



Helsinki  
Center  
of  
Economic  
Research

Discussion Papers

# Biodiversity Conservation in Boreal Forests: Optimal Rotation Age and Volume of Retention Trees

Erkki Koskela  
University of Helsinki, RUESG and HECER

and

Markku Ollikainen  
University of Helsinki

and

Timo Pukkala  
University of Joensuu

Discussion Paper No.2  
April 2004

ISSN 1795-0562

# Biodiversity Conservation in Boreal Forests: Optimal Rotation Age and Volume of Retention Trees\*

## Abstract

The paper examines the economics of biodiversity conservation at the stand level. We extend the Hartman model to take into account green tree retention as a means of creating new structural elements of old and decaying wood, capable of supporting variety of species in commercial forests. We first characterize qualitatively the socially optimal choice of harvest volume and rotation age, and their dependence on exogenous parameters. We then assess empirically the optimal volume of retention trees and optimal rotation ages in a simulation model calibrated to Finnish forestry for pine. We find that biodiversity conservation may increase the socially optimal rotation age far beyond the Faustmann rotation age. The optimal volume of retention trees is, naturally, sensitive to timber price and biodiversity valuation being about ten cubic meters per hectare under Finnish biodiversity valuation estimates and current timber prices.

**JEL Classification:** D62, H21, Q23

**Keywords:** rotation, tree retention, biodiversity management.

Erkki Koskela  
Department of Economics  
P.O. Box 17 (Arkadiankatu 7)  
University of Helsinki  
FI-00014 University of Helsinki  
FINLAND  
e-mail: [erkki.koskela@helsinki.fi](mailto:erkki.koskela@helsinki.fi)

Markku Ollikainen  
Department of Economics and Management  
P.O. Box 27  
University of Helsinki  
FI-00014 University of Helsinki  
FINLAND  
e-mail: [markku.ollikainen@helsinki.fi](mailto:markku.ollikainen@helsinki.fi)

Timo Pukkala  
Faculty of Forestry  
P.O.Box 111  
University of Joensuu  
FI-80101 Joensuu  
FINLAND  
e-mail: [timo.pukkala@joensuu.fi](mailto:timo.pukkala@joensuu.fi)

\* Koskela thanks the Research Unit on Economic Structures and Growth (RUESG) in the University of Helsinki funded jointly by the Academy of Finland, University of Helsinki, Bank of Finland, Nokia Group, and the Yrjö Jahnsson Foundation, for financial support. Ollikainen thanks the Academy of Finland for the grant No.204476 for the position of Senior Researcher.

## 1. Introduction

The Convention on Biological Diversity (UNCED 1992) implies multiple dimensions for biodiversity maintenance in forestry. Species, habitats and genetic diversity must be conserved at many scales: at the stand level, landscape level, and the level of larger forest regions (Hunter 1999). At all these levels forest ecosystems are hierarchical and it should be treated so. Disturbance dynamics has a crucial role for the diversity of species and habitats and it should be mimicked in forestry management. (Franklin 1993, Holling 2001, Bergeron et al. 2002, Kuuluvainen 2002).

Understanding forests as hierarchical and structural landscapes stresses a need to manage forests as a connected network (see Lindenmayer and Franklin 2002). At the landscape level this network consists of different types of forests and stands. Entirely preserved forest areas are the core of the biodiversity conservation network. Around this core should be built a pattern of buffer zone forests, commercial forests with restricted management and regular commercial forests in which biodiversity conservation is actively taken into account. All parts of the network are linked to each other so as to ensure the interconnection and continuum of forest landscapes (see also Franklin and Forman 1987).<sup>1</sup>

Given that the preserved core of the network is of limited size (preservation becomes increasingly costly), biodiversity management in commercial forests is necessary for a successful biodiversity maintenance network. In commercial forests, harvesting and

---

<sup>1</sup> The most popular models of the hierarchical structure of forests are (i) the corridor-patch-matrix model (Forman 1995) and (ii) the landscape continuum model (McIntyre and Hobbs 1999). The corridor-patch-matrix model constructs the forest landscape with the help of three concepts: habitat patches, corridors connecting them and the surrounding areas that are unsuitable for species in question. This model draws on the theory of island biogeography (see MacArthur and Wilson 1967), but has been supported recently by the theory of metapopulation (see e.g. Hanski 1999). This model has a variety of applications to biodiversity management in commercial forests, for instance, in the U.S. and in Scandinavian countries. The landscape continuum model treats the landscape with the help of environmental factors as a forest cover, which changes smoothly in terms of its structure. This model suits well to forests where different habitat patches are not distinct. This situation holds true, especially, in naturally developed forest areas (Wiens 1997).

management has substituted for natural disturbances typical to virgin forests. Prevailing harvesting and thinning methods, aiming at producing valuable timber, traditionally take away altogether the old tree generation and create a new even-aged stand. Biodiversity maintenance in the management of commercial forests implies a focus on the structural elements, such as the volumes of dead and decaying wood, on the distribution of all tree species, and on the biodiversity benefits of lengthened rotation ages. Moreover, biodiversity conservation requires application of multiple harvesting methods so as to mimic more closely forest structure and disturbance dynamics.<sup>2</sup> Selective harvesting and design of small clearing areas are the means of keeping spatially heterogeneous and uneven-aged forestry structures and at ensuring forest cover continuum.

The above-mentioned aspects concern both management at the stand level and management of bundles of stands, that is, structuring the stands in a landscape. While the analysis at the landscape level is crucial on the economics of biodiversity networks, it must be based on stand level analysis, which is enlarged to cover spatial aspects (that can be analyzed within the framework of stand interdependency). Thus far the stand level analysis has been missing in forest economics literature, where biodiversity is usually analyzed within the confines of species preservation in site selection models or within the ecosystem management and other interdependence models.<sup>3</sup>

In this paper we will examine the economics of biodiversity conservation in commercial forests at the stand level. We specifically focus on green tree retention as the basic means of promoting structural elements vital to biodiversity promotion in

---

<sup>2</sup> Traditional clear-cutting retains its role, as it works like strong forest fires. However, given that forest fires with very strong impacts are not as common as previously thought, other methods are needed as well.

<sup>3</sup> Most of the stand-based literature focuses on species preservation in the site selection framework, see Ando et al. 1998 for general approach, and Stockland 1997 and Juutinen et al. 2004 for applications to forestry. Another strand of the literature is ecosystem management in forestry, see for instance, Montgomery et al. 1994, Hopf and Raphael 1993, Haight 1995, Haight and Travis 1997, Bevers and Hof 1999. For interdependence literature, see Swallow and Wear 1993, Swallow et al. 1997, Koskela and Ollikainen 2001b and Amacher et al. 2004.

even-aged forest management. The term “retention trees” refers to trees that are permanently left unharvested to die and decay in the forest to provide habitats to many kinds of species. Leaving retention trees has been recently adopted in some countries like Finland, Sweden and the U.S. Together with enlarged rotation ages and, possibly, selective harvesting methods retention trees are the important means for biodiversity in commercial forests.

Our extension of the Hartman model leads to the following research questions: at what age and in what volume a stand should be harvested so as to bring harvest revenue and biodiversity benefits? To provide answers to these questions, we analyze a simultaneous choice of the rotation age and the amount of retention trees, that is, trees left unharvested. For simplicity we assume that biodiversity benefits are additive in the rotation age of trees becoming harvested, and in the age and volume of retention trees. We solve the socially optimal rotation age and the volume of retention trees, and compare them with the private solutions when the landowner behaves according to the Faustmann or Hartman model. By using Finnish forestry data we also assess empirically the socially optimal volumes of retention trees and biodiversity-adjusted rotation ages, and compare them to conventional Faustmannian commercial harvesting solution. This is carried out to a typical pine stand in Finland by using a complex forestry simulation model.

To anticipate the results, we show that the socially optimal rotation age is longer and the volume of unharvested trees larger than in the private optimum, where the private optimum plausibly does not entail retention trees at all. In our empirical application to the Finnish forestry we demonstrate that biodiversity conservation increases the optimal rotation age relative to the Faustmann age. The difference in rotation ages depends on the chosen interest rate, varying from 10 years for 1% interest rate to 6 years for 4% interest rate. Under our Finnish estimate of biodiversity valuation in commercial forests, i.e., outside forest conservation areas, the volume of retention trees per ha is 10 cubic meters and consists of about 30 trees. The optimal green tree retention increases (decreases) rapidly with biodiversity valuation (timber price).

The rest of the paper is structured as follows. Section 2 deals with our description of the retention trees and their decaying process and provides an analysis of both the socially optimal choice of rotation age and retention trees. In Section 3 we provide a numerical application for the case of Finnish forestry. Finally, there is a brief concluding discussion in Section 4.

## **2. Biodiversity Management in Boreal Forests: the Social Optimum**

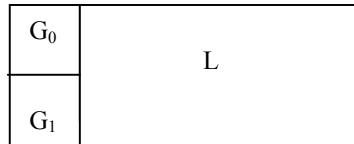
In this section we introduce biodiversity benefits into the Hartman model (for its general properties, see Hartman 1976, Strang 1893, Bowes and Krutilla 1985, 1989, Koskela and Ollikainen 2001a). We focus first on a case, where the landowner has an initial stand of age  $A$ , and examine how the original choice of the volume of retention trees and of the rotation age should be made. Although the time span even between the first and the next choices of harvesting is very long, and research knowledge on the development of biodiversity benefits is scarce, we start by focusing on the steady state considerations following the logic of conventional rotation analysis. Throughout this section we analyze the socially optimal biodiversity management.

### **2.1 Modeling Biodiversity Benefits**

Consider a forest plot comprising an original growing stand, of age  $A$ . In order to promote biodiversity, in addition to harvest revenue, the social planner wishes to create *structural elements*, such as decaying and dead trees and increasing tree species diversity, in stand management. This can be made by creating green tree retention ( $G$ ), that is trees left unharvested, in the forest area and letting it to decay. This green tree retention serves as a simplified aggregate description of the structural elements. Depending on the properties of biodiversity benefits, the volume of tree retention may be small or high – thus creating a continuum of selective harvesting, small clearing areas, and traditional clear-cutting.

In order to simplify the analysis, we assume that the decay of retention trees, which have achieved financial maturity and approach their biological maturity, takes place during one (the second) rotation period, while the other trees are harvested and new trees are planted after harvesting. The problem of choosing harvest volume, in addition to rotation age, also deals with the land use, because it means that part of the forest land is allocated to unharvested retention trees. We illustrate this in Figure 1, where the whole land area is divided into three parts and denoted by their sum,  $L + G_0 + G_1$ .

**Figure 1:** *Green tree retention as a land use decision*



Originally, the whole area is under forest cover with one stand of age  $A$ . The choice of the first harvest time and the volume of retention trees ( $G_0$ ) divides this land area into two parts: (i) bare land ( $L + G_1$ ) and (ii) an area of trees of age  $T(G_0)$ . When the new rotation period has elapsed the following holds:  $G_0$  has entirely decayed, and the rest of the area ( $L + G_1$ ) has a stand of age  $T$ . The planner leaves new retention trees ( $G_1$ ), harvests the rest, and plants the harvested and decayed areas ( $L + G_0$ ). While not specified in the model we assume that based on the site-specific characteristics, the choice of the location of retention trees is chosen spatially optimally.

Next we model biodiversity benefits from rotation age and retention trees as a simple way by modifying the felicity function of the basic Hartman model. More specifically, we assume that biodiversity benefits can be expressed as a sum of the benefits accruing from the stand becoming harvested, and benefits from the unharvested retention trees which reach their biological maturity and decay during the next (second) rotation period. We denote the current values of rotation age and retention tree by  $T$  and  $G$ , respectively. Thus biodiversity benefits,  $BB$ , can be expressed as

$$BB = a(T) + v(T, G) = \int_0^T F(x)e^{-rx} dx + \int_T^{2T} B(x, G)e^{-rx} dx \quad (1)$$

The first term in equation (1),  $a(T) = \int_0^T F(x)e^{-rx} dx$ , is conventional amenity valuation but is here applied to biodiversity. The properties of this amenity valuation function have been thoroughly discussed in the previous literature. Because our focus is on biodiversity, we may legitimately assume that older stands yield higher biodiversity benefits.<sup>4</sup> Thus we assume that  $F'(T) > 0$ .<sup>5</sup>

The second term,  $v(T, G) = \int_T^{2T} B(x, G)e^{-rx} dx$ , is our new extension. It describes the biodiversity benefits from retention trees, chosen during the current rotation period and accruing during the next rotation period. Reflecting the long rotation periods in northern boreal forests, the time between  $T$  and  $2T$  is assumed to be long enough for the retention trees to decay to a point where they provide biodiversity benefits but their land area can, nevertheless, be replanted.<sup>6</sup> Thus, biodiversity considerations make an explicit link between two stands and thereby two rotation periods.

We make the following assumptions concerning biodiversity benefits:

$$v_T = \hat{B}(T, G, r) \equiv e^{-rT} [B(2T, G)e^{-rT} - B(T, G)] > 0 \quad (2a)$$

$$v_{TT} = \hat{B}_T(T, G, r) < 0 \quad (2b)$$

---

<sup>4</sup> See Calish et. al (1978) for examples of cases where valuation of younger stands might be relevant.

<sup>5</sup> In what follows, derivatives of a function with one argument are denoted with primes, while partial derivatives of functions with more than one argument are denoted by subscripts.

<sup>6</sup> Actually, for instance, pine can stay alive and standing over several rotation ages. Allowing for additional rotation ages would be straightforward to do. It does not change qualitative results, however, but would considerably increase the number of terms in the analysis. In our simulation model we have a more explicit and detailed description of the dying and decaying process of the trees, so that this is allowed to affect over a higher number of subsequent rotations.

$$v_G = \int_T^{2T} B_G(x, G)e^{-rx} dx > 0; \quad v_{GG} = \int_T^{2T} B_{GG}(x, G)e^{-rx} dx < 0 \quad (2c)$$

$$v_{TG} = v_{GT} = e^{-rT} [B_G(2T, G)e^{-rT} - B_G(T, G)] > 0, \quad (2d)$$

where  $\hat{B}_T = e^{-rT} [e^{-rT} 2(B_T(2T, G) - rB(2T, G)) - B_T(T, G)]$

Interpretation of the derivatives of biodiversity benefit function is as follows. Marginal biodiversity benefit in (2a) is defined as a positive difference in diversity value of green retention between the beginning and the end of the second rotation period. Together assumptions in (2a) and (2b) are conventional meaning that intertemporal benefits from the age of retention trees over the second rotation have decreasing marginal benefits. According to (2c), the same is assumed to hold true for marginal benefits from the volume of green tree retention,  $G$ . We neglect here the possibility of threshold effects, i.e., a discontinuous increase in biodiversity benefits due to reserved trees, which may be important in some cases. Finally, the cross-derivative assumption in (2d) indicates that increasing the number (volume) of standing trees increases the marginal utility derived over time from these trees.

## 2.2 Biodiversity Management at the Stand Level

In this section we analyze biodiversity management both in the steady state and in the case where society has an old growth initial stand. While the former case is relevant for regeneration practices that promote developing biodiversity, the latter is highly important for the very current policy aiming at conserving the existing biodiversity tied to old growth forests.

### A. Social Planner's Problem in the Steady State

In the steady state, the social planner starts with bare land, which, however, has a given amount of retention trees  $G$ . The volume of timber as a function of rotation age

(with is calendar age in the steady-state) is defined by the forest growth function  $f(T)$ . Let  $p$  denote timber price,  $r$  real interest rate and  $c$  regeneration costs, which we assume to be constant like in the Faustmann and Hartman models. The social planner's economic problem includes now two decision variables: (i) the choice of the rotation age  $T$  and (ii) the determination of the volume of retention trees,  $G$ , so as to maximize the following objective function

$$SW = \left[ pe^{-rT} [f(T) - G] - c + \int_0^T F(x)e^{-rx} dx + \int_T^{2T} B(x, G)e^{-rx} dx \right] (1 - e^{-rT})^{-1}. \quad (3)$$

The first-order conditions for this problem read as

$$SW_G = -pe^{-rT} + \int_T^{2T} B_G(x, G)e^{-rx} dx = 0 \quad (4a)$$

$$SW_T = pf'(T) - rp[f(T) - G] + F(T) + B(2T, G)e^{-rT} - B(T, G) - rSW = 0 \quad (4b)$$

Equation (4a) can be interpreted as follows. The volume of retention trees is chosen so as to equate the present value of the marginal loss of the harvest revenue with the present value of sum of the marginal utility of retention trees over their whole decaying process. According to equation (4b), the optimal rotation age is chosen so that the marginal return of delaying the harvest by one unit of time equals the opportunity cost of delaying the harvesting. While the former is defined by the sum of the harvest revenue and biodiversity benefits during the first and the second rotation period, the latter includes the interest cost on standing timber and on land. It is clear that equations (4a) and (4b) differ in many ways from the Faustmann and Hartman rotation analysis. We shall return to this question in section 2.3.

Using the notation for marginal biodiversity benefits from green retention adopted in equation (2a), the second-order conditions can be expressed as

$$SW_{GG} = \int_T^{2T} B_{GG}(x, G)e^{-rx} dx < 0 \quad (5a)$$

$$SW_{TT} = e^{-rT} [pf''(T) - rpf'(T) + F'(T) + \hat{B}_T(T, G, r)] < 0 \quad (5b)$$

$$D = SW_{GG}SW_{TT} - SW_{TG}^2 > 0, \quad (5c)$$

where  $SW_{TG} = SW_{GT} = e^{-rT} [rp + \hat{B}_G] > 0$ , and  $\hat{B}_G = [B_G(2T, G)e^{-rT} - B_G(T, G)] > 0$ .

We assume that (5a) – (5c) hold.

Now we turn to analyze the qualitative properties of the volume of retention trees and rotation age in terms of exogenous parameters. Comparative statics will differ partly from the conventional rotation analysis, as now we have 2x2 equation system with symmetrical and positive cross-derivatives between the rotation age  $T$  and the volume of retention trees  $G$ . However, it turns out that in terms of the rotation age we have qualitatively similar outcomes as in conventional rotation models. We characterize the steady state comparative statics of the model in terms of timber price  $p$ , regeneration cost  $c$  and interest rate  $r$ .

The effects of timber price can be expressed as follows:

$$\frac{\partial G^*}{\partial p} = D^{-1} \{-SW_{Gp}SW_{TT} + SW_{Tp}SW_{TG}\} < 0 \quad (6a)$$

$$\frac{\partial T^*}{\partial p} = D^{-1} \{-SW_{Tp}SW_{GG} + SW_{Gp}SW_{GT}\} < 0 \quad (6b)$$

where  $SW_{Gp} = -e^{-rT} < 0$  and  $SW_{Tp} = f'(T) - r[(f(T) - G)(1 - e^{-rT})^{-1}]$ . Using the first-

order condition (4b) we get  $f'(T) - r[(f(T) - G)(1 - e^{-rT})^{-1}] =$

$p^{-1}[-(F(T) - rE) - (\hat{B}(T, G, r) - rH) - c(1 - e^{-rT})^{-1}] < 0$ , where the adopted notation is

$$E = (1 - e^{-rT})^{-1} \int_0^T F(x)e^{-rx} dx \quad \text{and} \quad H = (1 - e^{-rT})^{-1} \left[ \int_0^T F(x)e^{-rx} dx + \int_T^{2T} B(x, G)e^{-rx} dx \right]$$

We know from Koskela and Ollikainen (2001a) that  $(F(T) - rE) > 0$  if and only if  $F'(T) > 0$ , and we can show in an analogous way that  $\hat{B}(T, G, r) - rH > 0$ . Thus,  $SW_{Tp} < 0$  and we have from (6b) that the rotation age shortens and from (6a) that the amount of retention trees decreases. The latter effect results from the fact that higher timber price increases the profitability of harvesting and makes it more costly to conserve timber for biodiversity purposes.

The comparative statics of regeneration costs is:

$$\frac{\partial G^*}{\partial c} = D^{-1}\{SW_{Tc} \ SW_{GT}\} > 0 \quad (7a)$$

$$\frac{\partial T^*}{\partial c} = -D^{-1}\{SW_{Tc} \ SW_{GG}\} > 0, \quad (7b)$$

where  $SW_{Tc} = r(1 - e^{-rT})^{-1} > 0$  (note that  $SW_{Gc} = 0$ ). Hence, higher regeneration costs make timber production less profitable and, therefore, increase biodiversity maintenance in the form of higher volume of reserved trees and longer rotation period.

Finally, the effects of real interest rate can be described as

$$\frac{\partial G^*}{\partial r} = D^{-1}\{-SW_{Gr}SW_{TT} + SW_{Tr}SW_{TG}\} < 0 \quad (8a)$$

$$\frac{\partial T^*}{\partial r} = D^{-1}\{-SW_{Tr}SW_{GG} + SW_{Gr}SW_{GT}\} < 0. \quad (8b)$$

In (8a) and (8b)  $SW_{Tr} = e^{-rT} \left[ -p(f(T) - G) - TB(2T, G)e^{-rT} - SW - \frac{d}{dr}SW \right] < 0$ , and

$SW_{Gr} = Te^{-rT} p - \int_T^{2T} xB_G(x, G)e^{-rx} dx = \int_T^{2T} (T - x)B_G(x, G)e^{-rx} dx$ , where we have used

the first-order condition (4a).  $SW_{Gr}$  is clearly negative because it starts from negative value at  $2T$  and approaches zero at  $T$ .

In the expression of  $SW_{Tr}$ , the first two terms are clearly negative, but also the last two terms are negative (see e.g. Johansson and Löfgren 1985 and Koskela and Ollikainen 2001a). Thus, from (8b), a higher real interest rate increases the marginal benefits of delaying harvests more than its opportunity costs. This implies a shorter rotation period like in conventional Faustmann and Hartman models. In (8a), a higher interest rate increases the present value of the opportunity cost of green tree retention, tending thus to decrease its volume. This is reinforced by a decrease in the present value marginal biodiversity benefits. Thus, the overall outcome leads to smaller green tree retention.

Next we focus on the case where the society has an initial old growth stand with existing high level of forest biodiversity. This is an important issue for the current policy aiming at conserving the existing biodiversity associated with the current old-growth forests.

## **B. Social Planner's Problem under an Initial Stand**

Let us denote the growth function of the original stand of age  $A$  by  $Q(A)$ . The increase of the volume of this stand to the point of the first harvest is  $Q(T - A)$ , and the volume that is harvested can be expressed as a difference between the overall volume and the volume of reserved trees,  $[Q(T - A) - G]$ . Differing from the case of the steady state, the initial biodiversity benefits can be expressed over time as a sum of the whole existing stand

$$BB^0 = \int_A^T U(x - A)e^{-r(x-A)} dx , \quad (9)$$

where we use  $U$  instead of  $F$  to reflect the fact that, because there are no retention trees, initial stand covers a larger land area than subsequent steady-state stands.

The social planner's economic problem includes, again, the choice of two decision variables: the rotation age and the volume of retention trees. Given the original stand, this problem has to be studied in two phases. First, the society decides upon the use of the original stand. Then, from that onwards the choice reduces to conventional steady-state choice of rotation age and reserved trees. The steady-state rotation age and retention tree problem was defined in equation (3).

The objective function of the social planner for the initial choice of the optimal rotation age and of the volume of retention trees is given in

$$SW^0 = pe^{-r(T-A)}[Q(T-A) - G] + \int_A^T U(x-A)e^{-r(x-A)} dx + e^{-r(T-A)}SW \quad (10)$$

Equation (10) accounts for the fact that the sooner the initial harvest is made, the sooner the harvest benefits from subsequent rotations become.<sup>7</sup> The difference  $(T - A)$  tells how long the society will wait for the first harvesting time (naturally, if it turns out that  $T < A$  then the stand is harvested immediately).

The first-order conditions for the initial choices of the volume of rotation trees and rotation age are

$$SW_G^0 = -p + \int_T^{2T} B_G(x, G)e^{-rx} dx (1 - e^{-rT})^{-1} = 0 \quad (11a)$$

$$SW_T^0 = pQ'(\cdot) - rp[Q(\cdot) - G] + U(\cdot) - rSW = 0 \quad (11b)$$

---

<sup>7</sup> Note that the choice of the first  $G$  will marginally affect the regeneration cost  $c$ , which from that onwards remains constant. For simplicity we, however, abstract from this feature.

Equation (11a) can be interpreted as follows. The amount of retention trees is chosen so that the harvest revenue lost from the last cubic meter equals to present value of the sum of savings in regeneration costs and the marginal utility of biodiversity generated by this volume. Note that even though the marginal benefits are derived during the next rotation period, they affect the original decision phase. From (11b) we can conclude that the optimal rotation age is chosen so that marginal return of delaying the harvest by one unit of time equals the opportunity cost of delaying the harvesting. While the former is defined by the sum of the harvest revenue and biodiversity benefits, the latter include the interest cost on standing timber and on the land. From (11b), retention trees affect the rotation age at the margin, by changing both marginal return and the opportunity cost of harvesting.

Comparative statics of the initial stand will not differ qualitatively from that of the steady state (see Appendix, which also reports the second-order conditions). Therefore, we just condense the results here as

$$T^0 = T(\underset{-}{p}, \underset{-}{r}, \underset{+}{c}) \quad (12a)$$

$$G^0 = G(\underset{-}{p}, \underset{-}{r}, \underset{+}{c}). \quad (12b)$$

Thus, at the interior solution, the comparative static effects of price, real interest rate and regeneration costs on the optimal rotation age  $T$  are conventional. The effects of these variables on the volume of retention trees  $G$  are natural. Higher timber price and interest rate make timber production more profitable relative to biodiversity benefits and decrease the volume of retention trees, while a higher regeneration costs will increase it.

### 2.3. Private landowners' behavior

How do private landowners value forest amenities? Traditionally two hypotheses have been presented and used. The most common assumption is that landowner maximizes the present value of harvest revenue from timber production over infinite series of rotations. In this case the landowner behaves as described in the Faustmann rotation model. An alternative approach – which lies in conformity with some indirect empirical evidence (see e.g. Binkley 1981 and Kuuluvainen et al. 1996) - is to assume that the landowner maximizes the present value of the sum of harvest revenue and amenity services over infinite time horizon, behaving thus like the landowner in the Hartman model.

For the purpose of this paper, we ask: is there evidence on a possibility that landowners value biodiversity from their own point of view? Unfortunately there are no empirical studies concerning this issue. While landowners may sometimes put value on some species or land areas, it is plausible to think that typically they do not take into account the whole spectrum of biodiversity. Given these considerations we will here focus on both basic types of private landowner preferences in this section. Thus, landowners are assumed to behave either in the Faustmannian or Hartmanian way.

When the landowner follows the Faustmann model, he maximizes the present value of harvest revenue from timber production over infinite cycles of rotations. Thus the objective function in the steady state can be written as

$$V = (pe^{-rT} f(T) - c)(1 - e^{-rT})^{-1} \quad (13)$$

The solution to this problem is well-known (see e.g. Johansson and Löfgren 1985). The following first-order condition characterizes the privately optimal rotation age:  $V_T = 0 = pf'(T) - rpf(T) - rV$ . Thus the Faustmann behavior produces a solution pair

$(T^F, G^F)$ , for which it holds that  $G^F = 0$ . Comparing this to equations (4a) and (4b) suggests  $T^F < T^*$  and  $G^F = 0 < G^*$  where the variables with asterisk refer to the socially optimal choices.

Under the Hartman behavior, the landowner maximizes the present value of the sum of harvest revenue and monetary value of amenity services over infinite time horizon. The objective function is now given by

$$W = (pe^{-rT} f(T) - c + \int_0^T A(x)e^{-rx} dx)(1 - e^{-rT})^{-1} \quad (14)$$

In (14) we denote the private valuation of amenities by  $A(x)$  to indicate that this valuation function may not be related to age dimension of biodiversity but other types of amenities. The first-order condition for the private optimum is now  $W_T = pf'(T) - rpf(T) + A(T) - rW = 0$ . This condition implicitly defines the solution pair  $(T^H, G^H)$  for which it also holds that  $G^H = 0$ . Therefore, also in the Hartman framework we have that  $G^H = 0 < G^*$  and, if the landowner values young stands, we also have that  $T^H < T^*$ . However, if the landowner values old stands the Hartman rotation age may be longer or shorter than the biodiversity benefits based rotation age depending on how biodiversity (age) valuation function  $F(T)$  and the private amenity valuation function  $A(T)$  relate to each other. To conclude, both solutions fail to achieve the socially optimal rotation age and the socially optimal volume of retention trees. Thus there is scope for government intervention.

Our theoretical analysis of the socially optimal choice of rotation age and retention trees in our framework has now been completed. We ask next: How does our model behave empirically, i.e. how great is the optimal amount of retention trees, how does the socially optimal rotation age relate to private rotation age under plausible description of actual forestry and social valuation of biodiversity. These are the issues we study in the next section by using a complex forestry simulation model.

### **3. An Empirical Application to Finnish Forestry**

In this section we illustrate our model by using a complex numerical simulation developed for Finnish Forestry. First, we describe the model and drawing on Finnish empirical studies develop our estimates of biodiversity valuation. We then assess empirically the length of the rotation period and the volume of retention trees, and their behavior when exogenous parameters change.

#### **3.1 The numerical simulation – optimization system**

A simulation – optimization system was developed for numerical optimization of the rotation length and amount of retention trees. The simulation system calculates the value of the objective function with the combination of our decision variables, while the optimization system gradually modifies the values of decision variables based on the feedback from the simulation system, and eventually finds the optimal rotation length and basal area of retention trees. The algorithm developed by Hooke and Jeeves, and adopted from Osyczka (1984), for non-linear derivative-free optimization was used (see Pukkala and Miina 1997 for more details).

Simulation of stand development is based on individual trees. The simulation begins with bare land with no retention trees and no deadwood. The stand establishment is predicted with the models of Miina and Saksa (2004). The models predict the number of surviving planted trees per hectare, as well as the amount of naturally regenerated pine, spruce, birch, and hardwood coppice. Stand development is simulated in 5-year time steps. Various Finnish models are used to predict the juvenile height growths and diameters of seedlings from the seedling stage to the sapling stage (dbh 5 cm), after which the individual-tree growth models of Nyysönen and Mielikäinen (1971) are used. A tending treatment is simulated at a stand age of 5 to 20 years (depending on site and planted tree species). It removes all coppices and regulates the frequencies of other trees. The stand establishment and tending costs, used in the simulator, are based on cost statistics.

The self-thinning models of Pukkala and Miina (1997) are used to calculate the maximum stand density for a given mean tree diameter. Mortality occurs when this limit is passed, creating one or several cohorts of standing deadwood (snags). During a time step, a part of a snag cohort forms a down-wood cohort, its relative frequency being equal to the probability of falling down. Both snag and down-wood cohorts decompose with time, the decomposition rate being clearly higher for down-wood than for snags.

Stand development is simulated until the rotation age is reached, after which a final cut is simulated. Retention trees may be left to continue growing, depending on the current input value of the retention tree parameter. The roadside value of the removed volume (gross income) is calculated using user-supplied unit prices of different timber assortments. The assortment volumes are calculated using the taper functions of Laasasenaho (1982). The harvesting cost is calculated with the models of Valsta (1992).

Simulation is continued for three additional rotations, keeping the deadwood cohorts and retention trees of the previous rotation(s). The simulation is otherwise similar as during the first rotation except that there are now initial retention tree cohorts and initial deadwood. The growth of retention trees is simulated using the growth models of Nyssönen and Mielikäinen (1971). A part of a retention tree cohort is wind-thrown and another part may die of senescence during a time step, the relative frequencies of these new cohorts depending on the probabilities of these events. Dead retention tree cohorts decompose with the same rate as the other deadwood cohorts. A standing deadwood cohort originating from a retention tree cohort falls down with the same probability as other snags.

Retention trees are assumed to reduce the growing space that is available to the other trees: their effect to the other growing stock is simulated through an area multiplier. The share of growing space taken by retention trees is equal to the ratio of the basal

area of retention trees to the maximum stand basal area that the site can sustain. If the basal area of retention trees decreases due to mortality, the growing space available to other trees increases creating accelerated growth. It is assumed that the other trees can fully utilize the growing space left by dead retention trees. This kind of simulation is reasonable when retention trees occur in dense and small groups, which is the current practice.

In addition to costs and incomes, the simulator calculates a biodiversity index for the stand at every time point. The biodiversity index is as a weighted sum of scaled values of various structural elements present in the stand. The structural elements are: volumes of different tree species, volumes 10-cm diameter classes, and volumes deadwood components (standing deadwood and down-wood of different tree species). Each element increases the index fast up to certain level (“satisfactory amount”) after which its additional contribution becomes very small.

The monetary value ( $\text{€ha}^{-1}\text{a}^{-1}$ ) of the maximum biodiversity index is a user-supplied parameter. We used Finnish estimates for valuation of biodiversity conservation as a part of normal practices in commercial forestry. A contingent valuation study by Rekola and Pouta (1999) suggests that the mean of WTP for an increase of retention trees from current 15 to 30 would be 40 euros. We calibrate our quadratic biodiversity valuation function to reflect this estimate as follows. The value (VAL) of biodiversity index was calculated from equation  $VAL = WTP(BD/BD_{max})$ , where WTP is the value of the maximum biodiversity index ( $BD_{max}$ ) of the stand.

The forestry simulation model was operated as follows. In a steady state optimization, the objective function value was calculated from the last simulated rotation, which was assumed to be repeated to the infinity. The other rotations were used to initialize the steady-state amounts of deadwood and retention tree cohorts, present in the beginning of the last rotation.

### 3.2 Simulation Results

Our results are solved for pine under typical growth conditions in Southern Finland without thinning treatment. We report first our commercial reference point: the rotation age, harvested timber volume and net income solved from the Faustmann rotation model in Table 1. We report our results for a set of real interest rates ranging from 0.01 to 0.04.

**Table1.** *The privately optimal rotation age in the Faustmann model*

	r = 1%	r = 2%	r = 3%	r = 4%
Rotation Age	76	66	60	56
Retention m <sup>2</sup> /ha	0	0	0	0
Mean annual harvest	4.43	4.42	4.21	3.90
Mean annual net income	179	170	150	135
Timber benefit	11135	3298	984	106
BD benefit	0	0	0	0
Total benefit (SEV)	11135	3298	984	106

Using the value of 0.03 as our benchmark interest rate, in the absence on biodiversity (any amenity) valuation, the socially optimal rotation age is 60 years. Under our forest growth function, this implies 4.2 cubic meters as the mean annual harvest and 150 euros/ha as the respective net income. The total site expectation value is 984 euros per ha. Naturally, the volume of retention tree is zero in the Faustmann model. The privately optimal rotation age decreases in the real interest rate, as the model suggests. Note, finally, that relative to current forestry practice the rotation ages in Table 1 (and also in subsequent Tables 2 and 3) are rather short. One reason for this is that, following our theoretical models, we omit commercial thinning, which tends to postpone the optimal age for final felling (see e.g. Pukkala et al. 1998).

Table 2 provides result of the empirical counterpart of our biodiversity model.

**Table 2.** *The socially optimal rotation age and retention volume*

	r = 1%	r = 2%	r = 3%	r = 4%
Rotation age	86	71	64	62
Retention m <sup>2</sup> /ha	1.3	1.1	0.8	0.1
Mean annual harvest	4.24	4.29	4.20	4.14
Mean annual net income	177	168	157	147
Timber benefit	10403	3027	945	54
BD benefit	2252	618	246	148
Total benefit (SEV)	12655	3644	1191	202

We use, again, the value of 0.03 of the real interest rate as our benchmark. The socially optimal rotation age is 64 years being longer than the privately optimal age. The most important difference between Tables 1 and 2 is, naturally, that the amount of retention trees is now positive, being 0.8 m<sup>2</sup>/ha. Multiplying it by 10 gives roughly the retention volume and assuming that the diameter size is about 21 centimeters (which both reflect adequately forest statistics) gives about 25 retention trees. Thus, given that at the final felling there are 800-900 trees per ha, this means that 3% of wood biomass is left to stand in small groups in the area. The mean annual harvest slightly decreases because retention trees, but the mean annual net income slightly increases because of increased size of harvested timber due to longer rotation age (recall, the Faustmann solution lies below the MSY, hence longer rotation implies higher volume). The site expectation value is higher reflecting the fact that the social value of forests is higher than private valuation. The difference to Table 1 is 207 euros per ha. Biodiversity benefits account about 25% of timber benefits. Like in Table 1, the optimal rotation age is decreasing in the interest rate as long as the value of land remains positive. Also, retention tree volume is decreasing in the interest rates, as was demonstrated in the theoretical model.

We conducted sensitivity analysis with respect to timber prices, biodiversity valuation and regeneration costs. The most illuminating results are collected in Table 3. They are calculated by using 3% interest rate as our basic choice.

**Table 3.** *Sensitivity analysis: biodiversity valuation, prices and costs*

	BD-valuation 80 E	1.2(timber price)	1.2 (regener. cost)
Rotation age	67	63	65
Retention m2/ha	2.7	0.01	1.0
Mean annual harvest	3.85	4.23	4.20
Mean annual net income	150	201	154
Timber benefit	826	1472	739
BD benefit	1288	206	255
Total benefit (SEV)	2113	1678	995

All variables affect the socially optimal rotation age and retention volume as one could expect in the light of the theoretical model. A considerably higher biodiversity valuation increases the socially optimal rotation age by 3 years and the volume of green retention becomes more than three times higher – turning to about 80 trees per ha, which means that about 9-10% of the wood biomass is left to stand in the final felling. Both the mean annual harvest and mean annual net income decreases relative to Table 2, because a considerable part of the growing space is used by retention trees. Biodiversity benefits account for half of the overall benefits. Hence, we can conclude naturally that the more biodiversity valuation increases, the more one approaches the case of selective harvesting and small amounts of harvested timber.

The effect of a 20% increase in timber price has dramatic effects on retention volume, which goes down, almost to zero, remaining though positive. Hence, the socially optimal volume of retention is very sensible to timber price. As expectable, harvest volume and net income are now higher than in Table 3. A higher regeneration cost may increase the rotation age, or retention tree volume, or both. In the example reported in Table 3, higher regeneration costs will increase the rotation age leaving the volume of retention trees the same as in Table 2. The total economic benefits are, however, lower than in Table 2.

Findings reported in Table 3 have an important implication. Given that biodiversity conservation is very sensitive to timber price, changes in market demand may quickly undermine efforts to conserve biodiversity in commercial forests – unless the valuation of biodiversity increases with higher timber prices. This result stresses the role on entirely preserved forest areas as the core of the biodiversity conservation network.

#### **4. Conclusion**

We have analyzed biodiversity conservation at the stand level by extending the standard Hartman model to account for the biodiversity effects of retention trees left at final felling. This volume of retention tree creates new structural elements in commercial forests in the forms of old and decaying trees, and deadwood, which promote species diversity. Depending on the properties of biodiversity benefits, the volume of tree retention may be small or very high – thus creating a continuum of selective harvesting, small clearing areas, and traditional clear-cutting.

In theoretical analysis preliminary answers were provided to the following questions: at what age and in what volume a stand should be harvested so as to bring harvest revenue, amenity services and biodiversity benefits. By assuming that biodiversity benefits are additive in the rotation age and in trees left unharvested, we showed that biodiversity maintenance requires a simultaneous choice of the rotation age and trees left unharvested. We solved the socially optimal rotation age and the volume of retention trees, and compared them with the private solution when the landowner behaves according to either Faustmann or (the basic) Hartman model.

In our empirical application to the Finnish Forestry we demonstrated that biodiversity conservation increases the optimal rotation age relative to the Faustmann case, the difference varying from 20 to 4 years depending on the chosen interest rate. Under our empirical estimate of biodiversity valuation in commercial forests, i.e., outside forest conservation areas, the volume of retention trees per ha is almost 10 cubic meters and

consists of about 25 trees. Moreover, the optimal green tree retention increases (decreases) rapidly with biodiversity valuation (timber price).

Our extension raises many interesting research topics. First, extending our stand level analysis to cover biodiversity management at the landscape level is an interesting, but demanding challenge. This extension could be made by introducing interdependency between stands for instance along the lines, presented in Amacher et al. (2004), Koskela and Ollikainen (2001b), Swallow and Wear (1993) and Swallow et al. (1997). Second, it would be important to analyze what kind of policy instruments our model suggests the social planner should use to promote biodiversity conservation in commercial forests.

## Literature Cited

- Amacher, G.S., Koskela, E. and M. Ollikainen 2004. Forest Rotation and Stand Interdependency: Ownership Structure and Timing of Decisions. Forthcoming in *Natural Resource Modeling* 17, 1-43.
- Anderson, S.P., de Palma, A. and B. Kreider. 2001. The Efficiency of Indirect Taxes under Imperfect Competition. *Journal of Public Economics* 81, 231-251.
- Ando A, J. Camm, S. Polasky and A. Solow 1998. Species Distribution, Land Values and Efficient Conservation. *Science* 279, 2126-2128.
- Beavers M. and J. Hopf 1999. Spatially Optimizing Wildlife Habitat Edge Effects in Forest Management Linear and Mixed-Integer Programs. *Forest Science* 45, 249-258.
- Bergeron, Y., A. Leduc, B. Harvey and S. Gauthier 2002. Natural Fire Regime: a Guide for Sustainable Management of the Canadian Boreal Forests. *Silva Fennica* 36, 81-95.
- Binkley, C. 1981. *Timber Supply from Private Nonindustrial Forests*. Yale University. School of Forestry and Environmental Studies. Bulletin No. 92.
- Bowes, M. and J Krutilla 1985. Multiple-Use Management of Public Forestlands, in *Handbook of Natural Resource and Energy Economics*. vol. II. (A.V. Kneese and J.L. Sweeney Eds). North-Holland, Amsterdam.
- Bowes, M. and J. Krutilla. 1989. *Multiple-Use Management: The Economics of Public Forests*. Washington D.C., Resources for the Future. 353 p.
- Calish, S., R. Fight and D. Teeguarden. 1978. How Do Nontimber Values Affect Douglas-Fir Rotations? *Journal of Forestry* 76, 217-221.
- Englin, J. and M. Klan 1990. Optimal Taxation: Timber and Externalities, *Journal of Environmental Economics and Management* 18, 263-275.
- Forman, R. 1995. *Land Mosaics: The Ecology of Landscapes and Regions*. New York, Cambridge University Press.
- Franklin, J. 1993. Preserving Biodiversity: Species, Ecosystems or Landscapes, *Ecological Applications* 3, 202-205.
- Franklin, J., and R. Forman 1987. Creating Landscape Patterns by Forest Cutting: Ecological Consequences and Principles. *Landscape Ecology* 1, 5-18.
- Haight, R. 1995. Comparing Extinction Risk and Economic Cost in Wildlife Conservation Planning. *Ecological Applications* 5, 767-775.
- Haight R., L. Travis 1997. Wildlife Conservation Planning Using Stochastic Optimization and Importance Sampling. *Forest Science*, 43, 129-139.
- Hanski, I. 1999. Habitat Connectivity, Habitat Continuity, and Metapopulations in Dynamic Landscapes. *Oikos* 87, 209-219.
- Hartman, R. 1876. The Harvesting Decision When A Standing Forest Has Value, *Economic Inquiry* 14, 52-58.
- Holling, C. 2001. Understanding the Complexity of Economic, Ecological and Social Systems. *Ecosystems* 4, 390-405.
- Hooke, R and T. Jeevens 1961. "Direct search" Solution of Numerical and Statistical Problems. *Journal of Association of Computational Machinery* 8, 212 – 229.

- Hopf, J. and M. Raphael 1993. Some Mathematical Programming Approaches for Optimizing Timber Age-Classes Distributions to Meet Multispecies Wildlife Population Objectives. *Canadian Journal of Forest Research* 23, 828-834.
- Hunter, M. 1999. *Maintaining Biodiversity in Forest Ecosystems*. Cambridge University Press.
- Innes, R., S. Polasky and J. Tschirhart 1998. Takings, Compensation and Endangered Species Protection on Private Lands. *Journal of Economic Perspectives* 12, 35-52.
- Juutinen, A., E. Mäntymaa, M. Mönkkönen and J. Salmi 2004. Cost-effective Selection of Boreal Old Forest Reserves: A Case from Northern Fennoscandia. *Forest Science*, forthcoming.
- Koskela, E. and M. Ollikainen. 2001a. Forest Taxation and Rotation Age under Private Amenity Valuation: New Results. *Journal of Environmental Economics and Management* 42, 374-384.
- Koskela, E. and M. Ollikainen. 2001b. Optimal Private and Public Harvesting under Spatial and Temporal Interdependence. *Forest Science* 47, 484-496.
- Koskela, E. and M. Ollikainen. 2003. Optimal Forest Taxation under Private and Social Amenity Valuation. *Forest Science* 49, 596-607.
- Kuuluvainen, J., H. Karppinen and V. Ovaskainen 1996 Landowner Objectives and Nonindustrial Private Timber Supply, *Forest Science*, 42, 300-309.
- Kuuluvainen, T. 2002. Natural Variability of Forests as a Reference for Restoring and Managing Biological Diversity in Boreal Fennoscandia. *Silva Fennica* 36, 97-125.
- Laasasenaho, J. 1982. Taper Curve and Volume Functions for Pine, Spruce and Birch. *Communications Institutum Forest Fennicum* 108. 74 pp.
- Lindenmayer, D. and J. Franklin 2002. *Conserving Forest Biodiversity. A Comprehensive Multiscaled Approach*. Island Press, Washington DC.
- MacArthur R. and E. Wilson 1967. *The Theory of Island Biogeography*. Princeton University Press, Princeton.
- McIntyre, S. and R. Hobbs 1999. A framework for Conceptualizing Human Effects on Landscapes and Its Relevance on Management and Research Models. *Conservation Biology* 13, 1282-1292.
- Miina, J. and T. Saksa 2004. Predicting the Regeneration Establishment in Norway Spruce Plantations Using a Multivariate Multilevel Model. (Manuscript).
- Montgomery, C., G. Brown and D. Adams 1994. The Marginal Costs of Species Preservation: The Northern Spotted Owl. *Journal of Environmental Economics and Management* 26, 111-128.
- Nyysönen, A. and K. Mielikäinen 1978. Estimation of Stand Increment. *Acta Forestalia Fennica* 163. 40 pp. In Finnish with an English summary.
- Oszczka, A. 1984. Multicriterion Optimization in Engineering with FORTRAN Programs. Ellis Horwood, Chichester. 178 pp.
- Pasanen, K. 1995. Integrating Variation in Tree Growth into Forest Planning. M.Sci. thesis in Forestry. *University of Joensuu*, Finland. 31 pp.
- Pukkala, T., S. Kellomäki, and E. Mustonen 1988. Prediction of the Amenity of a Tree Stand. *Scandinavian Journal of Forest Research* 3, 533-544.

- Pukkala, T. and J. Miina 1997. A Method for Stochastic Multi-objective Optimization of Stand Management. *Forest Ecology and Management* 98, 189-203.
- Pukkala, T., J. Miina, M. Kurttila, and T. Kolström 1998. A Spatial Yield Model for Optimizing the Thinning Regime of Mixed Stands of *Pinus Sylvestris* and *Picea Abies*. *Scandinavian Journal of Forest Research* 13, 31-42.
- Rekola, M. and E. Pouta 1999. Valuing Environmental Effects of Forest Regeneration Cutting. In Roper C. and A. Pard (eds): *The Living Forest. Non-market Benefits of Forestry. Proceedings of an International Symposium on the Non-Market Benefits of Forestry. Edinburgh.*
- Stockland, J. 1997. Representativeness and Efficiency of Bird and Insect Conservation in Norwegian Boreal Forest Reserves. *Conservation Biology* 11, 101-111.
- Strang, W. 1983. On the Optimal Forest Harvesting Decision. *Economic Inquiry* 21, 576-583.
- Swallow, S., P. Talukdar, and D. Wear. 1997. Spatial and Temporal Specialization in Forest Ecosystem Management under Sole Ownership. *American Journal of Agricultural Economics* 79: 311-326.
- Swallow, S. and D. Wear. 1993. Spatial Interactions in Multiple-Use Forestry and Substitution and Wealth Effects for the Single Stand. *Journal of Environmental Economics and Management* 25: 103-120.
- Valsta, L. 1992. An Optimization Model for Norway Spruce Management Based on Individual-tree Growth Models. *Acta Forestalia Fennica* 232. 20 pp.
- Wiens, J. 1997. The Emerging Role of Patchiness in Conservation Biology. In Pickett, S., Ostfeld, R., Shachak, M, and Likens, G. (eds.) *The Ecological Basis of Conservation: Heterogeneity, Ecosystems and Biodiversity*. Chapman & Hall, New York.

## Appendix: Second-order conditions of the initial stand case

The second-order conditions are

$$SW_{GG}^0 = \int_0^T B_{GG}(T, G)e^{-rx} dx (1 - e^{-rT})^{-1} < 0 \quad A1a$$

$$SW_{TT}^0 = pQ''(\cdot) - rpQ'(\cdot) + U'(\cdot) \left[ \int_0^T B_{TT}e^{-rx} dx - r \int_0^T B_T e^{-rx} dx \right] (1 - e^{-rT})^{-1} < 0 \quad A1b$$

$$\Delta = SW_{GG}^0 SW_{TT}^0 - (SW_{TG}^0)^2 > 0 \quad A.1c$$

where  $SW_{GT}^0 = SW_{TG}^0 = \int_0^T B_{GT}e^{-rx} dx (1 - e^{-rT})^{-1} > 0$ . Note first that the second-order conditions A1a – A1c are qualitatively similar to those reported for the steady state. Hence, it suffices to ascertain that by perturbing the first-order conditions with respect to exogenous variable yields qualitatively similar marginal effects. By differentiation we have

$$SW_{Gp}^0 = -e^{-rT} < 0; \quad SW_{Tp}^0 = Q'(T) - r[(Q(A+T) - G_0)] \quad A2$$

$$SW_{Gc}^0 = 0; \quad SW_{Tc}^0 = r(1 - e^{-rT})^{-1} > 0. \quad A3$$

$$SW_{Gr}^0 = -Te^{-rT} (1 - e^{-rT})^{-2} \int_0^T F_G(T, G)e^{-rx} dx - (1 - e^{-rT})^{-1} \int_0^T xF_G(T, G)e^{-rx} dx < 0;$$

$$SW_{Tr}^0 = -(Q(A'T) - G_0) - SW^0 - r \frac{dSW^0}{dr} < 0 \quad A4$$

Comparing A2 – A4 with the respective derivatives reported in equations 6 – 8 confirms our result.