information science (GISc) are geometrical representations with regularly or irregularly distributed points, which may be supported by feature points and lines. The earliest definition of a *digital terrain*

*model* (DTM) as ‘a statistical representation of the continuous surface of the ground by a large number of selected points with known xyz coordinates in an arbitrary coordinate field’ dates back to the 1950s (Miller & Laflamme 1958). However, in the following decades the use of the concepts *digital elevation model* (DEM), DTM and, more recently, *digital surface model* (DSM) has been confusing (Table 1). Nowadays, it is apparent that these terms have a context-dependent implication and that there may be differences in meaning from one country to

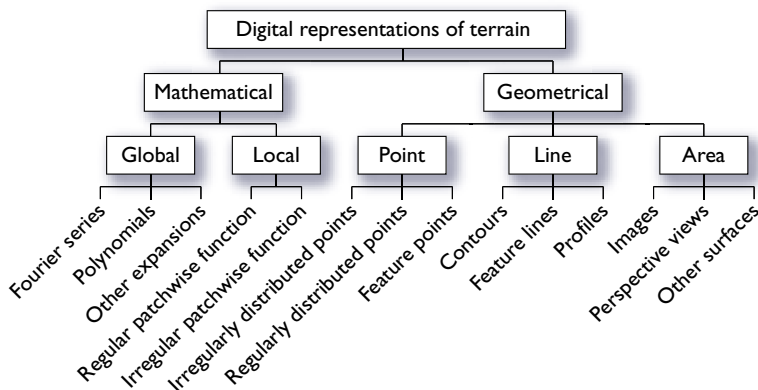


Figure 1. Digital representations of terrain (Li et al. 2005).

Table 1. Definitions of DEM, DSM and DTM.

Term	Definition
Digital elevation model (DEM)	<p>A) Generic term covering digital topographic (and bathymetric) data in all its various forms as well as the method(s) for interpreting implicitly the elevations between observations. Normally implies elevations of bare earth without vegetation and buildings, but may include other manmade features, such as road embankments. Elevations of hydrological features (e.g. lakes and rivers) normally imply a free water surface (Maune et al. 2001). DEM is the most fundamental component of DTM (Li et al. 2005). As used in this thesis.</p> <p>B) Numeric representation of a topographic surface arranged as a set of regularly spaced points, normally in a square grid or hexagonal pattern, expressed as three-dimensional coordinates (Kennie &amp; Petrie 1990, USGS 2000).</p>
Digital terrain model (DTM)	<p>A) Umbrella concept covering models of elevations and other geographical elements and natural features, such as rivers and other break lines. May also include derived data about the terrain, such as slope, aspect, curvature, visibility, etc. (Kennie &amp; Petrie 1990, Li et al. 2005). As used in this thesis.</p> <p>B) DTM is a synonym of bare-earth DEM (Maune et al. 2001).</p>
Digital surface model (DSM)	Model depicting elevations of the top of reflective surfaces, such as buildings and vegetation (Maune et al. 2001). Used widely in works related to airborne laser scanning.

another (Maune et al. 2001). Throughout the thesis DEM is considered the most important component of DTM (Li et al. 2005).

Elevation has been recognized as one of the most essential and fundamental variables in GIS (Atkinson 2002). The reasons for this lie in the geomorphological importance of DEM as the reference surface for gravitation-driven material flow, as well as the wide range of DEM uses and applications (Table 2).

As with any other geospatial dataset, DEMs are produced at a number of spatial scales (Table 3), each of which has its own cost-effective techniques for data acquisition. While the DEMs with the highest accuracy

and correspondingly the smallest spatial extent, are typically based on tachymeter or GPS field surveys, decreased production costs can be achieved for larger areas but at a lower accuracy by using photogrammetry, map digitization or interferometric synthetic aperture radar (InSAR) (Li et al. 2005). Since the mid 1990s, topographic light detection and ranging (LIDAR), also known as airborne laser scanning, has offered a combination of the aforementioned extremes: high accuracy with rapid data collection and high cost-effectiveness (Fowler 2001). For a detailed description of the methods, readers are referred to Li et al. (2005) and Maune (2001).

This thesis focuses on the fine topographic DEMs, which are typically produced by national mapping agencies (NMAs) from topographic data (e.g. USGS 2000, OS 2001, NLS 2002b, BKG 2004). They have a regular lattice

representation and are used at scales 1:5000–1:50000 (Hutchinson & Gallant 2000). So far, fine topographic DEMs have been the most detailed countrywide elevation databanks and are characteristically the basis for landscape level

Table 2. Examples of common DEM uses and applications (modified from Heuvelink 1999, Gallant & Wilson 2000 and Sulebak 2000). Due to the vast number of articles on DEM applications, the given references should be treated as representative examples. For complementary lists, see e.g. Weibel & Heller (1991), Florinsky (1998a), Burrough & McDonnell (1998), Hutchinson & Gallant (1999) and Pike (2002).

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#### DEM uses and applications

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1. Country-wide elevation databanks, which may be primary (based on direct elevation measurements), such as AHN of the Netherlands (AHN 2005) and DOM/DTM-AV from the areas of Switzerland lying below 2000 m above mean sea level (Swisstopo 2006), or secondary (based on map data), as in most European countries (e.g. OS 2001, LM 2002, NLS 2002b, BKG 2004)
  2. Orthorectification of aerial photographs (Petrie 1990)
  3. Cut and fill analysis in civil engineering (Brown & Arbogast 1999, Aruga et al. 2005)
  4. Cartographic purposes, such as contour maps, hypsographic maps and relief shadings (Jenny 2001, Oksanen & Sarjakoski 2005)
  5. 3-D visualisation (Patterson 2001, Lisle 2006)
  6. Geomorphological analysis and modelling (Rowbotham & Dudycha 1998, Kuhni & Pffiffner 2001, Hjort & Luoto 2006)
  7. Biogeographical analysis and modelling (Vuorela 2001, Luoto et al. 2002)
  8. Hydrological and hydraulic applications (Maidment 1993, Moore 1996, Tucker et al. 2001, Alho et al. 2005)
  9. Studies of landscape dynamics (Mitasova et al. 2005, Wangenstein et al. 2006)
  10. Climate and climate impact studies (New et al. 1999, Dirnböck et al. 2003)
  11. Water and wildlife management (Hunsaker & Levine 1995, Dettmers & Bart 1999)
  12. Geological applications (Borga et al. 1998, Chorowicz et al. 1999, Van Dijk et al. 2000)
  13. Agricultural applications (Pilesjö et al. 2006)
  14. Paleogeographical mapping (Drzyzga et al. 2002, Tikkanen & Oksanen 2002, Mäkiaho 2005)
  15. Planning support, e.g. cross-country visibility analysis (Fisher 1991, Ehlschlaeger 1998), road and dam planning, automatic drainage basin delineation (Fried et al. 2000, Turcotte et al. 2001, Vogt et al. 2003) and flood-risk analysis (Garrote & Bras 1995, Cobby et al. 2001, Sane et al. 2006)
  16. Geophysical models (Virtanen 2001)
  17. Telecommunication planning (Wells & Shears 1996)
  18. Hydrological and meteorological services (Yates et al. 2000, Jasper et al. 2002)
-

terrain analysis (Wechsler 2003). However, the situation may change when models based on topographic LIDAR overlap the market for DEMs.

## 1.2 Terrain analysis

Geomorphometry, the quantitative analysis of terrain forms, combines elements from a number of fields, among them earth sciences, engineering and other applied sciences, mathematics, statistics and computer science (Figure 2). Digital geomorphometry based on the use of DEMs is nowadays covered by the concept of terrain analysis (Wilson & Gallant 2000). Even though the roots of geomorphometry date back to the 1800s and the founders of academic geography, Alexander von Humboldt

and Carl Ritter (Pike 2002), the revolution in the discipline during the last 35 years can be explained by advances in computing capacity and, especially, the mass production of DEMs (Pike 1995, Pike 2000).

Topographic attributes have been classified according to their characteristics and spatial extent. Typically, primary attributes (e.g. slope, aspect and curvature) that are derived directly from the DEM are distinguished from secondary attributes (e.g. indices for topographic wetness and radiation), which are based on two or more primary attributes and possibly on physically based or empirically derived indices (Wilson & Gallant 2000). The calculation of the primary attributes is typically based on numerical approximations of the first and the second partial derivatives of the DEM heights in the directions of the lattice axes. Concise

Table 3. Typical application scales of digital terrain modelling (modified from Hutchinson & Gallant 2000).

Scale	DEM Resolution	Examples of Data Sources	Examples of Applications
Microscale	0.1 – 5 m	Field survey LIDAR	Civil engineering Large-scale mapping Orthorectification of aerial photographs Detailed hydrological modelling Precision agriculture
Fine toposcale	5 – 50 m	Photogrammetry Map digitization InSAR LIDAR	Spatial hydrological modelling Spatial analysis of soil properties Orthorectification and radiometric corrections of aerial photographs
Coarse toposcale	50 – 200 m	Photogrammetry Map digitization InSAR	Broad scale hydrological modelling Subcatchment analysis for lumped parameter hydrological modelling and assessment of biodiversity
Mesoscale	200 m – 5 km	Map digitization	Elevation-dependent representations of surface temperature and precipitation
Macroscale	5 – 500 km	Map digitization	Global circulation models

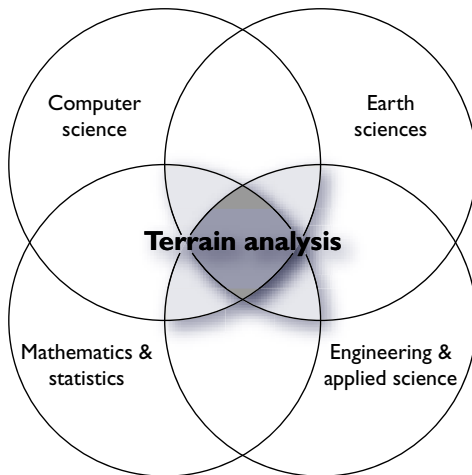


Figure 2. Terrain analysis positioned between other sciences (Pike 1995).

reviews of the approximation methods are represented in Gallant & Wilson (2000) and Zhou & Liu (2004a). Approximation is usually done within the calculation window of  $3 \times 3$  lattice points, but the window may be of any size that produces multi-scale parameterisation of the terrain (Wood 1996a). For a comprehensive review of the primary and secondary attributes, readers are directed to Florinsky (1998a), Wilson & Gallant (2000) and Shary et al. (2002). From the error propagation analysis perspective, it is useful to group surface

derivatives into local (constrained) and global (unconstrained) categories. In local derivatives, such as slope and aspect, the size of the calculation window is explicitly defined. In contrast, global derivatives are based on unpredictable and far-reaching interactions between DEM heights. For example, automatic drainage basin delineation is based on iterative processing of flow direction data and the final extent of the result is unknown until the analysis is finished (Jenson & Domingue 1988).

### 1.3 DEM uncertainty and error propagation analysis

The thesis follows the taxonomy in which 'error' is considered to be the well-defined and measurable part of 'uncertainty' (Figure 3). This choice is justifiable from the DEM quality perspective because the semantics of elevation do not suffer from the conceptual ambiguities common in, for example, defining the error in area class maps.

The tasks typically involved in digital terrain modelling, from elevation data capture to geospatial applications, tend to introduce uncertainty (Figure 4). Understanding the characteristics of this uncertainty is therefore essential to ensure scientific progress in GIS theory and tool development (Chrisman 1991). From

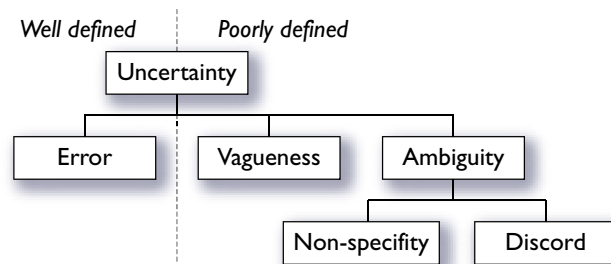


Figure 3. Taxonomy of uncertainty in GISc (adapted from Fisher 1999, 2003).

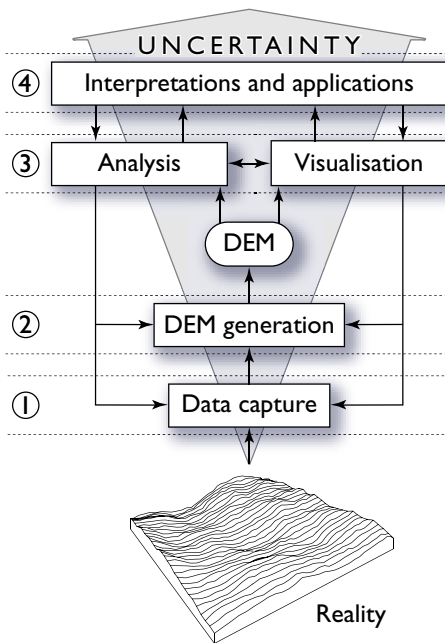


Figure 4. The main tasks of digital terrain modelling and the phases (1-4) in which uncertainty is introduced into the decision-making based on terrain analysis (modified from Hutchinson & Gallant 2000).

the modelling perspective, uncertainty in fine topographic DEMs originates from two sources: 1) gross, systematic and random errors in the lattice points, and 2) accuracy loss due to lattice representation of the terrain (Li et al. 2005), which sets the lower bound on overall accuracy of the DEM (Chrisman 1991). Further refinement is possible from the process perspective. Error in lattice points originates primarily at phase 1 (Figure 4), which covers random and human error in data acquisition including errors caused by measurement equipment (e.g. tachymeters, GPS, stereoplotters or LIDAR). There may also be errors due to cloud or forest cover, instable remote sensing equipment, the Earth's curvature, atmospheric refraction, GPS and inertial navigation systems, image

processing, geometric transformations and cartographic generalisation (Florinsky 1998a, Scherzinger et al. 2001). Error can also occur in phase 2 (Figure 4), which comprises the selected method of terrain representation, the method of DEM approximation and the level of detail (e.g. mesh size or TIN density). Phase 3 (Figure 4) consists of uncertainty originating from the algorithms selected for calculating topographic attributes (Skidmore 1989, Florinsky 1998c, Zhou & Liu 2004a) and the related assumptions made, such as the calculation of flow directions without considering sub-surface flows. Finally, uncertainty in phase 4 (Figure 4) may be caused by a possible communication gap in human-computer interaction, which may be the result of poorly-implemented error propagation analysis or misinterpretation of the crisp boundaries in deterministic terrain analysis.

When DEMs are used in terrain analysis, the aforementioned errors may occur and will consequently affect the results. Because output of terrain analysis is always a function of input, the output error will be a function of the input error (phases 1-2, Figure 4) and the model error (phase 3, Figure 4). Investigation of this phenomenon is known as error propagation analysis, which has a direct influence on any decision-making process based on interpretations and applications of terrain analysis. Additionally, it may have an indirect influence on data acquisition and the DEM generation phases.

Throughout the thesis, the DEM error has been assumed to be independent of the actual elevations and topographic attributes, which rationalises the direct modelling of the DEM error using geostatistical methods. However, it is possible to apply the methods in models where the DEM error correlates with secondary information, such as slope. For building a stochastic model of DEM error (Figure 5),

it is useful to express the error by applying regionalised variable theory (Burrough & McDonnell 1998). Thus, the outcome  $Z(s)$  of the stochastic process at location  $s$  is the sum of a structural component with a constant drift  $m(s)$ , a random but spatially correlated component  $\varepsilon'(s)$ , and a spatially uncorrelated Gaussian noise term  $\varepsilon''$  (Equation 1).

$$Z(s) = m(s) + \varepsilon'(s) + \varepsilon'' \quad (1)$$

The original definitions of the variables may be relaxed for the purposes of DEM error modelling. For example, while the drift  $m(s)$  is strictly the systematic component of the error, it may also contain systematic-like effects of the error if it is possible to isolate and model such effects. The spatially correlated component  $\varepsilon'(s)$  might also correlate with secondary information. If a relationship between the DEM error and secondary information is established, the effect can be removed from the error data before the experimental semi-variogram modelling and put back after the geostatistical simulation resulting in heterosce-

dastic realisation of the error model. The first two terms together are the elevation difference between the points of the DEM and reality at the same locations (Figure 5). The spatially uncorrelated noise term  $\varepsilon''$  is caused by the lattice representation of the DEM and its inability to record the surface forms between the mesh points. In the geostatistical DEM error model, this is the nugget variance. However, appropriate determination of  $\varepsilon''$  from the experimental semi-variogram is impossible if the reference data has an interpoint distance larger than the DEM mesh size (Isaaks & Srivastava 1989).

After 20 years of active research within the field of spatial uncertainty, one might question why GIS software developers have still overlooked the importance of error propagation analysis making such tools an exception rather than a rule (Heuvelink et al. 2006). Firstly, error of the input data and used models needs to be characterised, which is often a difficult and expensive activity. Secondly, error propagation analysis is costly, and may be difficult to justify financially. Thirdly, the complexity of error modelling calls for specific expertise. Finally,

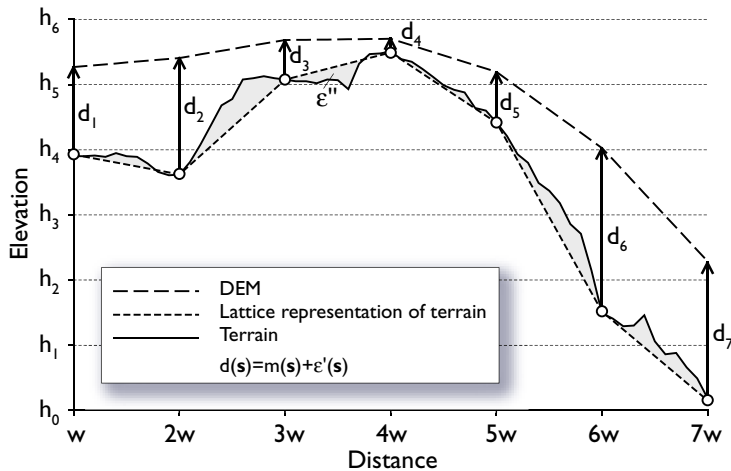


Figure 5. Components used for developing a geostatistical error model of lattice DEM.

the users of geographic information are not interested in the uncertainty of their analysis results, because crisp decisions have to be made anyway and communicating the uncertainty is challenging (Heuvelink et al. 2006).

#### 1.4 Aims of the study

While it has been acknowledged that our understanding of the impact of uncertainty on geospatial applications is still far from complete (Shi et al. 2004), this thesis is an attempt to stitch together some of the gaps in our understanding of the complexity of fine toposcale DEM error and its propagation in terrain analysis. Thus, the thesis presents a consistent set of methods for DEM error propagation analysis, beginning with preliminary quality evaluation of DEMs and going on to interpretation of the analysis results.

The aims of the thesis can be summarised in the following three questions:

1. *How should the DEM error be evaluated and reported so as to serve the needs of DEM users as well as possible?*
2. *How can the metadata concerning DEM error be taken into account in terrain analysis?*
3. *What is the influence of DEM error on terrain analysis, especially in the example cases of slope, aspect and automatic drainage basin delineation?*

The hypothesis of the thesis is that geostatistics offers a suitable theoretical framework for modelling the DEM error as a stochastic process. Based on this hypothesis, the thesis offers means for evaluating DEM quality and inspecting the propagation of error in ter-

rain analysis. The thesis presents a consistent framework for exploring DEM suitability for a specific terrain analysis task (Figure 6). From the perspective of the geospatial data quality standard (ISO 2002), the thesis covers only the aspect related to positional accuracy of the dataset. Other factors, including completeness, logical consistency, temporal accuracy and thematic accuracy were left outside the scope of the study. However, the deficiencies in logical consistency or completeness can be detected using, for example, the methods given in **I**.

The decision whether to accept a DEM for further terrain analysis can be made at any of steps A-C within the framework (Figure 6). For example, step A may show that the DEM contains morphological gross errors at critical locations for terrain analysis purposes. The advantage in step A is that no reference data is needed and therefore it should always be used routinely at the beginning of any terrain analysis task. Step B may reveal that the accuracy of the DEM does not meet the quality requirements set before the analysis. Step C may reveal that the uncertainty introduced by the DEM errors is too high for the specific analysis task. Step C can also be performed without reference data (as in **II** and **III**), because in many situations it is better to base the choice of spatial autocorrelation model on qualitative interpretation and reasoning than on the pattern of spatial continuity shown by too few and poorly distributed samples (Isaaks & Srivastava 1989). In contrast, successful completion of step B imposes high quality and density requirements on the reference data.

After the review in Section 2 and the summary of the materials and methods used (Section 3), Section 4.1 presents the results of **I** regarding visualisation of the morphological gross errors of DEM. In Section 4.2, a look at the geostatistical characterisation of fine toposcale DEM error represented in **IV** is

given, while Section 4.3 reviews the analytical and Monte Carlo simulation methods for DEM error propagation analysis and summarises the results of **II** and **III**. Finally, the

pros and cons of the methods and results of the thesis are discussed and directions for future work given.

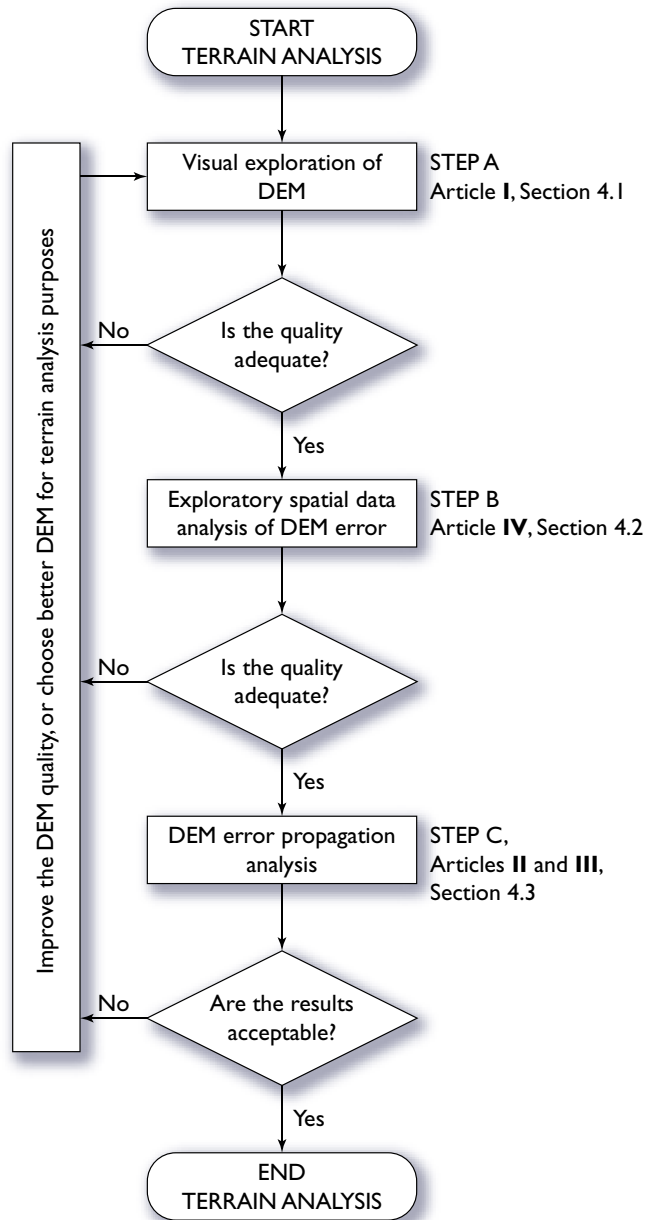


Figure 6. The framework for exploring DEM suitability for terrain analysis.



## CHAPTER 2

### REVIEW

The quality of DEMs has been studied from two perspectives in geography and remote sensing. One of these emphasizes the DEM quality as an intrinsic value, and focuses on formulating a general definition of DEM accuracy for given input data, surface approximation method, elevation representation and topographic properties of the landscape (e.g. Ackermann 1979, Torlegård et al. 1986, Li 1988, 1993a, 1993b, Kumler 1994, Li 1994, Carrara et al. 1997, Weng 2002). The other perspective, which is adopted in this thesis emphasizes the influence of DEM error in specific applications, in which the information of the product quality serves the purposes of error propagation analysis.

The long-term vision of the research in spatial data uncertainty has been to develop a general purpose 'error button' for geographical information systems (Openshaw et al. 1991), which, according to one suggestion, could be implemented by incorporating the 'button' into the product metadata (Goodchild et al. 1999). In more sophisticated definitions, the functionality of the 'error button' is seen as user-dependent, which offers various possibili-

ties for refining the error model according to the user's level of expertise (Heuvelink 2002). The first steps towards the vision becoming a reality have been taken in the data uncertainty engine, which implements the general framework for characterising uncertain environmental variables with probability models (Heuvelink & Brown 2005).

Irrespective of the practical implementation and the user interface of the 'error button', the core of the functionality is the error propagation analysis. While the analysis has been elementary in geodesy for decades, analysing error propagation in GIS-based geospatial data processing dates back to the 1970s (see references from Heuvelink et al. 1989, Heuvelink 1999, and Fisher 1991). The final breakthrough of the idea within the GISc community occurred in the late 1980s (Heuvelink et al. 1989). Nowadays, the research within GISc has separated into three branches focusing on the uncertainty of 1) continuous variables, 2) categorical variables and 3) spatial objects (Zhang & Goodchild 2002). The prevailing practice in error propagation analysis has been to use an analytical method (often with

numerical integration using a Taylor series of finite order), or Monte Carlo simulation (e.g. Heuvelink et al. 1989, Florinsky 1998b, Kyriakidis et al. 1999, Wise 2000). The basis of the analytical method is well established in the general law of propagation of variances (Mikhail & Ackermann 1976). In Monte Carlo simulation, adding the realisation of the error model to the original data is repeated a number of times, and during each simulation run geospatial analysis is performed and the results are stored for further statistical analysis (Heuvelink et al. 1990, Fisher 1991, Openshaw et al. 1991). It has been concluded that analytical approaches are suitable for situations where the sources of uncertainty are not significantly correlated and when the terrain analysis is sufficiently simple to permit arithmetic operations (Zhang & Goodchild 2002). On the other hand, the analytical method has been considered unsuitable for decision-based local derivatives (Raaflaub & Collins 2006), such as the D8 algorithm for determination of flow directions (O'Callaghan & Mark 1984). The Monte Carlo simulation has been regarded more versatile and better for solving real world problems of uncertainty (Zhang & Goodchild 2002).

Wide availability of DEMs together with increased computing capacity in the 1990s lead to a richness of articles focusing on error propagation in terrain analysis, which will be reviewed in the following sections. The chronological order of the review emphasizes the increase in understanding of the DEM error's spatial characteristics and the methodological advances in the error propagation analysis. In addition, large amount of references indicates that research on DEM error propagation is an active area within the field of GISc. For further insight on the topic, readers are referred to the reviews in Shortridge (2001) and Fisher & Tate (2006) summarising the development

in DEM error propagation analysis over the last two decades. However, the review here starts with an introduction into the theoretical basis of geostatistics, which forms the core of modern DEM error propagation analysis. In addition, work related to gross error detection has so far been handled separately from the work on DEM error propagation analysis, which is unjustified because both are an essential part of the framework for exploring the suitability of DEMs for terrain analysis (Figure 6). Therefore, a concise summary of morphological gross error detection in DEMs is also represented, followed by a look at the principal literature on spatial characterisation of DEM error and error propagation analysis.

## 2.1 Geostatistics and simulation of random fields

While the principles of geostatistics are established in a number of textbooks, the following review is based mainly on the work of Shabbenberger & Gotway (2005) and Burrough & McDonnell (1998). The subject is presented from the perspective, which aims at the development of a geostatistical model of DEM error for error propagation analysis purposes.

At the core of geostatistics is the concept of stochastic process, which is also known as random function or, especially in the spatial context, random field. The location-specific covariance  $C(\mathbf{s}, \mathbf{h})$  (Equation 2) and correlation  $\rho(\mathbf{s}, \mathbf{h})$  functions (Equation 3) of a stochastic process  $Z(\mathbf{s})$  are spatial extensions of the corresponding functions for time series.

$$C(\mathbf{s}, \mathbf{h}) = Cov[Z(\mathbf{s}), Z(\mathbf{s} + \mathbf{h})] \quad (2)$$

$$= E[\{Z(\mathbf{s}) - \mu(\mathbf{s})\} \{Z(\mathbf{s} + \mathbf{h}) - \mu(\mathbf{s} + \mathbf{h})\}]$$

$$\rho(\mathbf{s}, \mathbf{h}) = \frac{C(\mathbf{s}, \mathbf{h})}{\sqrt{Var[Z(\mathbf{s})]Var[Z(\mathbf{s} + \mathbf{h})]}} \quad (3)$$

The vector notation accounts for the distance and direction of  $\mathbf{h}$ . In a typical case, use of distance binning and angular tolerances is necessary for the selection of observation pairs.

In the case of strict stationarity, all moments of  $Z(\mathbf{s})$  are invariant under translations. The stochastic process is said to be a second-order stationary or weakly stationary process if the expected value  $E[Z(\mathbf{s})]$  exists and is invariant within the study area and if the covariance  $C(\mathbf{h})$  exists and is dependent only on the vector  $\mathbf{h}$  (Equations 4-6). Under the decision of second-order stationarity, expressions for spatial autocorrelation will be less complex, because they are no longer dependent on the absolute location  $\mathbf{s}$ .

$$E[Z(\mathbf{s})] = \mu \quad (4)$$

$$\text{Cov}[Z(\mathbf{s}), Z(\mathbf{s} + \mathbf{h})] = C(\mathbf{h}) \quad (5)$$

$$\rho(\mathbf{h}) = \frac{C(\mathbf{h})}{\sigma^2} \quad (6)$$

The random function is said to be anisotropic if  $C(\mathbf{h})$  is dependent on the distance and direction of  $\mathbf{h}$ , and isotropic if  $C(\mathbf{h})$  depends only on the distance  $|\mathbf{h}|$ .

In geostatistics, the most widely used tool for investigating the second-moment structure of spatial data is the semivariogram  $\gamma(\mathbf{h})$  (Equation 7), which measures the average dissimilarity between data separated by  $\mathbf{h}$  (Matheron 1965, Goovaerts 1997). The semivariogram can be calculated even where the random function does not have a constant mean and finite variance. The adequate condition is intrinsic hypothesis, which means that the increments  $Z(\mathbf{s}) - Z(\mathbf{s} + \mathbf{h})$  are stationary of order two (Goovaerts 1997).

$$\gamma(\mathbf{h}) = \frac{1}{2} \text{Var}\{Z(\mathbf{s}) - Z(\mathbf{s} + \mathbf{h})\} \quad (7)$$

Under the decision of stationarity, the relation between semivariogram  $\gamma(\mathbf{h})$ , covariogram  $C(\mathbf{h})$  (Equation 8) and correlogram  $\rho(\mathbf{h})$  (Equation 9) can be established (Figure 7):

$$\gamma(\mathbf{h}) = C(0) - C(\mathbf{h}) \quad (8)$$

$$\rho(\mathbf{h}) = 1 - \frac{\gamma(\mathbf{h})}{C(0)} \quad (9)$$

Experimental semivariogram is typically represented by a semivariogram model, which guarantees that the autocovariance function used in further geostatistical analysis is positive definite. Often used semivariogram models are

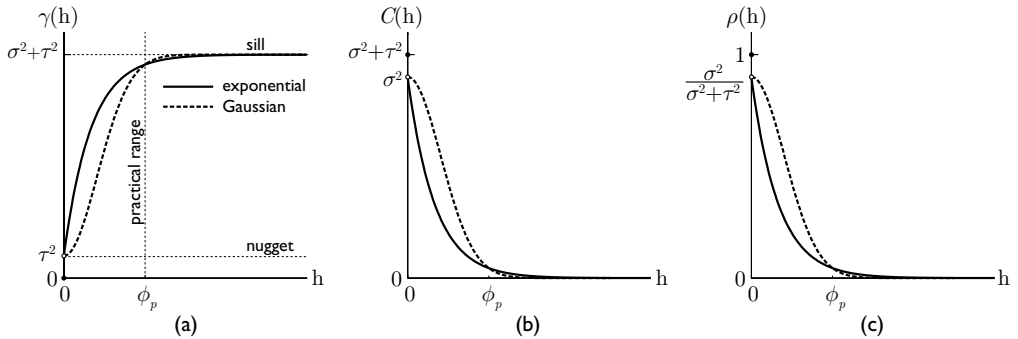


Figure 7. Relation between (a) semivariogram, (b) covariogram and (c) correlogram of a second-order stationary random field.

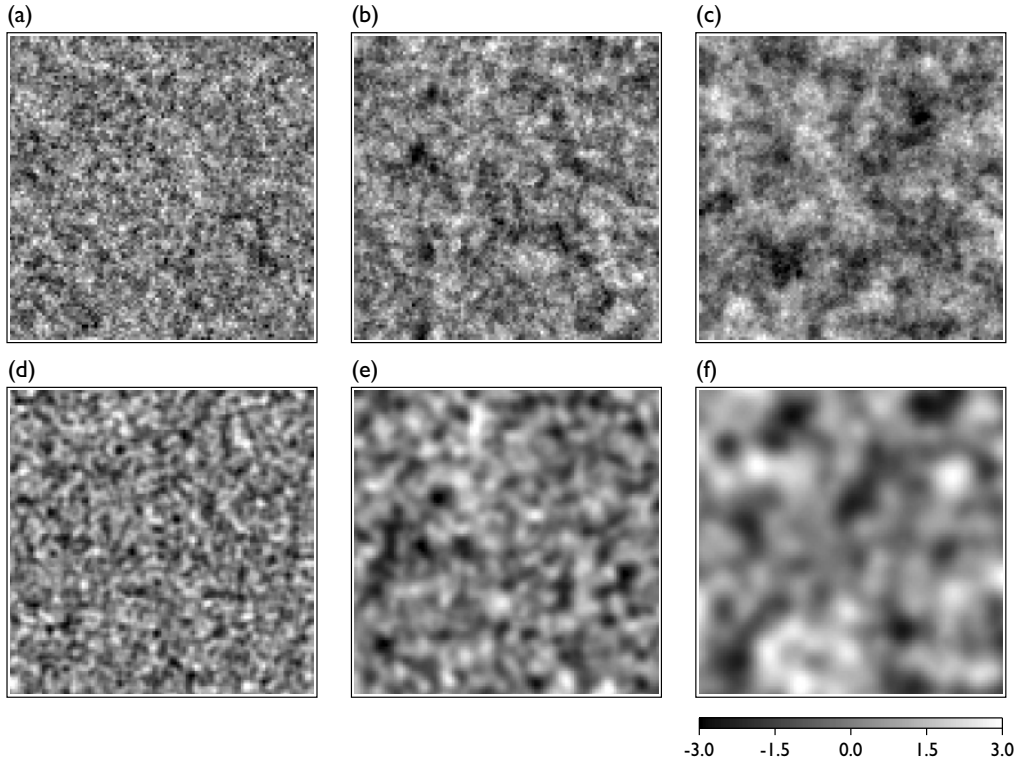


Figure 8. Realisations of 100x100 unit stationary Gaussian random fields with exponential (a–c) and Gaussian (d–f) spatial autocorrelation. Sill ( $\tau^2 + \sigma^2$ )=1 and nugget variance  $\tau^2=0$  for all models. Practical range ( $\phi_p$ )=3 (a and d), 6 (b and e), and 12 (c and f). The range (-3.0 – 3.0) is set for visualisation purposes.

spherical, power, exponential (Equation 10) or Gaussian (Equation 11), of which the last two were used in this thesis (Figure 8).

$$\gamma_{\text{exp}}(h) = \begin{cases} 0, & h = 0 \\ \tau^2 + \sigma^2 \left( 1 - e^{-\frac{h}{\phi}} \right), & h > 0 \end{cases} \quad (10)$$

$$\gamma_{\text{Gaussian}}(h) = \begin{cases} 0, & h = 0 \\ \tau^2 + \sigma^2 \left( 1 - e^{-\left(\frac{h}{\phi}\right)^2} \right), & h > 0 \end{cases} \quad (11)$$

The exponential and Gaussian semivariogram models are characterised by three parameters: sill ( $\tau^2 + \sigma^2$ ), range ( $\phi$ ) and nugget

variance ( $\tau^2$ ). The semivariogram may increase indefinitely if the variability of the spatial process has no limit at long distances. However, if the semivariogram reaches a limit value, the sill, this indicates the separation distance beyond which the process is spatially uncorrelated. The lag distance for which  $C(\mathbf{h})=0$  is called the range and it describes the ‘zone of influence’. If the semivariogram reaches the sill asymptotically, as in the exponential and Gaussian semivariance models, then the practical range ( $\phi_p$ ) is defined as the lag distance at which the semivariogram reaches 95% of the sill. The magnitude of the discontinuity near the origin is termed the nugget effect, which arises due to the microstructure (component of

the phenomenon with a range shorter than the sampling support or smallest interpoint distance) and measurement or positioning errors.

The most well-known family of methods in geostatistics is based on kriging, which is a set of generalised linear regression methods for minimising estimation variance defined from a prior covariance model (Deutsch & Journel 1998). Kriging is named after the South African mining engineer D.G. Krige, who developed methods for determining true ore-grade distributions from sample ore grades (Cressie 1991). Commonly used kriging methods are simple kriging (estimation with known mean), ordinary kriging (unknown but constant mean), universal kriging (presence of external trends), indicator kriging (estimation of probabilities of thresholds) and co-kriging (estimation with a secondary attribute). In addition to 'best' unbiased estimates, the method provides kriging variances at unsampled locations. Thus, the method can be used as a contouring algorithm, as well as a tool for obtaining conditional probability distributions (Deutsch & Journel 1998). However, even though the kriged surface is the best estimate in a least-squares sense, there are tasks in which the resulting surface is unrealistically smooth and continuous, restricting the method's usability. In addition, smoothing depends on the data configuration being at a minimum close to the data locations and increasing as a function of distance from the data locations. This leads to gross underestimation of the short-range structure of the random field (Goovaerts 1997).

A solution to the above limitation is stochastic simulation, which is the process of determining alternative, but equally probable, realisations of random variables from a random function model (Deutsch & Journel 1998). If the simulation is unconditional, it will honour only the semivariogram model, whereas a conditional simulation will honour

the existing observations as well. In theory, kriging-based interpolation will produce the same result as the average of a large number of conditional simulations (Englund 1993). Geostatistics recognises a number of simulation methods (e.g. Goovaerts 1997, Chilès & Delfiner 1999, Olea 1999), of which those applied in the DEM error propagation analysis, namely simulated annealing, the spatially autoregressive model, spatial moving averages, and sequential Gaussian simulation, are concisely reviewed below.

In **simulated annealing** (Deutsch & Journel 1998), which is also known as pixel swapping (Fisher 1991), the basic idea is to repeatedly perturb an initial grid until it matches the predefined characteristics of an objective function. Each perturbation is accepted only when it carries the perturbed grid toward the objective. The procedure is repeated as long as the objective is achieved within the acceptable tolerance. Although the technique is computationally intensive, it can be applied successfully when decisions as to whether to keep or reject the perturbation are optimised (Deutsch & Journel 1998).

The **spatially autoregressive model** represents one type of Markovian field (Zhang & Goodchild 2002) and is defined as

$$\mathbf{X} = \rho\mathbf{W}\mathbf{X} + \varepsilon, \quad (12)$$

where  $\mathbf{X}$  is a vector of grid values in the random field,  $\rho$  is a parameter of spatial autocorrelation (0-0.25 in square grid),  $\mathbf{W}$  is a matrix of weights (1 for rook's case neighbours, zeros otherwise) and  $\varepsilon$  is a vector of Gaussian random deviates. The solution to Equation 12 requires the inversion of  $\mathbf{I}-\rho\mathbf{W}$  ( $\mathbf{I}$  is the unit matrix), which becomes impractical even with simulation of small areas (Equation 13). However, algorithms with reduced computational complexity for solving  $\mathbf{X}$  with adequate

numerical accuracy do exist and have been applied in generating realisations of Gaussian random processes (Goodchild et al. 1992, Heuvelink 1998, Heo 2003). The spatial autoregressive method is considered suitable, especially for cases where the shape of the semivariogram is not explicitly known, even though the relation between the Bessel semivariogram model and parameter  $\rho$  has been established (King & Smith 1988).

$$\mathbf{X} = (\mathbf{I} - \rho\mathbf{W})^{-1} \varepsilon \quad (13)$$

Use of **spatial moving averages** is based on the fact that filtering the uncorrelated noise with a low-pass filter increases the amount of autocorrelation in the random field (Cressie & Pavlicová 2002). A special case of spatial moving averages is process convolution, in which the spatial structure of the target random field is controlled by the a priori correlogram (Thiébaux & Pedder 1986, Higdon 1998, Schabenberger & Gotway 2005). The process convolution is filtering of an uncorrelated Gaussian process using a unimodal and symmetric kernel  $k$  that produces a match between the correlogram  $R$  of the outcome and the a priori determined correlogram. The solution for defining the appropriate kernel  $k$  lies in the fact that convolving dense Gaussian noise with an appropriately scaled kernel yields a zero-mean Gaussian random field with a correlation structure that is the same as the convolution of the kernel with itself (Kern 2000). Kern (2000) presented a general solution for the task by applying a convolution theorem, which states that the Fourier transform of the convolution of two functions is proportional to the product of the Fourier transformations of the individual functions. Thus, if the correlogram  $R$  has been defined, the kernel that satisfies  $(k*k) \propto R$  may be obtained by indentifying the Fourier transform  $\mathcal{F}[R]$  of  $R$ , and

calculating the inverse Fourier transform of the square root of  $\mathcal{F}[R]$  when  $k$  is continuous and integrable (Kern 2000).

**Sequential Gaussian simulation** (SGS) starts by defining a random path for visiting each node of the grid once. At each node of the dataset, a specified number of original observations and previously simulated nodes are selected for conditioning purposes, and kriging is used to determine the location-specific mean and variance of the conditional cumulative distribution function (CCDF). Finally, a random value is drawn from the CCDF, the value is added to the dataset, and the procedure is repeated until all nodes have been visited (Goovaerts 1997, Deutsch & Journel 1998). When numerous realisations are needed, a significant decrease in computational load can be achieved by using the same random path for visiting the nodes many times, since the kriging weights then need to be solved only once. However, the danger is that the realisations become too similar (Deutsch & Journel 1998).

## 2.2 Tracing gross errors in DEMs

As well as exceptionally large or small legitimate observations, outliers include gross errors which are a result of poor experimental technique or mistakes in the measurement process (Milton & Arnold 1995). The decision to remove illegitimate outliers from further statistical analysis is typically based on an explorative inspection of the data distribution. In digital terrain modelling, the decision is not as straightforward since observations, which might be unexceptional at the global level, may be exceptional locally. In terms of their relative heights, these morphological gross errors are small but significant surface features. While all legitimate observations should be included in further analysis, the detection and fixing or

removing of gross errors is necessary, since they may introduce significant bias in the statistical or terrain analysis (Wise 2000).

In traditional cartography, gross errors in surface representation, especially in contours, were often a result of the work of an unskilled topographer, who may have accidentally confused the interpretation of the terrain surface, over-generalised the representation or recorded an erroneous height value (Imhof 1982). In addition to these types of gross error, contour-based fine topographic DEMs can also feature a number of other gross errors, such as straight lineaments caused by poor edge matching, triangular facets caused by poorly selected mesh size in relation to the generalisation level of the underlying TIN, flat terraces, under- and overshoots and undulation of nearly flat surfaces. Practically all these gross errors are the result of incorrect use of the DEM generation method.

Detecting morphological gross errors in DEM's, such as systematic striping (Polidori et al. 1991) or under- and overshoots of the approximation method (Florinsky 2002), has been based on two alternative methods: 1) automatic methods based on the divergent anisotropy and fractal dimensions of the DEM (Polidori et al. 1991, Brown & Bara 1994) or anomalous slopes (Hannah 1981, Li et al. 2005 cit. Li 1990), and 2) manual methods based on DEM visualisation (Carter 1989, Wood & Fisher 1993, Wood 1994, 1996b, Wise 1998). The drawback of the automatic methods is their sensitivity only to exceptionally high and low elevations and systematic striping, ignoring a number of other types of morphological gross error (I: Table 2).

The capacity of human vision to collect and synthesize information and recognize spatial patterns efficiently is fundamental to the use of visualisation as a scientific method. Visualisation can be seen as a descriptive and explorative method, which on the one hand

is used for representing the uncertainty of geospatial information (MacEachren 1992, MacEachren et al. 2005, II, III), and on the other hand supports explorative spatial data analysis (MacEachren 1995, I). The problem in direct visualisation of DEM heights is that the result is not discriminating, and surface features within a certain relative elevation threshold remain invisible. To overcome this limitation, a number of methods have been used, including shaded relief maps, slope and aspect maps, maps based on edge detection filters (e.g. Laplacian filter), profile and plan curvature maps, root mean squared error (RMSE) maps, modulo maps and aspatial representations, such as hummock plots (Wood & Fisher 1993, Wood 1996b).

A real-world example of visual detection of gross errors is the production of the seamless National Elevation Dataset (NED) by the United States Geological Survey (USGS). This used fully automated procedures, followed by manual inspection of the shaded relief image to verify the successful processing of the data (Gesch et al. 2002). Nevertheless, a well-known problem in shaded relief images is that the choice of direction for the light source has a major influence on the visualisation result (e.g. Imhof 1982). In addition, the method is not discriminating on gently outlined morphological gross errors on flat terrain. Use of modulo maps (Wood 1996b) resolves the problem in part, but such maps are visually hard to interpret, and the distinction between convex and concave surface forms is impossible due to lack of depth cues (MacEachren 1995).

### **2.3 Increasing the understanding of DEM error propagation**

Among the extensive early work with the aim of revealing the various aspects of DEM

error was a research project by the International Society of Photogrammetry and Remote Sensing, in which the accuracy of a number of DEMs was investigated in an international joint effort (Torlegård et al. 1986). Based on the results, the first steps were taken in representing methods for describing the spatial characteristics of DEM error and considering the error in terrain analysis (Östman 1987). Despite the strong intuition concerning the spatial autocorrelation of DEM error, the phenomenon has been difficult to explore in practice, mostly due to inaccurate, sparse and topographically biased reference data (e.g. Monckton 1994, Fisher 1998, Holmes et al. 2000).

DEM error propagation analysis was introduced to the GISc community in the early 1990s. In pioneering work by Heuvelink et al. (1990), error propagation in calculating slope and aspect was represented using Monte Carlo simulation and Taylor series approximation. It was shown that standard deviations of slopes and aspects were higher than expected, and the authors recognised the sensitivity of runoff and erosion models for error in DEMs. In addition, the study launched a prolonged assumption, whereby DEM error without spatial autocorrelation could be used as a worst-case scenario (e.g. Heuvelink 1998, Wechsler 2000, Van Niel et al. 2004, Raaflaub & Collins 2006). A pioneering study was also carried out by Fisher (1991), in which the focus was on the accuracy of the viewshed area. This was based on Monte Carlo simulation using four scenarios for spatial autocorrelation of the DEM error. The magnitude of spatial autocorrelation was expressed using Moran's I (e.g. Bailey & Gatrell 1995), and 19 realisations of each scenario were generated with pixel swapping. The work showed that the size of a viewshed calculated from the original DEM, without considering the potential errors in it,

could be significantly overestimated. The work was extended in articles applying similar principles, for example, in analysing the effect of DEM errors in floodplain extraction (Lee et al. 1992). Further improvements in the analysis of DEM uncertainty were offered by focusing on the differences in viewshed algorithms (Fisher 1993) and the semantics of probable and fuzzy viewsheds (Fisher 1994).

Hunter and Goodchild (1995) applied probability theory to the examination of elevation thresholds. The novel aspect of their work was the visualisation of specified elevation as a probability zone, in which it was possible to see the most likely location of the height threshold together with the associated uncertainty. In addition, they paid attention to the regularly spaced gross approximation errors in the USGS's 7.5 minute DEMs, which showed up using contour plots and relief shading. A significant refinement of the idea was published soon after (Hunter & Goodchild 1997), when they focused on the influence of DEM elevation error on the calculation of slope and aspect. For the first time, they derived and published analytical variance equations for the numerical approximations of the first partial derivatives of the DEM. However, due to the nonlinearity of the slope and aspect equations, they applied Monte Carlo simulation using spatial autoregressive model in further error propagation analysis. The authors generated ten realisations of the DEM error model with 40 values of  $\rho$  (see Equation 12). The results showed that even without the empirical knowledge of the spatial structure of the DEM error,  $\rho=0.20$  could be used as a worst-case scenario showing the outer limit of variation in slope and aspect.

Simultaneously, a study on the effect of DEM errors on the determination of flow-path direction (Veregin 1997) presented a critical view of the earlier error propagation

studies. With good justification, the author recognised that when a realisation of modelled error is added to the original DEM, the result is not a DEM with an added error component, but rather a realisation of an error-free DEM. Veregin also questioned the use of a small number of simulation runs, as this results in a high risk of non-converged results. A new perspective in error modelling was the inclusion of cross-correlation of DEM error and slope in addition to the error's spatial autocorrelation. Furthermore, Veregin concluded that 500 simulation runs was adequate in his applications and that the actual number of realisations required depends on the specific statistics of interest. The experiment was done using eight combinations of RMSE, autocorrelation, and cross-correlation values. Realisations of the DEM error model were generated with pixel swapping. At the end, the author advised against using USGS 30 m DEMs for derivation of flow-paths, because in some cases the path reliability was only marginally better than the level that would be attained when assigning the paths on a random basis.

In 1998, geostatistical concepts were introduced in DEM error modelling and error propagation analysis for the first time (Fisher 1998). Even though the approach taken was a major breakthrough for future DEM error propagation studies, the small-scale structure of the error in fine toposcale DEMs used in the case study still remained poorly defined due to sparse reference data.

Another major improvement in 1998 was the publication of variance equations for a number of surface derivatives (Florinsky 1998b). The author stressed that the accuracy of, for example, slope and aspect cannot be determined by using reference data from topographic maps (e.g. Skidmore 1989) or field surveys (e.g. Bolstad & Stowe 1994, Giles & Franklin 1996), because the actual land surface

is not mathematically smooth and therefore it cannot have unambiguous derivatives. Florinsky found out that the propagated variance in approximating the first partial derivatives of the DEM was smallest in 'the Evans method' (Evans 1979), and showed that mapping was the most convenient way to represent the results of the variance equations. It was also demonstrated that high variances of slope, aspect and curvature were typical for nearly flat areas. However, by overlooking the DEM error's spatial autocorrelation, the research was still on an unsound basis.

The first monograph on error propagation analysis in GIS was published by Heuvelink (1998). While covering the whole range of spatial analysis, the author also included chapters focusing only on error propagation in DEMs. He presented a map series showing standard deviation of slope and aspect based on second- and third-order finite difference methods (Burrough & McDonnell 1998) using Monte Carlo simulation with 5000 runs. The method for generating the realisations was not documented. It was concluded that accuracy of slope increased as the spatial autocorrelation of the DEM error model increased (in contradiction with Hunter & Goodchild 1997 and II), and that the third-order finite difference method was more accurate than the second-order finite difference method. It was also concluded that variance of aspect was very high in nearly flat areas.

In his doctoral thesis, Ehlschlaeger (1998) presented simulation methods for modelling spatial application uncertainty. The themes included uncertainties in continuous fields and area-class maps. The author presented case studies of DEM error propagation on a least-cost path and visibility analysis. For generating realisations of the DEM error model, Ehlschlaeger presented a three-parameter method based on spatial moving averages with

an interactive calibration phase. Justifiably he concluded that the method was favourable for spatial uncertainty studies compared with the earlier methods used (spatial autoregression and simulated annealing based on Moran's I), since they allow only one parameter to define both the large- and small-scale structure of the error model and are computationally intensive.

Kyriakidis et al. (1999) presented methods for building a stochastic DEM, which conflated the sparse and accurate observations with exhaustive but inaccurate data. Unlike any other article focusing on DEM error propagation analysis, the authors used geostatistical modelling and simulation directly on the elevations, not on their errors. The aim was to generate equi-probable realisations of the unknown high-accuracy elevation surface using sparse measurements (hard data), a DEM covering the whole study area (soft data) and auto- and cross-covariance models for them. Thus, it was possible to obtain the realisation of DEM error indirectly by subtracting the original DEM from the realisation of the geostatistical model of the DEM. The realisations were generated using SGS, in which local CCDF were derived using co-kriging on the hard and soft data. The indirect simulation of DEM error allows the surface to be heteroscedastic and non-stationary, but the drawback is that since the error model was not explicitly defined, the causality between DEM error properties and terrain analysis results remained unclear, and the method is troublesome to implement in analytical error propagation analysis. Despite the rigorous methodology, the complexity of the model development and low correlations between the DEM error and secondary data has allowed the use of simpler error models. The research by Holmes et al. (2000) presented a coherent study of DEM error propagation analysis for the first time start-

ing from documented semivariogram modelling of USGS DEM error based on reference data from GPS, and ending with the Monte Carlo simulation, in which 50 realisations of the error model were generated with SGS. The focus was on the number of surface derivatives, including for example slope, aspect, plan and profile curvature, and flow accumulation. In addition, a case study of the effect of DEM error on hillslope failure prediction showed how the use of original USGS DEM coarsely underestimates potential landslide hazard areas. Unfortunately, the small-scale spatial structure of the DEM error remained hidden due to sparse reference data.

In 2000, practical implementation of DEM error propagation analysis for commercial GIS software using the Monte Carlo method was undertaken as a part of a doctoral thesis (Wechsler 2000). In addition to programming work, the focus was also on the effects of uncertainty on elevation and derived topographic parameters, DEMs of different scale, and in flat and rough terrain. Realisations of the DEM error were generated with uncalibrated spatial moving averages, which led to a lack of control in standard deviation and spatial autocorrelation of the random fields.

In 2002, a methodology for inter-map cell swapping heuristics for fitting a number of application-specific goal statistics simultaneously was presented (Ehlschlaeger 2002). Instead of swapping cells within one realisation of the DEM error model, the method swaps co-located cells in all realisations. The advantage was that heteroscedastic properties of the initial random fields were preserved. In Ehlschlaeger's example, the 'spatial data uncertainty model' included statistics for the proportion of artificial pits and the water gradient in the study area. However, mixing components from the specific terrain analysis with the DEM error model brings a risk of making error propa-

gation analysis unnecessarily complex. The utmost fine-tuning of the error model parameters may also be seen as unjustifiable if only a few realisations of the model can be generated within a reasonable time.

The second comprehensive monograph on error propagation analysis in spatial data processing was published by Zhang and Goodchild (2002), which is the first publication in which an analytical variance equation taking spatial autocorrelation into account for calculation of slope was presented. However, the presentation was considered complex and unsuitable for practical purposes. This led to the conclusion that analytical approaches are suitable for situations in which the sources of uncertainty are not significantly correlated and when the terrain analysis is sufficiently simple to permit arithmetic operations.

A detailed investigation of the accuracy of slope and aspect was published by Zhou and Liu (2004a, 2004b). They focused on the errors of slope and aspect associated with DEM precision (vertical resolution), horizontal resolution and grid orientation.

Despite the large number of works based on the use of Monte Carlo simulation, only a few authors after Veregin (1997) have paid attention to the convergence of error propagation analysis results. According to Nackaerts et al. (1999), the probabilistic visibilities based on a spatially uncorrelated model of DEM error appeared to converge after 60 simulation runs. In an application based on cost-path

analysis the minimum number of simulation runs for converged analysis results was found to be 400-500 (Aerts et al. 2003).

Carlisle (2002, 2005) changed the focus from spatial autocorrelation to cross-correlations of DEM error with number of surface derivatives. In the concept of 'accuracy surface', he presented a method for applying multivariate regression modelling in defining the relationship between DEM error and geomorphometric characteristics. The method permitted generation of a heteroscedastic DEM error model, even though the causality between the DEM error and 12 terrain parameters was left unclear and only minor attention was paid to the spatial autocorrelation of the error.

To summarise the review, the general picture of DEM error propagation analysis is characterised by the heterogeneity and inconsistency of the earlier works. Because of the methodological shortcomings, the spatial structure of fine toposcale DEM error has remained unclear. The geostatistical simulation methods commonly used have been computationally very demanding. Thus, the typical Monte Carlo -based error propagation analysis using less than 100 simulation runs has obviously been a consequence of limited computing capacity (Heuvelink 1998). In addition, the choice between the use of analytical and simulation methods has been confusing, with the latter often being used even though an analytical solution would have been more feasible.

## CHAPTER 3

# MATERIALS AND METHODS

The DEMs and reference datasets used in the study are listed in Table 4. The methods for exploring the suitability of DEM for ter-

rain analysis are listed in Table 5 and described in the original publications.

Table 4. DEMs and reference data used in the thesis.

Used in	DEMs	Reference data
<b>I</b>	<p>NLS DEM 25, NLS (NLS 2002b)</p> <ul style="list-style-type: none"> <li>- Source data: NLS topographic database (NLS 2002a)</li> <li>- Method: linear interpolation from TIN</li> <li>- Grid: 25 m</li> </ul> <p>Land-Form PROFILE®, OS (OS 2001)</p> <ul style="list-style-type: none"> <li>- Source data: OS contours (1:10000)</li> <li>- Method: undocumented</li> <li>- Grid: 10 m</li> </ul> <p>7.5 Minute DEM, USGS (USGS 2000)</p> <ul style="list-style-type: none"> <li>- Method/source data: 1) interpolation of the elevations from stereomodel digitised contours or DLG hypsographic and hydrographic data, 2) the Gestalt Photo Mapper II, and 3) manual profiling from photogrammetric stereomodels</li> <li>- Grid: 10/30 m</li> </ul>	-
<b>II</b>	<p>Self-made DEM</p> <ul style="list-style-type: none"> <li>- Source data: office map contour sheets (1:2000) by Urban Planning Department of the City of Turku and Department of Geography, University of Turku (Vuorela 2001)</li> <li>- Method: LIDIC (Oksanen &amp; Jaakkola 2000)</li> <li>- Grid: 10 m</li> </ul>	-
<b>III</b>	<p>Self-made DEM</p> <ul style="list-style-type: none"> <li>- Source data: NLS topographic database (NLS 2002a)</li> <li>- Method: LIDIC (Oksanen &amp; Jaakkola 2000)</li> <li>- Grid: 10 m</li> </ul>	Manual drainage basin delineations made by 5 hydrology experts
<b>IV</b>	<p>NLS DEM, NLS</p> <ul style="list-style-type: none"> <li>- Source data: NLS topographic database (NLS 2002a)</li> <li>- Method: HIFI (Ebner et al. 1980)</li> <li>- Grid: 10 m</li> </ul>	<p>LIDAR DEM</p> <ul style="list-style-type: none"> <li>- Produced by the author from Toposys FALCON II LIDAR data (TopoSys 2004)</li> <li>- Grid: 2.5 m</li> </ul> <p>Field Survey</p> <ul style="list-style-type: none"> <li>- 1363 reference points measured with RTK-GPS using either self-set up or virtual reference station</li> </ul>

Table 5. The methods used in the thesis and the steps in the framework for exploring DEM suitability for terrain analysis (Figure 6).

Methods	References	Step	Described and used in			
Adaptive TIN densification for LIDAR ground point classification	(Axelsson 1999, 2000)	B				<b>IV</b>
Agglomerative nesting	(Kaufman & Rousseeuw 1990)	B				<b>IV</b>
Automatic drainage basin delineation	(Jenson & Domingue 1988, ESRI 2005)	C	<b>II</b>	<b>III</b>		
Calculation of slope and aspect	(Horn 1981, ESRI 2005)	C	<b>II</b>			<b>IV</b>
Coordinate and height system transformations	(Ollikainen 2002, Anon. 2003)	B				<b>IV</b>
Correlation analysis	(e.g. Milton & Arnold 1995, R Development Core Team 2006)	C	<b>II</b>			<b>IV</b>
Descriptive statistics	(e.g. Milton & Arnold 1995, R Development Core Team 2006)	B				<b>IV</b>
Filtering methods	(Wood 1994)	A	<b>I</b>			
General law of propagation of variances	(Mikhail & Ackermann 1976)	C	<b>II</b>			
Generation of lattice DEM from LIDAR ground points	(Soininen 2003)	B				<b>IV</b>
Global and local semivariogram analysis	(e.g. Dennis & Schnabel 1983, Cressie 1991, Ribeiro & Diggle 2001)	B				<b>IV</b>
Methods for summarising Monte Carlo simulation results		C	<b>II</b>	<b>III</b>		
Modulo maps	(Wood 1996b)	A	<b>I</b>			
Monte Carlo simulation	(e.g. Openshaw et al. 1991)	C	<b>II</b>	<b>III</b>		
Multi-band shading	(Hobbs 1999)	A	<b>I</b>			
Process convolution	(e.g. Higdon 1998)	C			<b>III</b>	
Relief shading	(Horn 1981)	A	<b>I</b>			<b>IV</b>
Remainder images		A	<b>I</b>			
Sequential Gaussian simulation	(e.g. Goovaerts 1997, Pebesma 2004)	C	<b>II</b>			
Visualisation of error maps	(e.g. Fisher 1991, Florinsky 1998b)	C	<b>II</b>	<b>III</b>	<b>IV</b>	



## CHAPTER 4

# RESULTS - FRAMEWORK FOR EXPLORING DEM SUITABILITY FOR TERRAIN ANALYSIS

### 4.1 Visualisation as a tool for gross error detection (Step A)

Article **I** presents explorative visualisation tools for detecting morphological gross errors potentially affecting terrain and error propagation analysis. The tools are helpful in the product development and documentation processes from the data and the metadata producer's viewpoint. In addition, the methods give valuable additional information on the fitness for use of the DEM for specific terrain analysis purposes for the individual DEM user.

While the idea of using visualisation as a tool for detecting DEM errors is not new, article **I** reviews the earlier methods, such as filtering methods (Wood 1994) and RGB multi-band shading (Hobbs 1999), and presents the remainder image technique, which is an update of the 'modulo distribution maps' introduced by Wood (1996b). The basis for remainder image consists of slicing a DEM into narrow vertical bands and this is achieved by calculating the remainder  $M_{i,j}$  of the terrain elevation divided by some artificial height  $C$  (Equation 14). The height can be, for ex-

ample, the contour interval of input data used in DEM approximation. If the DEM is not based on contour data, the divisor needs to be adjusted so that the best visual impression of the remainder image is obtained.

$$M_{i,j} = DEM_{i,j} \bmod C \quad (14)$$

Visualisation of the remainder surface is based on knowledge of the remainder values, which are distributed between 0 and the divisor  $C$ . The first and last colour of the scale should be set to be the same, and the number of additional colours between the extremes is dependent on the scale of the final map and the amount of detail in the topography (Figure 9). Adding transparent hill shading as a depth cue is necessary to make a distinction between convex and concave surface forms (Figure 10).

The method has been used successfully in the accuracy assessment of Finnish DEMs (Oksanen & Jaakkola 2000, Oksanen et al. 2002, Oksanen & Sarjakoski 2003). While Wood (1994) saw the method 'as a complementary set of tools to be drawn upon where necessary', visualisation is considered more as

an essential part of defining DEM quality in this thesis. Even though the focus of the thesis is on fine toposcale DEMs, the methods in **I** are suitable for DEMs at any application scale (Table 3).

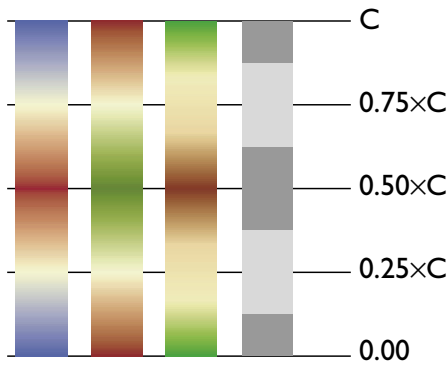


Figure 9. Examples of colour scales suitable for visualisation of the remainder images.

## 4.2 Characterisation of DEM error (Step B)

The most important part of DEM error propagation analysis is the appropriate characterisation of the error, including information about its distribution and spatial structure (Shorridge 2001). While the aspatial description of DEM quality might be seen as adequate from the data producer's perspective, knowledge of the spatial structure is necessary from the perspective of error propagation analysis. Alternatively, one has to be aware of consequences of ignoring spatial autocorrelation of DEM error (**II**).

To date, the limiting factor in characterising fine toposcale DEM error has been the unfeasible collection of high-quality reference elevation datasets covering large areas. In **IV**, where the error of NLS DEM was characterised, the problem was solved by using high-quality LIDAR DEM, which completely covered the study area. This obviously does

not correspond to the real-world task of accuracy assessment. However, the results can be applied in, for example, sampling design of operational DEM quality evaluation.

The comparison of NLS DEM error statistics with corresponding parameters for NLS contour data errors reveals that the quality of contours has the most significant impact on NLS DEM accuracy (**IV**: Table 2). In addition, the reported standard deviations of the NLS DEM height errors had a high variation depending on the subjective definition of the outliers (**IV**: Table 2 & Figure 6), which confirms that a more robust dispersion measure, such as inter quartile range (IQR), should be used in addition to conventional error statistics. Visually, it was possible to divide the outliers into three classes according to their size and underlying topography: large patches of outliers existing because of gross errors in the contour data; undershoot errors in the HIFI surface approximation method (Ebner et al. 1980); and small regions of outliers existing because of steep slopes.

Empirical tests were performed to ascertain whether it was possible to represent the error distribution without the extreme outliers using a weighted combination of two normal distributions. All attempts to separate these groups either statistically or spatially from the NLS DEM error distribution failed, which suggests that there are no external means to separate these groups from the full error distribution.

A global empirical directional semivariogram covering lag distances up to 6000 m of the NLS DEM error (**IV**: Figure 7[a]), and an omni-directional variogram for lag distances up to 500 m (**IV**: Figure 7[b]) revealed the nested spatial structure of the error with three distinct zones. Since the NLS DEM error appeared to become spatially unstructured at lag distances  $> 300$  m, the whole study area was split into calculation windows (800 m x 800 m

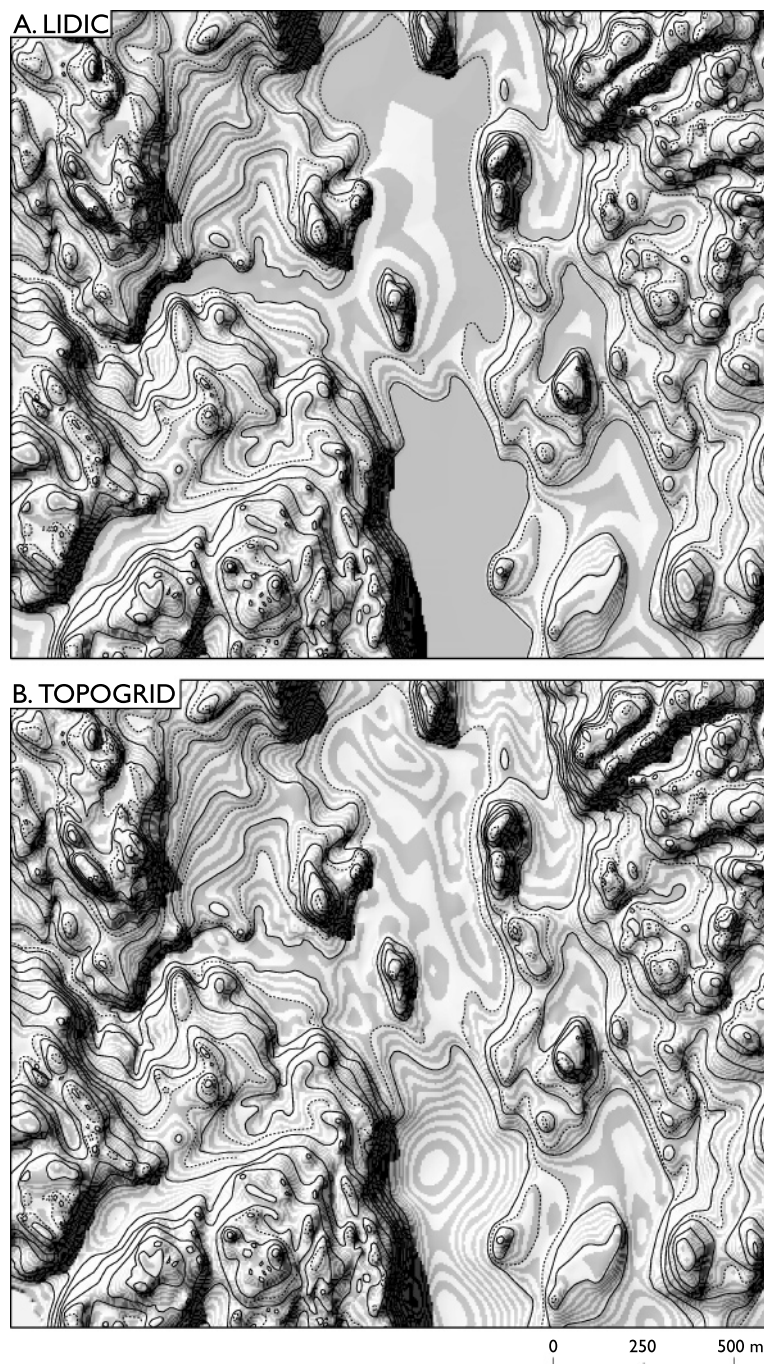


Figure 10. Comparison of DEMs based on LIDIC (Oksanen & Jaakkola 2000) and TOPOGRID (Hutchinson 1989, ESRI 2005) using remainder images. Input: Contour interval 5.0 m (solid line) and 2.5 m (dashed line). Output: DEM represented by vertical classes 0.5 m in height (grey stripes). Contours: © National Land Survey of Finland, Licence 049/MYY/06.

with a 600 m overlap) for local semivariogram analysis purposes (**IV**: Figures 8[a] & 8[b]).

To obtain evidence of the relationship between topography and the semivariogram model parameters of the NLS DEM error, a comprehensive correlation analysis of the parameters and numerous terrain attributes was conducted (**IV**: Table 3). It appeared that the greater the terrain ruggedness, aspect bias and vegetation index, the higher the sill. In contrast, increasing terrain ruggedness was weakly associated with shorter spatial autocorrelation ranges. The terrain attributes featuring the highest correlations with the semivariogram parameters and the lowest correlations with each other were average slope, standard deviation of curvature, and vegetation index. The correlations of these three terrain attributes formed the basis for clustering, in which the study area was divided into regions with homogeneous topography (**IV**: Figure 9). The clusters separated the granite and gneiss zone of the area, as well as the flat field areas and rugged terrain within the granite zone (**IV**: Figures 1 and 10). Experimental semivariogram analysis of the NLS DEM errors in the terrain clusters revealed that a high sill and small range was typical for rugged terrain, whereas a low sill and long range characterised the semivariogram of the flat field area. However, the local semivariogram analysis indicated that the variation of semivariogram parameters within each of the clusters was high (**IV**: Figure 11).

Since there were three zones in the nested spatial structure of the NLS DEM error (**IV**: Figure 7), this led to the idea of decomposing the error surface by repeatedly filtering the error surface using spatial moving averages with radii of 1280, 640, 320, 160, 80, 40, 20 and 10 m (**IV**: Figure 12). A similar method has been used in geography for partitioning social data (Pigozzi 2004). Decomposition provided

the means for establishing the explanations for the non-stationarity of the DEM error. The largest spatial moving averages gave the broad low-frequency components of the NLS DEM error, and the residual represented the highest frequencies in the error field. While the highest frequencies appeared to be connected with terrain curvature and slope, the lowest frequencies seemed to exist independently of the topography. The aspect bias, which appears as topographic features in the DEM with over-estimated elevation on one side and underestimated on the other, was most clearly visible in the errors with medium frequencies.

The results showed that the spatial autocorrelation of the fine toposcale DEM error was the result of a complex combination of random and systematic-like components, and that its appropriate modelling by geostatistical methods is problematic because of the small extent of the areas in which the assumption of stationarity is valid. It was also impossible to describe the shape of the DEM error distribution precisely with a single parameter of dispersion. This was due to a large number of outliers, which suggests that more robust error descriptors should be used in addition to conventional error statistics.

### 4.3 DEM error propagation analysis (Step C)

The following sections summarise the results given in **II** and **III**. The articles give two alternative methods for investigating the error propagation in terrain analysis and the influence of the error in the calculation of slope, aspect and automatic drainage basin delineation. The error propagation analysis was done by using an analytical method (**II**) or the Monte Carlo method (**II** and **III**), which both applied the geostatistical DEM error model character-

ised by semivariogram and correlogram model parameters.

#### 4.3.1 Analytical method

In **II**, the variance equations for slope and aspect were derived by applying the general law of propagation of variances (Mikhail & Ackermann 1976). The application of the law is possible for local surface derivatives because the variance-covariance matrix of the DEM error is explicitly defined for the calculation window. Even though the variance equations (**II**: Equations 27-29) are only for slope and aspect based on the third-order finite difference method (Horn 1981, ESRI 2005), it is possible to apply the methods to any local surface derivative and to any approximations of the partial derivatives of the surface.

The advantage of the analytical method compared with the Monte Carlo method is that after the derivation of variance equations, the computation load for generating error maps is minimal. In addition, investigation of the variance equations reveals causality between the DEM error characteristics and variance of surface derivatives (**II**: Figures 3–5).

#### 4.3.2 Monte Carlo method

In the Monte Carlo method, the derivation of variance equations is replaced with ‘brute force’ by repeating the terrain analysis a number of times on a DEM in which the realisations of the error model are added (**II**: Figure 2, **III**: Figure 2). The method offers flexibility at the cost of heavy computational load. The thesis presents a tool based on process convolution that offers a powerful framework for investigating DEM error propagation with a large number of terrain analysis simulation runs.

In terrain analysis three factors that are critical from the performance perspective in the Monte Carlo simulation are the generation method of DEM error model realisation, computation of the surface derivatives and the simulation runs needed for converged analysis results. While in **III** derivatives were computed by using tools that were readily available (ESRI 2005), the article presented the application of a process convolution for simulating Gaussian random fields. The method was applied in DEM error propagation analysis for the first time in **III**, in which a numerical solution for resolving the convolution kernel from the a priori correlogram was presented. The method was found to be functional for Gaussian spatial autocorrelation models, and the validity of the procedure was verified with a number of numerical experiments (**III**: Figure 3).

Convergence of the error propagation analysis results in Monte Carlo simulation is still largely unknown, even though some application-specific thresholds have been given (cf. Section 2.3). The analysis has often been done with less than 100 simulation runs (e.g. Fisher 1991, Openshaw et al. 1991, Kyriakidis et al. 1999, Holmes et al. 2000), even though the risk of unreliable results is high (e.g. Heuvelink 1998). The problem is that convergence analysis is always case-specific, as it is dependent on the terrain analysis task, error model parameters and topography. For the case study in **III**, the average of the Boolean drainage basins representing probable drainage basin appeared to converge after 500 simulation runs (Figure 11).

#### 4.3.3 Influence of DEM error on terrain analysis

In **II** and **III** the authors presented the results of an error propagation analysis of slope, aspect and automatic drainage basin

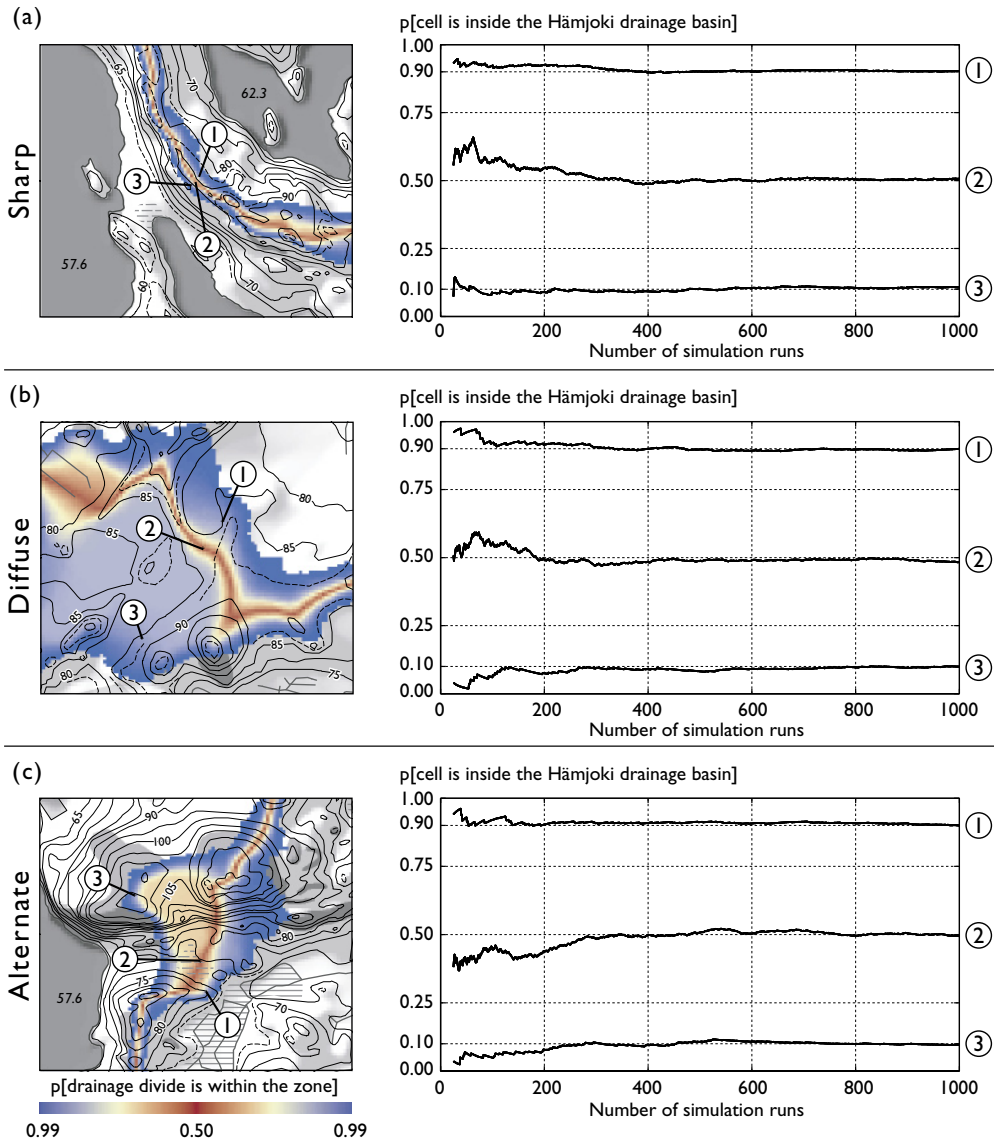


Figure 11. Convergence of the probable drainage basin at different parts within a) sharp, b) diffuse, and c) alternate delineations. The thick lines of the diagrams represent the evolution of the probable drainage basins during the simulation runs. Contour interval: 5.0 m (solid line), 2.5 m (dashed line). Contours: © National Land Survey of Finland, Licence 049/MYY/06.

delineation. In addition, the uncertainty in DEM-based drainage basin delineation was compared with the uncertainty of manual delineations made by five experts in hydrology and physical geography. In II, the theoretical

part of the work was put into practice in a case study in which 32 realistic scenarios of DEM error models were used. The scenarios were based on exponential and Gaussian spatial autocorrelation models with four sills (0.0625,

0.25, 1.00 and 4.00 m<sup>2</sup>) and practical ranges (0, 30, 60 and 120 m). With this arrangement, it was possible to trace the sensitivity of error propagation analysis on the correct choice of error model parameters. In **III**, the error model parameters were fixed and the focus was on the uncertainty of the automatic drainage basin delineation, and especially on the use of process convolution in the Monte Carlo method. Several of the figures depicting the results of **II** are reproduced in the thesis (Appendix A). The key findings of articles **II** and **III** are summarised below.

- a) An increase in the DEM vertical error will increase the error in the terrain analysis (**II**).
- b) The variance of slope measured in angular units is inversely proportional to the steepness of the terrain, but when the slope is measured in percentage units, the variance of slope equals the variance of the approximations of the surface's first partial derivatives (**II**: Equations 27 and 28).
- c) The choice of correct parameters for the DEM error model was critical for error propagation analysis in the calculation of slope, because the variation in the slope's standard deviation appeared to be higher between the scenarios than within the study area. The variance of slope appeared to be more sensitive to the appropriate definition of the DEM error model's sill than to the practical range (**II**: Figure 9, Appendix A: Figure 1).
- d) The variance of aspect is also inversely proportional to the steepness of the terrain, and it increases sharply when the terrain is nearly flat. In flat terrain, aspect and its variance are undefined (**II**: Equation 29).
- e) Since topography dominates the variance of aspect, the correct choice of DEM error model parameters is less sensitive than in the calculation of slope (**II**: Figure 10, Appendix A: Figure 2).
- f) With an increase in mesh size in the DEM, the variance in slope and aspect decreases (**II**: Equation 23). It should, however, be noted that this does not mean that slopes and aspects derived from coarse DEMs are more accurate than derivatives based on DEMs with a finer mesh.
- g) The role of the appropriate shape in a spatial autocorrelation model, either exponential or Gaussian, was not as important as the choice of appropriate autocorrelation parameters: practical range and sill. However, the shape of the spatial autocorrelation model appeared to have more influence on the calculation of slope and aspect than on the drainage basin delineation (**II**: Figures 3, 4 and 8, Appendix A: Figure 3).
- h) In local surface derivatives, such as slope and aspect, the maximum error in the results appeared to occur when the practical range of the error's spatial autocorrelation was roughly equal to the size of the surface derivative's calculation window. In unconstrained terrain analysis, such as drainage basin delineation, the variance of the results in the test setting increased when the spatial autocorrelation range increased. The study did not reveal the morphology-related limit at which delineation uncertainty begins to decrease (**II**).
- i) The uncertainty of drainage basin delineation could be represented in three alternative ways: as 1) a probable drainage basin (**III**: Figure 4), 2) a probable drainage divide (**III**: Figure 4, Appendix B), and 3) the probability

zone of the drainage divide (**III**: Figure 5, Appendix C). The optimal representation method was found to be context dependent.

j) The drainage basin delineations were sharp, diffuse or alternate. Sharp delineations were typically found on ridges outlined by low-lying areas or by bodies of water. In addition, they were found in areas where the divide was clearly above the other potential thresholds. In diffuse delineations, the divide was spread over a wide unclear zone, which was usually a result of the flatness of the terrain. Along the alternate delineation, there were two or more bands of higher probabilities indicating the most likely location of the drainage divide. They were found in areas where peaks were near each other and were separated only by shallow depressions (**III**: Figure 8).

k) When the DEM error model's practical range and sill increase, the number of locations with diffuse or alternate delineations also increases. However, diffuse and alternate delineations were found even with the smallest tested

DEM error model parameters (**II**: Figure 7, Appendix A: Figure 4).

l) Uncertainty in both DEM-based and manual delineation was quite similar, and the most significant differences were in those areas in which the delineation experts interpreted the existence of subsurface flows or where the experts had clearly been in error (**III**, Appendix B & C).

m) It was shown that error propagation analysis could be used to find out the lowest limit for the size of drainage basins that can be delineated with sufficient accuracy (**III**). However, the results include uncertainty due to simplification of the error model (**IV**).

n) For error propagation analysis purposes, an analytical approach appears to be more useful for constrained derivatives, while the Monte Carlo method is appropriate for analysing both constrained and unconstrained derivatives (**II**).

## CHAPTER 5

### DISCUSSION

The thesis offers a three-step framework for exploring the suitability of DEM for terrain analysis. The DEM error was characterised consistently in all steps with the use of geostatistical modelling. However, there are a number of critical issues within each of the steps and these are discussed below.

#### 5.1 Visualising morphological gross errors

The explorative visualisation methods presented in **I** confirmed that fine toposcale DEMs contain a number of morphological gross errors that are ignored in the conventional statistics describing quality of a geospatial dataset. However, these morphological gross errors are not included in typical error propagation analysis and may, even so, result in significant bias in terrain analysis (Wise 1998). In addition to methods presented earlier (Wood & Fisher 1993, Wood 1996b), a new remainder image technique was introduced, which adds hill shading as a depth cue for the modulo maps. Even though the visualisation

methods in **I** highlight the potential problem areas of DEMs, a lot of manual work and the careful attentiveness of the user is still needed during the interactive process of exploring the visualisations.

#### 5.2 Geostatistical modelling of DEM error and the universe of discourse

Building a geostatistical model of DEM error includes a number of factors affecting the error propagation analysis. These factors include the decision of stationarity and its implications, the universe of discourse in characterising the error, and the complexity of fine-tuning the DEM error model parameters.

The results of **IV** and **I** challenge the appropriateness of error propagation analysis based on modelling DEM error directly as second-order stationary stochastic processes. The hypothesis that geostatistics offers a suitable theoretical framework for modelling DEM error was shown to be a simplification of reality, even though the feasibility of implementing the model in the analytical and simulation-

based error propagation analysis makes it tempting. In **IV** we showed that global geostatistical characterisation of DEM error is a gross generalisation of the complex reality, and even local semivariogram modelling of DEM error was not trouble-free. However, the possibility of isolating and modelling systematic-like effects of the error surface and using geostatistical simulation only for the residual process should be investigated.

It has been claimed that no professional GIS software currently in use can provide the user with information about the confidence limits that should be associated with the results of the analysis (Heuvelink 1999). However, from the decision-making perspective, one might question whether it is wiser to ignore the uncertainty of DEM-based terrain analysis or to believe the results of error propagation analysis indiscriminately without understanding the deficiencies of DEM error modelling. An improved understanding of the incompleteness of DEM error models is necessary to minimise the risk of misusing error propagation analysis.

The universe of discourse is a representation of the real or hypothetical world that includes everything of interest (ISO 2002), and the error in the geospatial dataset is defined with respect to the universe of discourse. The choice made in **IV** was to compare the DEM with reference data at the same horizontal resolution. Thus, the uncertainty introduced by the lattice representation remained unknown. In an alternative approach, the uncertainty caused by the lattice representation could be implemented in the DEM error model as a nugget effect.

Until now the use of spatially uncorrelated DEM error models has been considered as a 'worst-case scenario' (e.g. Heuvelink 1998, Wechsler 2000, van Niel et al. 2004, Raafflaub & Collins 2006), but this opinion may now be

challenged because none of the DEM derivatives investigated in the study had maximum variation with spatially uncorrelated random error. In local surface derivatives, such as slope and aspect, the maximum error in the results appeared to exist when the practical range of the error's spatial autocorrelation was roughly equal to the size of the surface derivative's calculation window. In global terrain analysis, such as drainage basin delineation, the variance of the results in the test setting increased with the increase in the spatial autocorrelation range. The study did not reveal the morphology-related limit at which delineation uncertainty begins to decrease.

Finally, perhaps we have to accept that the results from fine toposcale DEM error propagation analyses should be treated qualitatively rather than quantitatively (**IV**), because the choice of appropriate parameters sets high quality and density demands for the reference data (**IV**) and fine-tuning the DEM error model parameters has a significant impact on the error propagation analysis results (**II**). The use of a 'worst-case scenario', which overcomes the complexity of building a realistic error model and represents maximum variance of the terrain analysis, might be an alternative approach for local surface derivatives. In the case of global surface derivatives, a similar approach is unreasonable, since the 'worst-case' parameters are clearly related to local topography and the result would be unrealistically pessimistic.

### **5.3 Process convolution in simulating DEM error**

The motivation for applying a process convolution method for generating realisations of a DEM error model (**III**) was its efficiency and easy implementation in any GIS software featuring filtering functionality for raster data.

Table 6. Performance comparison of process convolution and SGS showing duration of generating one realisation with Gaussian autocorrelation, practical range = 6 units. Test equipment: AMD Athlon™ XP 2100+, 500 MB RAM, Microsoft Windows 2000. SGS: R's (R Development Core Team 2006) geostatistical package gstat (Pebesma 2004). Process convolution: **II**, MatLab 6.0.0.88 (R12).

Method	Area (nodes)			
	100x100	200x200	500x500	1000x1000
SGS, conditioned on <20 observations	1.5 s	11 s	440 s	6368 s
SGS, conditioned on <50 observations	6.1 s	31 s	553 s	6842 s
Process convolution	0.05 s	0.13 s	0.66 s	2.6 s

In addition, the direct relation between the correlogram model and the convolution kernel bypasses the typical calibration of spatial moving averages. Comparison with SGS (Table 6) shows that there has been significant improvement in the performance of process convolution for generating the realisations. It should, however, be noted that SGS is a conditional simulation method honouring the original observations and it is needed in a number of geostatistical applications. The efficiency of process convolution can be utilised in either increasing the number of simulation runs, increasing the number of simulation nodes (micro scale DEM error propagation analysis), or

producing a small number of realisations very quickly. Thus, the method presented makes it possible for interactive terrain analysis web services to be set up and the concept of an 'error button' (Openshaw et al. 1991, Goodchild et al. 1999) could realistically be incorporated into these services.

Even though the numerical solution for calculating the convolution kernel (**III**) is suitable only for random processes with Gaussian spatial autocorrelation, it has been shown that process convolution applying Fourier transformation for defining the convolution kernel can also be used on an exponential spatial autocorrelation model as well (Ukkonen 2006).



## CHAPTER 6

# CONCLUSIONS

The concluding remarks of the thesis are set out with reference to the three questions given as the aims of the study.

*1. How should the DEM error be evaluated and reported so as to serve the needs of DEM users as well as possible?*

The thesis shows that appropriate and exhaustive reporting of various aspects of fine toposcale DEM error was not a simple task for three reasons. Firstly, the number of outliers in DEM error distribution is high and their objective elimination is difficult (**IV**). Secondly, morphological gross errors, which are detectable only by appropriate visualisation methods, occur frequently (**I**). Finally, use of global characterisation of DEM error is a gross generalisation of reality due to the small extent of the areas in which the decision of stationarity is not violated (**IV**).

Commonly used measures of dispersion (e.g. standard deviation) might be adequate for setting a quality requirement for the fine toposcale DEM, but are insufficient for reporting accuracy of a DEM. The reason is that the

measures may become arbitrary due to a high number of outliers, and subjective decisions on eliminating them from the error distribution will be necessary. Therefore, robust statistics (e.g. minimum, maximum, median, quartiles, deciles and IQR) should be used in addition to conventional measures of dispersion.

The visualisation techniques presented in **I** gave valuable additional information to the DEM user, revealing aspects of DEM quality which are hidden behind the conventional error statistics. Since morphological gross errors were not included in the model of DEM error presented, the methods given in **I** should always be used as the first step before any terrain analysis task. The advantage of the methods is that because no reference data is needed, the methods are usually readily available in DEM processing software.

The complexity of fine toposcale DEM error, including non-stationarity and morphological gross errors, is a challenge from two perspectives: sampling design for accuracy assessment and error propagation analysis. A number of earlier studies have shown that appropriate characterisation of the DEM error's

spatial structure is impossible with poor quality sampling design of reference data. Based on the results of **IV**, nested random sampling from a number of homogeneous terrain units might be an optimal solution. In addition, since the DEM error model parameters have a prominent effect on the error propagation analysis results and morphological gross errors are not included in the error model, understanding the uncertainty of error propagation analysis results becomes essential.

The extent to which characterising and reporting DEM quality should be within the remit of NMAs, and how much responsibility can be left for the DEM user, remains an open question. NMAs would be able to serve DEM users as well as possible if users are provided with the comprehensive information for error propagation analysis, but this may be beyond the scope of the data producer. NMAs could, however, improve the quality of their DEM products by learning from the error propagation studies.

## *2. How can the metadata concerning DEM error be taken into account in terrain analysis?*

The thesis presents a uniform three-step framework for investigating error propagation in DEM-based terrain analysis. The DEM error model was built using geostatistical methods. Even though the thesis focuses on fine toposcale DEMs, the framework is suitable for models at any spatial scale (Table 3).

While the analytical approach was found to be suitable for local surface derivatives, such as slope and aspect, the Monte Carlo method offers a flexible and general solution for any type of error propagation analysis. This challenges the generalisation that spatial autocorrelation of variables leads to use of Monte Carlo method (Zhang & Goodchild 2002). Modelling the DEM error as a second-order stationary sto-

chastic process made both of the approaches simple and feasible (**II** and **III**). However, the complexity of fine toposcale DEM error (**IV**) sets additional demands for the methodologically rigorous error propagation analysis. One way to overcome the limitations is to divide the research area into homogeneous clusters in which the decision of stationarity is valid. The method is adequate for error propagation analysis of local surface derivatives, but is not suitable for global derivatives such as automatic drainage basin delineation. In addition, the results in **IV** showed that such division is not trouble-free, and simplification of the actual error model parameters is apparent. To overcome the aforementioned statistical and spatial restrictions, alternative options for fine toposcale DEM error modelling include: 1) the use of process convolution (Thiébaux & Pedder 1986, Higdon 1998, Higdon et al. 1999, Higdon 2002, **III**), where independent Gaussian random variables are convolved with smoothing kernels varying as a function of location allowing non-stationary realisations of DEM error; 2) the use of conditional SGS based on dense reference data; 3) the use of simplified error models, in which low- and medium-frequency errors are ignored and only high-frequency errors are modelled; and 4) the use of a process-based approach, where modelling of the DEM error as a second-order stationary stochastic process is replaced by modelling the errors of the DEM production process. This method could possibly generate the most realistic realisations of the DEM error, but there is a danger that the error models will become too intricate for practical use. For options 1 and 2, the best solution might be the modelling and removal of systematic-like effects, including potential correlation with secondary information, prior to geostatistical analysis and simulation.

*3. What is the influence of DEM error on terrain analysis, especially in the example cases of slope, aspect and automatic drainage basin delineation?*

As expected, an increase in the DEM vertical error will increase the error in surface derivatives. However, contrary to expectations, the spatial autocorrelation of the model appears to have varying effects on the error propagation analysis depending on the application. The use of a spatially uncorrelated DEM error model has been considered as a 'worst-case scenario' (e.g. Heuvelink 1998, Wechsler 2000, Van Niel et al. 2004, Raaflaub & Collins 2006), but this opinion was now challenged, because none of the DEM derivatives investigated in the study had maximum variation with spatially uncorrelated random error **(II)**.

The study revealed that the importance of the appropriate shape of the spatial autocorrelation model, either exponential or Gaussian, was not as critical as the choice of appropriate autocorrelation parameters, practical range and sill. However, the shape of the spatial

autocorrelation model appeared to have more influence on the local derivatives, slope and aspect, than on the global drainage basin delineation **(II)**.

The results of error propagation analysis can be utilised in a number of ways. Firstly, error propagation analysis should be included routinely in terrain analysis for decision-making purposes. From the social perspective, for example, the inclusion of DEM error propagation analysis in flooding risk analysis would be significant. Secondly, the analysis of the probable catchment areas represented an example of an application-specific quality measure, which indicates how the error of the DEM sets the lowest limit for the size of drainage basins that can be delineated with sufficient accuracy. Thus, in some cases reporting the lowest limit instead of the DEM error could be more useful for a terrain analyst. Finally, if the purpose is to outline drainage basins with a high accuracy, the probable drainage divides can be used in focusing the field survey on only those areas that have the highest level of topography-related uncertainty **(III)**.



## CHAPTER 7

### FUTURE WORK

The use of exhaustive reference data and implementation of the process convolution method for DEM error propagation analysis in the thesis points to several potential areas for future study. Firstly, implementation of the process convolution method opens up two alternative routes for further developing the error propagation analysis tool. In one of these, aiming at flawless error propagation analysis, the process convolution method could be used for non-stationary modelling with spatially evolving convolution kernels (Higdon et al. 1999) and considering cross-correlations with secondary information (Carlisle 2005) for building heteroscedastic realisations of DEM error. The other route aims at the development of an 'error button' (Openshaw et al. 1991, Goodchild et al. 1999), which requires maximum computing power even at the cost of imperfections in the DEM error model and, possibly, a reduced number of simulation runs. By applying the methods of the thesis in a grid-computing environment and expanding the convergence analysis of the Monte Carlo simulation in other terrain analysis and study areas, it would be possible to build an interac-

tive web service in which the user can define the parameters of the DEM error model and this web service would then return the result of the error propagation analysis. In addition, the light computational load of process convolution makes the development of next generation error propagation analysis tools for micro-scale DEMs based on topographic LIDAR realistic.

Secondly, the thesis focused on analysing the propagation of errors, defined as elevation differences between DEM lattice points and reality, leaving open the question of uncertainty introduced by the lattice representation of the terrain. With exhaustive reference data, it would also be possible to investigate the uncertainty of the lattice representation and to fine tune the geostatistical DEM error model by including the nugget variance representing micro scale variation. Moreover, for characterising the DEM error the cumbersome use of GIS applications together with statistical and mathematical software should be replaced with an interactive and integrated explorative spatial analysis tool.

Thirdly, the thesis only scratched the surface in regard to the convergence of the Monte Carlo simulation results. It was concluded that convergence depends on the underlying topography, error model parameters and the specific terrain analysis task, but further systematic research is needed to increase understanding about the significance of the factors. A supplementary approach could be implemented in the terrain analysis web service in which the user could be given the interactive means to explore convergence of the error propagation analysis results during the Monte

Carlo simulation in real-time or within a short response time.

Finally, the exhaustive reference dataset enables exploration of appropriate sampling design for DEM error characterisation. As noted in the previous section, an optimal solution might be nested random sampling from a number of homogeneous terrain units, but more research is needed for a better understanding of the factors influencing the sample design for fine toposcale DEM error with apparent spatial autocorrelation.

## CHAPTER 8

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