Optimal carbon storage in even-and uneven-aged forestry

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Abstract

We study the effects of forest carbon storage on optimal stand management by applying a model where optimal harvests are partial cuttings, implying uneven-aged forestry, or both partial cuttings and clearcuts, implying even-aged forestry. Optimal carbon storage postpones partial cuttings and increases stand volume along the rotation. Carbon pricing may shorten or lengthen the rotation period depending on interest rate and speed of carbon release from wood products. If the carbon price is high, the shadow value of forest biomass is negative, implying that a higher interest rate leads to higher stand density. In empirically realistic examples, carbon pricing causes a switch from clearcuts to continuous cover management rather than *vice versa*.

Keywords: carbon sequestration, continuous cover forestry, Faustmann model, optimal rotation, uneven-aged forestry

1. Introduction

Forest ecosystems hold more than double the amount of carbon in the atmosphere (FAO 2006). Carbon storage can be enhanced by reducing deforestation and increasing afforestation, but also by changing forest management in existing forests. The latter option may help prevent price increases of agricultural land and products. As the present stand-level literature mostly deals with changing the rotation period, a wider set of management adaptation options has remained unexplored. Our study applies a generalized stand-level model, where optimal harvests are solely partial cuttings, or both partial cuttings and clearcuts. This model allows the joint optimization of wood production and carbon storage in uneven-aged and even-aged forestry. We show that, in addition to the rotation period, economically efficient carbon storage also changes stand density along the rotation and, in many cases, the optimal management regime from even-aged to uneven-aged management.

An extensive body of literature exists on carbon storage and afforestation potential on a national level (e.g. Lubowski et al. 2006, Mason and Plantinga 2013). At the stand level, a seminal paper by van Kooten et al. (1995) examines the effect of carbon taxes and subsidies on optimal rotation age. Their numerical results suggest that internalizing carbon benefits tends to increase rotation ages only moderately. The numerical study by Stainback and Alavalapati (2002) on slash pine forests in the southern U.S. suggests that carbon storage increases sawtimber yields but decreases pulpwood yields, and considerably increases the value of forestland. Guthrie and Kumareswaran (2009) study the effect of carbon credit schemes on the length of rotation period under stochastic timber prices. Olschewski and Benítez (2010) show that carbon storage increases optimal rotation length considerably in tropical fast-growing stands. Akao (2011) shows using an extended Faustmann model that carbon storage may shorten or lengthen the optimal rotation. Hoel et al. (2014) develop the approach in van Kooten et al. (1995) by including forests’ multiple carbon pools and the use of wood for bioenergy.

These studies (as well as many others) apply the generic Faustmann rotation model (Samuelson 1976), where forests can be harvested by clearcutting only. As such, this model is best suited for forest plantations, which, however, account for only 7% of global forest area (Payn et al. 2015). How well this model is suited for more natural forests is questionable. Moreover, risks induced by climate change may favor more diverse management practices in semi-natural forests and even in areas currently dominated by the rotation regime (Gauthier et al. 2015). The main alternative is uneven-aged or continuous cover forestry, which applies partial cuttings (i.e. thinnings) and relies on continuous natural regeneration. Compared to even-aged forestry, this regime is likely to be more favorable to many forest-dwelling species (Calladine et al. 2015) and more resilient against the many threats of climate change (Thompson et al. 2009). Thus, clear interest exists in exploring whether carbon storage favors uneven-aged forestry compared to even-aged forestry or *vice versa*.

Goetz et al. (2010) study uneven-aged forestry and carbon storage, but do not analyze the choice between management regimes. Pukkala et al. (2011) study the regime choice, but apply a model without sound economic basis. Gutrich and Howarth (2007) raise the question of management regimes with carbon storage, but the choice is analyzed without optimization. Thus, the question of whether carbon storage favors continuous cover or clearcut forestry is completely open. Our objective is to answer this question analytically using a model with sound economic basis and that covers both management alternatives simultaneously.

The economics of uneven-aged management have been studied since Adams and Ek (1974). While most models attempt to circumvent the dynamic complexities of optimizing uneven-aged management[1], Haight (1985) and Haight and Getz (1987) specify and numerically compute a theoretically sound dynamic optimization model for uneven-aged management. In another line of research, Chang (1981) and Chang and Gadow (2010) study optimal partial cutting cycles and growing stocks in uneven-aged stands, while Parajuli and Chang (2012) extend the model with carbon sequestration. Recently it has been shown that both clearcut and continuous cover management can

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[1] Various lines of research have been discussed in e.g. Getz and Haight (1989) and Rämö and Tahvonen (2014).

be covered by a single framework (Tahvonen 2015, 2016). In Tahvonen (2016), the Clark (1976, p. 263–269)[2] assumptions on forest aging are revised to account for natural regeneration. This leads to an analytically solvable model that includes both forest management regimes and their optimal choice. In the present paper, the model is further extended to include carbon storage.

Unlike all previous studies, our study analyzes optimal carbon storage without restrictions on the management system. This allows us to show that the set of economically efficient methods for enhancing carbon storage in forests is much wider than previously thought. We show analytically that carbon pricing may either increase or decrease the optimal rotation age, depending on assumptions on interest rate and carbon release from wood products. When carbon pricing increases rotation length, it tends to cause a switch from clearcuts to continuous cover forestry. This regime shift follows when the model is computed using empirically realistic parameter values. Additionally, we show that carbon storage postpones the beginning of thinning and increases stand volume before the possible clearcut. If the carbon price is very high relative to wood price, the shadow price of stand volume is negative, as the scarce resource is not wood but the remaining capacity for carbon sequestration. These effects have remained unnoticed in both studies based on the generic Faustmann model and studies that include thinning in numerically computed frameworks. All these results are new and reveal that carbon storage implies major changes in the established understanding of managing forest resources.

We continue by introducing the model and deriving the optimality conditions. This is followed by an analysis of optimal thinning, after which we present results on optimal rotation and management regime choice. Empirical and numerical examples are given alongside with analytical results. The proofs can be found in the appendix.

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[2] Recall that Faustmann (1849) included thinning in his bare land formula, but remained silent about the possibilities of harvesting trees by thinnings only. A similar route is followed by Clark (1976), while Samuelson (1976) neglects thinning altogether. Halbritter and Deegen (2015) extend the continuous-time thinning and rotation model with a focus on optimized artificial regeneration.

2. An economic model for wood production and carbon storage with endogenous management regimes

Let $x(t)$ denote the stand volume (m^3 ha^{-1}) and $h(t)$ the rate of harvested volume (m^3 a^{-1} ha^{-1}) in thinning. Regeneration cost is *w*, annual interest rate δ, and stumpage price *p*. At the initial moment t_0 the stand volume is x_0. Stand volume develops as a product of aging $g(t)$ and density-dependent growth $f(x)$:

$$\dot{x} = g(t)\, f\left(x(t)\right) - h(t), \quad x(t_0) = x_o. \tag{1}$$

Clark (1976, p. 264) assumes that f is single-peaked and that growth of old stands finally ceases independently of volume, i.e. $g'(t) < 0$ and $g(t) \to 0$ as $t \to \infty$. These assumptions on aging may be suitable if the model describes the growth of trees planted at t_0 and no natural regeneration occurs, i.e. a pure planted forest. Given this assumption, the outcome is a finite optimal rotation. However, if new saplings can emerge into the stand without planting, density-dependent growth may occur even in an “old” stand. To include natural regeneration, assume that $g(t)\, f(x)$ may remain strictly positive as $t \to \infty$. Suppose further that the aging function g and the growth function f are continuous and twice differentiable and

$$f(0) \geq 0, \;\; f(\underline{x}) = 0, \;\; f''(x) < 0, \;\; f'(\hat{x}) = 0, \;\; 0 < \hat{x} < \underline{x} \tag{A1}$$

$$g(0) > 0, \;\; g'(t) < 0, \;\; g''(t) > 0, \;\; \lim_{t\to\infty} g(t) = \tilde{g} > 0, \tag{A2}$$

$$\tilde{g}\, f'(0) > \delta, \tag{A3}$$

where $\underline{x}$ is the carrying capacity of the site, and $\hat{x}$ denotes the growth-maximizing stand volume. Assumption (A3) restricts the analysis to outcomes where continuous cover solutions are not *a priori* ruled out. Figure 1a) shows the original Clark (1976) growth function and 1b) a modified function that satisfies assumptions (A1) and (A2).

[Figure 1 here]

Let $p_c \geq 0$ denote the economic value of one CO_2 unit, $\mu > 0$ the amount of CO_2 per one unit of wood, and $p_c\mu\alpha$ the value of released CO_2 per one unit of harvested wood. If wood is burned immediately we set $\alpha = 1$, as implicitly assumed in the current New Zealand carbon credit system (Manley and Maclaren 2010, Adams and Turner 2012). If carbon storage in wood products is permanent, it may be possible to assume $\alpha = 0$. If CO_2 is instead gradually released as each wood product decomposes according to its specific qualities and usage, we set $0 < \alpha < 1$ (see Appendix 1).

The optimization problem takes the form

$$\max_{\{h(t),T,x(T)\geq 0\}} V = \frac{-w + \int\limits_0^T e^{-\delta t}\left[(p - p_c\mu\alpha)h(t) + p_c\mu g(t)\, f\left(x(t)\right)\right]dt + e^{-\delta T}\left[(p - p_c\mu\alpha)x(T)\right]}{1 - e^{-\delta T}} \tag{2}$$

subject to (1) and

$$h \in \left[0, h_{max}\right], \tag{3}$$

and where $T \in [0,\infty)$. Notice that the integral term in (2) yields the thinning revenues net of the value of released carbon. Choosing a finite rotation period implies a clearcut and even-aged forestry, while infinite rotation allows maintaining continuous forest cover and thinning without clearcut. Problem (2) can be solved in two steps (cf. Clark 1976, p. 265–269). Thus, we first solve optimal thinning while taking the rotation period as fixed, and given optimal thinning we solve for the optimal rotation period *T*.

The Hamiltonian function for the problem of optimizing thinning reads as

$$H = (p - p_c\mu\alpha)h(t) + p_c\mu g(t)\, f(x) + \varphi(t)\left[g(t)\, f(x) - h(t)\right]. \tag{4}$$

The Hamiltonian is linear in $h(t)$ and the necessary optimality conditions take the form (Seierstad and Sydsæter 1987, p. 178–182: theorem 1 and 3) if $p - p_c\mu\alpha - \varphi < 0, \quad h = 0$, (5a)

if $p - p_c\mu\alpha - \varphi = 0, \quad h \in [0, h_{max}]$ (5b)

if $p - p_c\mu\alpha - \varphi > 0, \quad h = h_{max}$, (5c)

$$\dot{\varphi} = \varphi\delta - H_x = \varphi\delta - g(t) f'(x)(p_c\mu + \varphi), \tag{6}$$

$$\varphi(T) - (p - p_c\mu\alpha) \geq 0, \; x(T) \geq 0, \; [\varphi(T) - (p - p_c\mu\alpha)]x(T) = 0. \tag{7}$$

To analyze the sufficiency of the necessary conditions, note that

$$H_{hh} = 0, \; H_{xx} = g(t) f''(x)(p_c\mu + \varphi), \; H_{xh} = 0, \; H_{xx}H_{hh} - H_{xh}{}^2 = 0 .$$

It is not possible to rule out $p_c\mu + \varphi < 0$ and $H_{xx} > 0$ *a priori*. Thus, Hamiltonian may not be concave in *h* and *x*. However, by the sufficiency theorem of Arrow (Sydsæter et al. 2008, p. 332), the necessary conditions are sufficient if maximized Hamiltonian is concave in *x*. By (5a, b) it follows that if optimal h remains either in the singular solution or $h = 0$ regime, the condition $0 < p \leq p_c\mu\alpha + \varphi \leq p_c\mu + \varphi$ holds true $(\alpha \leq 1)$, implying that necessary optimality conditions are sufficient when T is taken as fixed.

To obtain the optimality condition for the rotation period we differentiate (2) w.r.t. to *T*. Rearranging and utilizing (1) yields

$$\frac{\partial V}{\partial T} = \frac{e^{-\delta T}}{1 - e^{-\delta T}} \left\{ [p + (1-\alpha) p_c\mu] g(T) f(x(T)) - \delta[(p - p_c\mu\alpha)x(T) + V] \right\}. \tag{8a}$$

Since $e^{-\delta T} / (1 - e^{-\delta T}) > 0,$ for any finite *T* and $\delta > 0$ we obtain the necessary optimality condition for finite rotations as

$$y(T) \equiv [p + (1-\alpha) p_c\mu] g(T) f(x(T)) - \delta[(p - p_c\mu\alpha)x(T) + V] = 0. \tag{8b}$$

3. Results and discussion

3.1 Optimal thinning

The switching function for (4) is $\sigma \equiv p - p_c\mu\alpha - \varphi$, and $\dot{\sigma} = -\dot{\varphi} = 0$. By (6) and (5b),

$$\left(p - p_c\mu\alpha\right)\delta - g\left(t\right) f'\left(x\right)\left[p + (1-\alpha)p_c\mu\right] = 0. \tag{9}$$

Rearrange (9) into

$$f'\left(x\right)g\left(t\right) = \frac{p - p_c\mu\alpha}{p + (1-\alpha)p_c\mu}\delta. \tag{10}$$

By (10), $f'' < 0$ and $f'\left(\hat{x}\right) = 0$ in (A1), the singular solution satisfies the properties

$$\text{if } p > p_c\mu\alpha, \; f'(x) > 0 \Rightarrow x < \hat{x}, \tag{11a}$$

$$\text{if } p = p_c\mu\alpha, \; f'(x) = 0 \Rightarrow x = \hat{x}, \tag{11b}$$

$$\text{if } p < p_c\mu\alpha, \; f'(x) < 0 \Rightarrow x > \hat{x}. \tag{11c}$$

Hence the optimal stand volume on the singular path will be below (above) the growth-maximizing stand density if stumpage price exceeds (is smaller than) the value of released CO_2. Differentiating (9) with respect to time and utilizing (1) yields:

$$h\left(t\right) = g\left(t\right) f\left(x\right) + \left(p - p_c\mu\alpha\right)\frac{g'\left(t\right)\delta}{\left[p + (1-\alpha)p_c\mu\right]g^2\left(t\right) f''\left(x\right)}, \tag{12}$$

where the sign of the quotient is positive. Thus, if $p > p_c\mu\alpha$, the rate of thinning exceeds stand growth, and stand volume decreases on the singular path (Figure 2). If $p < p_c\mu\alpha$, the harvest rate is lower than stand growth, and stand volume increases, but harvest rate decreases on the singular path. The special case $p = p_c\mu\alpha$ implies $g(t)f'\left(\hat{x}\right) = 0$, i.e. thinning maximizes stand growth.

Given $x_0 = 0$, there exists an initial time interval $t \in [0, t_1]$ and a regime where $h\left(t\right) = 0$, i.e. the stand is initially left to grow undisturbed. The correct choice of $\varphi\left(0\right)$ implies that

$\varphi(t_1) = p - p_c\mu\alpha$ at the moment when the solution for $\dot{x} = g(t)\, f(x)$, $x_0 = 0$ intersects the singular solution in x, t plane, and h jumps to the singular-solution level defined by (12).

If carbon storage is omitted by setting $p_c = 0$, equation (10) reduces to $f'(x)g(t) = \delta$ as in Clark (1976, p. 265). Note that in (10), $p_c > 0$ implies $(p - p_c\mu\alpha)/[p + (1-\alpha)p_c\mu] < 1$. Hence it follows by the concavity of f that given a positive interest rate, the higher the carbon price, the higher the stand density at any stand age along the singular solution. Additionally, this implies that thinning must begin later under optimal carbon storage, i.e.

$$\frac{\partial t_1}{\partial p_c} > 0. \qquad (13)$$

Independently of carbon price, zero interest rate implies that optimal thinning maintains the stand density at the growth-maximizing level (Equation 10). The same result is obtained in the theoretical case with no CO_2 release from harvested trees $(\alpha = 0)$ and given $p_c \to \infty$. This is a natural outcome when only the maximization of carbon sequestration matters. If $0 < \alpha < 1$, the RHS of (10) approaches $-\alpha\delta/(1-\alpha) < 0$ from above as $p_c \to \infty$. Given $\alpha < 1$ this value is finite, implying that thinning may be optimal even with very high carbon price. If all CO_2 is released immediately $(\alpha = 1)$, we obtain $f'(x)g(t) = (p - p_c\mu)\delta / p \to -\infty$ as $p_c \to \infty$, implying that thinning becomes suboptimal.

By implicit differentiation, $\partial x / \partial \alpha > 0$ showing that the faster CO_2 is released from harvested wood, the higher is the optimal stand volume along the singular solution. If $p > p_c\mu\alpha$, it follows by (10) that $\partial x / \partial \delta < 0$, i.e. with a higher interest rate it is optimal to begin thinning earlier and keep the biomass level lower. However, given $p < p_c\mu\alpha$, we obtain $\partial x / \partial \delta > 0$ and that a higher interest rate postpones thinning and implies a higher biomass level along the singular solution. It becomes optimal to utilize the initial high-growth regime and carbon sequestration benefits longer, and to begin thinning only once the high stand volume begins decreasing stand growth and the immediate benefits from carbon sequestration.

When $p < p_c \mu \alpha$, we obtain by (5b) that $\varphi < 0$, i.e. if the value of released CO2 per harvested unit exceeds the stumpage price, the shadow price of the stand volume is negative. This is because any increase in stand volume decreases stand growth and thus the valuable sequestration of carbon. Further, an increase in stand volume provides no additional benefit, as the direct net revenues from harvesting are negative. In this case, the scarce resource is not wood but the remaining capacity for carbon sequestration. The negative shadow price explains the seemingly counterintuitive finding that the optimal harvest rate can be positive even when the direct net revenues from thinning are negative: stand volume is controlled to maintain a sufficient rate of stand growth and carbon sequestration. As stand growth exceeds the harvest rate, and the value of released CO2 cannot be larger than the value of sequestered CO2 ($0 \le \alpha \le 1$ implies $p_c \mu \alpha \le p_c \mu$), the combined net revenues from harvesting and carbon storage will actually be positive.

To present empirical examples, we apply a growth function specification calibrated using the growth model in Bollandsås et al. (2008), estimated for Norway spruce (average productivity site)[3]:

$$g(t)\,f[x(t)] = \left(\frac{1.6}{1+0.04t^{1.2}}+1\right)0.065[x(t)+8]\left[1-\frac{x(t)+8}{378}\right],\; x_0 = 0. \tag{14}$$

Given (14), the differential equation $\dot{x} = g(t)\,f[x(t)],\; x_0 = 0$ can be solved analytically, and the solution shows the development of stand volume without harvesting (solid line in Figure 2). If left undisturbed, the stand will approach a volume of 370 m^3 ha^{-1} ($= \underline{x}$). Stand growth is maximized at a volume of 181 m^3 ha^{-1} ($= \hat{x}$, dash-dotted line in Figure 2). The long-run maximum sustained thinning yield is approximately 6.2 m^3 a^{-1} ha^{-1} $(= \tilde{g}\, f(\hat{x}))$. Singular solutions with various carbon prices are

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[3] Our growth function specification yields similar long-run maximum sustained thinning yield, maximum volume, and maximum growth as the Bollandsås et al. (2008) model.

shown in Figure 2 (dotted line and dashed lines). We set the CO_2 mass per wood volume unit as $\mu =$ 0.7 $tCO_2\,m^{-3}$ following Niinimäki et al. (2013). Additionally, we set $\alpha = 0.7$, reflecting an assumption that half of the carbon content of harvested wood is released back to the atmosphere after 10 years (see Appendix 1).

[Figure 2 here]

The interceptions of the undisturbed growth path and the singular solutions show the switching moment where it is optimal to begin thinning (circle symbols in Figure 2). As shown analytically, a higher carbon price postpones the switch to thinning and increases the stand volume along the solution for optimal thinning. If the carbon price is sufficiently large to imply that $\varphi = p - p_c\mu\alpha$ is negative, stand volume exceeds the growth-maximizing level and continues to increase while the stand is thinned.

The effect of carbon storage on optimal thinnings has not been studied analytically in the previous literature. Thinnings have been included in the numerical analysis of the even-aged problem by e.g. Pohjola and Valsta (2007), Daigneault et al. (2010), and Niinimäki et al. (2013). According to these studies, carbon pricing postpones thinnings and increases stand volume along the rotation. We have derived this result analytically in addition to showing how optimal thinning depends on model parameters.

Numerical studies have not explicitly analyzed the situation where stumpage price is lower than the value of released CO_2. Our analysis reveals that in this case, optimal stand density increases with interest rate, and the shadow value of stand volume is negative. Moreover, while thinnings cause a direct cost, they are optimal as this optimizes the development of forest biomass as a carbon sink and a source of income from carbon storage. Given average stumpage prices in Nordic countries (around €40m^{-3}) and assuming instant release of CO_2 at harvest, such a situation occurs if the carbon price exceeds €60 tCO_2^{-1}. However, stumpage price approaches zero when the distance from the site to the nearest road and wood-processing plant increases toward the extensive margin. In contrast, the economic value of the carbon storage service is independent of site location. Hence, the value of released CO_2 may dominate the value of wood even with realistic carbon prices.

3.2 Optimal rotation age

The question of whether a clearcut regime is economically preferable to continuous cover management is ultimately a question of optimal rotation age. Finite optimal rotation implies clearcuts, while infinite rotation implies the optimality of continuous forest cover.

The necessary condition for a finite optimal rotation period is given by Equation (8b). To exclude the case where the stand is clearcut before the singular path is reached (cf. Clark 1976, p. 267), assume that $y(t_1) > 0$. Differentiating $y(T)$ and applying (10) and $V'(T) = 0$ yields

$$y'(T)_{|y(T)=0} = \left[p + (1-\alpha)\, p_c \mu\right] g' f < 0. \tag{15}$$

Thus, given any finite *T* satisfying (8b), the necessary conditions are sufficient for an optimal unique rotation age. The simplicity of this uniqueness result follows from the properties of the singular solution and it can be contrasted with the uniqueness result for the rotation model without thinning (Akao 2011, Gong and Löfgren 2016).

If $\lim_{t\to\infty} g(t) = \tilde{g}$ is very low and $(p - p_c\mu\alpha)x(T) + V > 0$, then $y(T)$ will become negative when T is high enough. Thus, the optimal rotation is finite if the long-term yield from thinning is low enough and the sum of clearcut net revenues and bare land value is positive. However, in (8b) $\left[p + (1-\alpha)\, p_c\mu\right] g(T)\, f\left(x(T)\right) - \delta\left[(p - p_c\mu\alpha)\, x(T)\right] > 0$ by (10) and the concavity of *f*. Thus, if the bare land value is negative or sufficiently small albeit positive, then $y(T) > 0$ for $T \in [t_1, \infty)$ and the optimal rotation will be infinite. In these cases, it is optimal to continue thinning forever without clearcuts, i.e. to apply continuous cover forestry.

For interpretation, write (8b) as

$$\left[p+(1-\alpha)p_c\mu\right]\left[h(T)+\dot{x}(T)\right]=\delta\left[(p-p_c\mu\alpha)x(T)+V\right]. \tag{16}$$

Thus, at the moment of clearcut, the rate of wood and carbon revenues net of their decrease equals the interest on the sum of clearcut net revenues and the value of bare land. Conversely, an infinite rotation and continuous cover management is optimal if $y(T)\geq 0$ for all $T\in[0,\infty)$ in (8b). Let us denote thinning satisfying (12) by $\lim_{t\to\infty}h(t)=\tilde{g}f(\tilde{x})=\tilde{h}$. By the uniqueness result (6), the condition

$$\left[p+(1-\alpha)p_c\mu\right]\tilde{h}\geq\delta(p-p_c\mu\alpha)\tilde{x}+V \tag{17}$$

implies that no solutions for finite rotation periods for (8b) exist, implying that the remaining optimal solution is to apply continuous cover forestry instead of clearcuts. Thus, continuous cover management is optimal, if the steady-state wood and net carbon storage revenues from thinning exceed the interest earnings for the values of clearcut net revenues and bare land.

3.3 The effect of carbon price on rotation age and choice of management regime

Factors implying a longer optimal rotation period favor continuous cover forestry and *vice versa*. Taking into account that the solution satisfies the singular condition (7), the derivative of (8b) with respect to carbon price can be given in the form

$$\frac{\partial y(T)}{\partial p_c}=(1-\alpha)\mu g(T)f(x(T))+\delta\mu\alpha x(T)-\delta\frac{\partial V}{\partial p_c}. \tag{18}$$

The first term $(1-\alpha)\mu g(T)f(x(T))\geq 0$ is the increase in net carbon storage revenues at T as carbon price increases. The second term $\delta\mu\alpha x(T)\geq 0$ is the decrease in the interest cost from postponing the clearcut, because a higher carbon price translates to a higher value of released CO_2 when the stand is clearcut. The third term $-\delta\,\partial V/\partial p_c<0$ reflects the increase in interest cost through increased bare land value. As carbon storage revenues are an additional source of income, they can only increase the bare land value.

Proposition 1. Given that $|p - p_c \mu \alpha|$ *is low enough,* α *close enough to one, and* $\delta > 0$*, an increase in carbon price lengthens optimal finite rotation periods.* Proof, Appendix 2.

The conditions specified in Proposition 1 are sufficient, but not necessary for the outcome that the rotation period increases with carbon price. Thus, at least under the given conditions, carbon storage and a higher carbon price support longer rotations and continuous cover forestry. Numerical examples, assuming the growth specification (14) and a 3% interest rate, show that optimal rotation length increases with carbon price regardless of the speed of CO_2 release from harvested wood (Figure 3). However, the magnitude of the effect varies substantially. Given $\alpha = 1$, i.e. assuming that all of the CO_2 is released immediately, a carbon price above €30 tCO_2^{-1} leads to a shift from clearcuts to continuous cover management. With slower CO_2 release from wood products $(\alpha = 0.7)$, carbon pricing implies continuous cover management when carbon price exceeds €45 tCO_2^{-1}. Given the case of no CO_2 release ($\alpha = 0$), carbon pricing does not reduce clearcut net revenues and increases bare land value considerably, implying that the incentive to postpone clearcutting is weak.

[Figure 3 here]

However, a higher carbon price does not always imply longer rotation:

Proposition 2: Given $\alpha = \delta = 0$ and $w > 0$, *an increase in carbon price shortens optimal finite rotation periods.* Proof, Appendix 3.

Under the given assumptions, rotation age decreases with carbon price. This can be understood by comparing economically optimal solutions to the solution that maximizes sustained yield (MSY). Given positive regeneration cost, zero interest rate, and no carbon pricing, the optimal rotation period is longer than the MSY rotation. When no carbon is released from harvested trees ($\alpha = 0$), a higher carbon price favors the volume-maximizing solution and thus shortens the rotation period. Numerical examples suggest that the result in Proposition 2 may also hold when interest rate is positive but low.

Given $\delta = 0.5\%$ and $\alpha = 0$, rotation age decreases from 129 to 120 years as carbon price increases from zero to €70 tCO_2^{-1} (Figure 4), while the MSY rotation is 100 years.

[Figure 4 here]

According to most of the literature (e.g. van Kooten et al. 1995, Gutrich and Howarth 2007, Pohjola and Valsta 2007, Olschewski and Benítez 2010, Pihlainen et al. 2014), valuing carbon storage increases optimal rotation age. However, Akao (2011) shows that within the generic Faustmann framework this is not always the case: carbon pricing may shorten finite optimal rotation periods if no carbon is released after harvesting. Hoel et al. (2014) obtain a similar result using a model without regeneration cost. Using a generalized framework with optimized thinning and the option of continuous cover forestry, we show that carbon pricing may indeed either increase or decrease optimal rotation age. However, we find that the decreasing effect requires strong assumptions: the interest rate must be close to zero, and carbon release after harvesting must be minimal.

Given more plausible assumptions, carbon pricing lengthens rotations and in many cases leads to infinite rotations, implying continuous cover forestry. Such regime shifts are beyond the scope of the existing studies that are restricted to the clearcut regime only. Both van Kooten et al. (1995) and Hoel et al. (2014) find that with a sufficiently high carbon price the clearcut is postponed indefinitely, i.e. harvesting becomes suboptimal. We show that when restrictions on the management system are relaxed, high carbon prices tend to imply continuous cover forestry in the form of partial cuttings, instead of total abandonment of wood production.

We showed analytically that continuous cover management becomes optimal when bare land value is low enough (albeit positive). Increasing the interest rate or the regeneration cost decreases the bare land value toward zero and below. This suggests that given some regeneration cost level, there may exist some interest rate that implies equal profitability of the continuous cover and clearcutting regimes. Moreover, such interest rate is the higher, the lower is the regeneration cost. This yields break-even curves where the two management regimes are equally profitable (Figure 5). A carbon price of €50 tCO_2^{-1} widens the optimal application area of continuous cover management compared to the case without carbon pricing. With a carbon price of €25 tCO_2^{-1} this effect is less clear: given a high interest rate, the continuous cover regime is less competitive than without carbon pricing. Thus, the interplay between the effects of carbon price, interest rate, and other economic parameters is far from straightforward.

Forest carbon storage is currently not included in the Emissions Trading Scheme (ETS) of the European Union (EU). However, New Zealand has integrated forest carbon storage in its ETS (Adams and Turner 2012), and in the future the EU may adopt a similar system to incentivize management adaptation and afforestation by landowners. The EU is committed to a 40% reduction of domestic greenhouse gas emissions from the 1990 level by 2030 (European Commission 2015). Depending on policy scenario, achieving this target would translate to an EU ETS carbon price in the range of €11–€53 tCO_2^{-1} by 2030, and €85–€264 tCO_2^{-1} by 2050 (European Commission 2014, p. 80–81). Based on our results, such carbon price levels would imply a switch to continuous cover management with high stand density.

The model used in this study can be augmented to include carbon emissions from the use of harvesting and hauling machinery. Soil carbon dynamics is another possible extension. Given that a clearcut exposes the forestland to sunlight and thus increases topsoil temperatures, it is likely that including carbon fluxes to and from the soil would further improve the relative profitability of continuous cover management under carbon pricing.

[Figure 5 here]

4. Conclusions

The global temperature target of the Paris agreement (UNFCCC 2015) is ambitious, and may not be attainable without increased carbon storage in forests. To this end, it is highly important to understand how the inclusion of carbon storage changes optimal stand management. Existing forest economic models typically apply the generic Faustmann model, which is limited to harvesting forest by clearcuts only, and thus excludes thinning and the possibility of continuous cover forestry. To adapt management of forest resources to the threat of climate change, it is necessary to generalize the classic forest economic models to incorporate more diverse management options. We apply a generalized model that can still be studied analytically.

Our analysis shows that optimal carbon storage postpones thinnings and increases stand volume along the rotation. Further, we show that carbon pricing may shorten or lengthen the rotation period, depending on interest rate and the speed of carbon release from wood products. Our empirical examples suggest that carbon pricing may well imply a switch from clearcuts to continuous cover management. We also demonstrate that the shadow value of forest biomass may be negative if carbon price is high. In such cases the economic value of a forest stand is dominated by the remaining capacity for carbon sequestration. This may have strong effects on the value of forest stands along the rotation, but we leave this subject to future studies. Further, as forest heterogeneity is vital for maintaining resilience under climate change, future research should develop understanding of optimal carbon storage in size-structured mixed-species forest stands.

Appendix 1. Emissions from wood products:

Denote the decay rate of wood products by $g\kappa$ and the harvested volume by *Y*. At moment τ, the increase in atmospheric carbon stock equals the initial stock minus what is left at moment τ **:**

$$A(\tau) = \mu Y - \mu Y e^{-g\tau},$$

implying that the emission level at moment τ is

$$\frac{dA(\tau)}{d\tau} = g\mu Y e^{-g\tau}.$$

Denote the social discount rate by *r*. As the economic present value of emissions occurring at τ equals $p_c g\mu Y e^{-g\tau} e^{-r\tau}$, the present value of emissions over an infinite time horizon is

$$\int_0^\infty p_c \mu Y\, g\, e^{-(g+r)\tau} d\tau = p_c \mu Y \frac{g}{g+r}.$$

Hence, per unit of wood product, the present value of emissions due to decay can be given as

$$\alpha = \frac{g}{g+r}.$$

This approach has previously been used in Pihlainen et al. (2014).

Appendix 2. Proof of proposition 1:

Write (8b) in the form

$$\begin{aligned} & y(T) \equiv \left[p + (1-\alpha) p_c \mu\right] g(T) f\left[x(T)\right] \\ & -\frac{\delta}{1-e^{-\delta T}} \left\{ -w + p_c \mu \int_0^{t_1} g(t) f\left[x(t)\right] e^{-\delta t} dt + \int_{t_1}^{T} e^{-\delta t} \left\{ (p - p_c \mu\alpha) h(t) + p_c \mu g(t) f\left[x(t)\right] \right\} dt + (p - p_c\mu\alpha) x(T) \right\} = 0. \end{aligned}$$

Thus,

$$\begin{aligned} & \frac{\partial y(T)}{\partial p_c} = (1-\alpha)\mu g(T) f\left[x(T)\right] + \left[p + (1-\alpha)\mu\right] g(T) f'\left[x(T)\right] \frac{\partial x(T)}{\partial p_c} \\ & - \frac{\delta}{1-e^{-\delta T}} \left\{ \begin{aligned} & \mu \int_0^{t_1} g(t) f\left[x(t)\right] e^{-\delta t} dt + \frac{\partial t_1}{\partial p_c} e^{-\delta t_1} p_c \mu g(t_1) f\left[x(t_1)\right] \\ & - \frac{\partial t_1}{\partial p_c} e^{-\delta t_1} \left\{ (p - p_c \mu\alpha) h(t_1) + p_c \mu g(t_1) f\left[x(t_1)\right] \right\} - \mu\alpha x(T) + (p - p_c\mu\alpha) \frac{\partial x(T)}{\partial p_c} \\ & + \int_{t_1}^{T} e^{-\delta t} \left\{ \mu \left\{ g(t) f\left[x(t)\right] - \alpha h(t) \right\} + (p - p_c\mu\alpha) \frac{\partial h(t)}{\partial p_c} + p_c \mu g(t) f'\left[x(t)\right] \frac{\partial x(t)}{\partial p_c} \right\} dt \end{aligned} \right\}. \end{aligned}$$

Given $\alpha \to 1$ and $|p - p_c\mu| \to 0$ and $\delta > 0$ we obtain by (10) and (13) that

$$\frac{\partial y(T)}{\partial p_c} \to \frac{-\delta}{1-e^{-\delta T}} \left\{ \mu \int_0^{t_1} e^{-\delta t_1} g(t_1) f\left[x(t_1)\right] dt - \mu x(T) \right\} > 0.$$

Because $\left.\frac{\partial y(T)}{\partial T}\right|_{y(T)=0} < 0$ it follows that $\frac{\partial T}{\partial p_c} > 0$. Q.E.D.

Appendix 3. Proof of proposition 2:

When $\alpha = \delta = 0$, equation (8b) is written as

$$\begin{aligned} y(T) &= (p + p_c\mu) g(T) f\left[x(T)\right] \\ &\quad - \frac{1}{T}\left\{-w + \int_0^{t_1} p_c \mu g(t) f\left[x(t)\right] dt + \int_{t_1}^{T} \left\{ph(t) + p_c \mu g(t) f\left[x(t)\right]\right\} + px(T)\right\} = 0. \end{aligned}$$

We obtain

$$\frac{\partial y(T)}{\partial p_c} = \mu g(T) f\left[x(T)\right] - \frac{1}{T}\left\{\int_0^{t_1} \mu g(t) f\left[x(t)\right] dt + \int_{t_1}^{T} \mu g(t) f\left[x(t)\right] dt\right\}.$$

Since $\int_0^{t_1} g(t) f\left[x(t)\right] dt = x(T)$ and $\int_{t_1}^{T} g(t) f\left[x(t)\right] dt = \int_{t_1}^{T} h(t) dt$ by (10), (12) and $\delta = 0$, $y(T)$ can be written as

$$(p + p_c\mu)\left\{g(T) f\left[x(T)\right] - \frac{1}{T}\left[\int_{t_1}^{T} h(t) dt + x(T)\right]\right\} = -\frac{w}{T}$$

showing that the sign of $\partial y(T) / \partial p_c$ is negative. Thus, $\frac{\partial T}{\partial p_c} < 0.$ Q.E.D.

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References

Adams, D.M., Ek, A.R., 1974. Optimizing the management of uneven-aged forest stands. Canadian Journal of Forest Research 4(3), 274–287.

Adams, T., Turner, J.A., 2012. An investigation into the effects of an emissions trading scheme on forest management and land use in New Zealand. Forest Policy and Economics 15, 78–90.

Akao, K.I., 2011. Optimum forest program when the carbon sequestration service of a forest has value. Environmental Economics and Policy Studies 13(4), 323–343.

Bollandsås, O.M., Buongiorno, J., Gobakken, T., 2008. Predicting the growth of stands of trees of mixed species and size: A matrix model for Norway. Scandinavian Journal of Forest Research 23, 167–178.

Calladine, J., Bray, J., Broome, A., Fuller, R.J., 2015. Comparison of breeding bird assemblages in conifer plantations managed by continuous cover forestry and clearfelling. Forest Ecology and Management 344, 20–29.

Chang, S.J., 1981. Determination of the optimal growing stock and cutting cycle for an uneven-aged stand. Forest science 27(4), 739–744.

Chang, S.J., Gadow, K.V., 2010. Application of the generalized Faustmann model to uneven-aged forest management. Journal of Forest Economics 16(4), 313–325.

Clark, C.W., 1976. *Mathematical Bioeconomics. The optimal management of renewable resources*. Wiley, New York. 352 p.

Daigneault, A.J., Miranda, M.J., Sohngen, B., 2010. Optimal Forest Management with Carbon Sequestration Credits and Endogenous Fire Risk. Land Economics 86(1), 155–172.

European Commission, 2014. Impact Assessment SWD (2014), 15 final. Accompanying the document Communication from the Commission to the European Parliament, the Council, the

European Economic and Social Committee and the Committee of the Regions. A policy framework for climate and energy in the period from 2020 up to 2030. 234 p.

European Commission, 2015. Intended Nationally Determined Contribution of the EU and its Member States. Submission by Latvia and the European Commission on behalf of the European Union and its Member States. https://ec.europa.eu/clima/sites/clima/files/docs/2015030601_eu_indc_en.pdf, accessed 14.8.2017.

FAO, 2006. *Global Forest Resources Assessment 2005. Progress towards sustainable forest management. FAO Forestry paper 147*. Food and Agriculture Organization of the United Nations, Rome. 320 p.

Faustmann, M., 1849. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen. Allgemeine Forst- und Jagd-Zeitung 25, 441–455.

Gauthier, S., Bernier, P., Kuuluvainen, T., Shvidenko, A.Z., Schepaschenko, D.G., 2015. Boreal forest health and global change. Science 349 (6250), 819–822.

Getz, W.M., Haight, R.G., 1989. *Population Harvesting: Demographic Models for Fish, Forest and Animal Resources*. Princeton University Press, New Jersey. 408 p.

Goetz, R-U., Hritonenko, N., Mur, R.J., Xabadia, A., Yatsenko, Y., 2010. Forest Management and Carbon Sequestration in Size-Structured Forests: The Case of Pinus sylvestris in Spain. Forest Science 56, 242–256.

Gong, P., Löfgren, K.G., 2016. Could the Faustmann model have an interior minimum solution? Journal of Forest Economics 24, 123–129.

Guthrie, G., Kumareswaran, D., 2009. Carbon subsidies, taxes and optimal forest management. Environmental and Resource Economics 43(2), 275–293.

Gutrich, J., Howarth, R.B., 2007. Carbon sequestration and the optimal management of New Hampshire timber stands. Ecological Economics 62, 441–450.

Haight, R.G., 1985. Comparison of Dynamic and Static Economic Models of Uneven-aged Stand Management. Forest Science 31, 957–974.

Haight, R.G., Getz, W.M., 1987. Fixed and equilibrium endpoint problems in uneven-aged stand management. Forest Science 33, 903–931.

Halbritter, A., Deegen, P., 2015. A combined economic analysis of optimal planting density, thinning and rotation for an even-aged forest stand. Forest Policy and Economics 51, 38–46.

Hoel, M., Holtsmark, B., Holtsmark, K., 2014. Faustmann and the climate. Journal of Forest Economics 20(2), 192–210.

Lubowski, R.N., Plantinga, A.J., Stavins, R.N., 2006. Land-use change and carbon sinks: econometric estimation of the carbon sequestration supply function. Journal of Environmental Economics and Management 51(2), 135–152.

Manley, B., Maclaren, P., 2010. Harvested Wood Products in the ETS. What would be the Impact? NZ Journal of Forestry 55(3), 20–26.

Mason, C.F., Plantinga, A.J., 2013. The additionality problem with offsets: Optimal contracts for carbon sequestration in forests. Journal of Environmental Economics and Management 66(1), 1–14.

Niinimäki, S., Tahvonen, O., Mäkelä, A., Linkosalo, T., 2013. On the economics of Norway spruce stands and carbon storage. Canadian Journal of Forest Research 43(7), 637–648.

Olschewski, R., Benítez, P.C., 2010. Optimizing joint production of timber and carbon sequestration of afforestation projects. Journal of Forest Economics 16(1), 1–10.

Parajuli, R., S.J. Chang, 2012. Carbon sequestration and uneven-aged management of loblolly pine stands in the Southern USA: A joint optimization approach. Forest Policy and Economics 22, 65–71.

Payn, T., Carnus, J. M., Freer-Smith, P., Kimberley, M., Kollert, W., Liu, S., Orazio, C., Rodriguez, L., Silva, L.N., Wingfield, M. J., 2015. Changes in planted forests and future global implications. Forest Ecology and Management 352, 57–67.

Pihlainen, S., Tahvonen, O., Niinimäki, S., 2014. The economics of timber and bioenergy production and carbon storage in Scots pine stands. Canadian Journal of Forest Research 44(9), 1091–1102.

Pohjola, J., Valsta, L., 2007. Carbon credits and management of Scots pine and Norway spruce stands in Finland. Forest Policy and Economics 9(7), 789–798.

Pukkala, T., Lähde, E., Laiho, O., Salo, K., Hotanen, J-P., 2011. A multifunctional comparison of even-aged and uneven-aged forest management in a boreal region. Canadian Journal of Forest Research 41(4), 851–862.

Rämö, J., Tahvonen, O., 2014. Economics of harvesting uneven-aged forest stands in Fennoscandia. Scandinavian Journal of Forest Research 29(8), 777–792.

Samuelson, P., 1976. Economics of forestry in an evolving society. Economic Inquiry 14(4), 466–492.

Seierstad, A., Sydsæter, K., 1987. *Optimal control theory with economic applications*. Elsevier, Amsterdam. 462 p.

Stainback, G.A., Alavalapati, J.R.R., 2002. Economic analysis of slash pine forest carbon sequestration in the southern US. Journal of Forest Economics 8, 105–117.

Sydsæter, K., Hammond, P., Seierstad, A., Strom, A. 2008. *Further Mathematics for Economic Analysis*. Prentice Hall, Essex. 632 p.

Tahvonen, O., 2015. Economics of Naturally Regenerating, Heterogeneous Forests. Journal of the Association of Environmental and Resource Economists 2(2), 309–337.

Tahvonen, O., 2016. Economics of rotation and thinning revisited: the optimality of clearcuts *versus* continuous cover forestry. Forest Policy and Economics 62, 88–94.

Thompson, I., Mackey, B., McNulty, S., Mosseler, A., 2009. *Forest Resilience, Biodiversity, and Climate Change. A synthesis of the Biodiversity / Resilience / Stability Relationship in Forest Ecosystems. Technical Series no. 43.* Secretariat of the Convention on Biological Diversity, Montreal. 67 p.

UNFCCC, United Nations Framework Convention on Climate Change, 2015. FCCC/CP/2015/L.9: Adoption of the Paris Agreement. https://unfccc.int/resource/docs/2015/cop21/eng/l09.pdf, accessed 18.4.2017.

van Kooten, G., Binkley, C., Delcourt, G., 1995. Effect of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services. American Journal of Agricultural Economics 77(2), 365–374.

Figures and captions

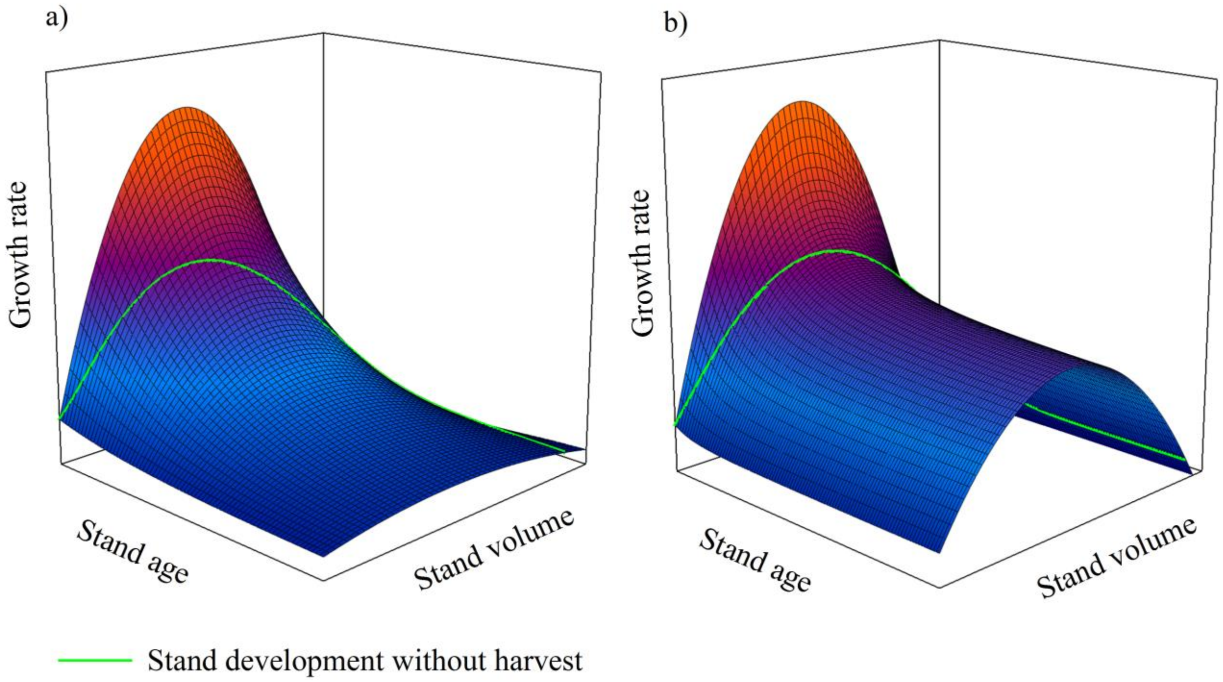

Figure 1. Stand development a) without natural regeneration and b) with natural regeneration.

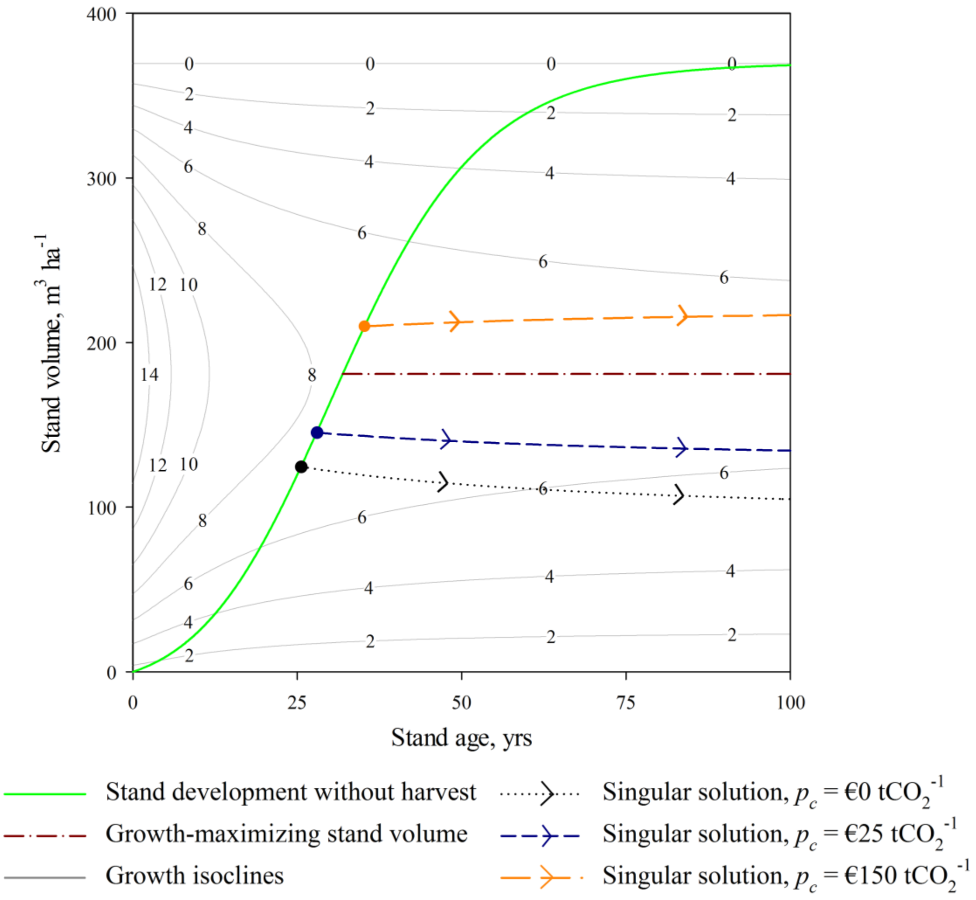

Figure 2. The growth function and singular solutions, with a 3% interest rate and carbon prices €0, €25, and €150 tCO$_2^{-1}$. Note: $p = €40\ m^{-3}$, $\alpha = 0.7$, $w = €1000\ ha^{-1}$.

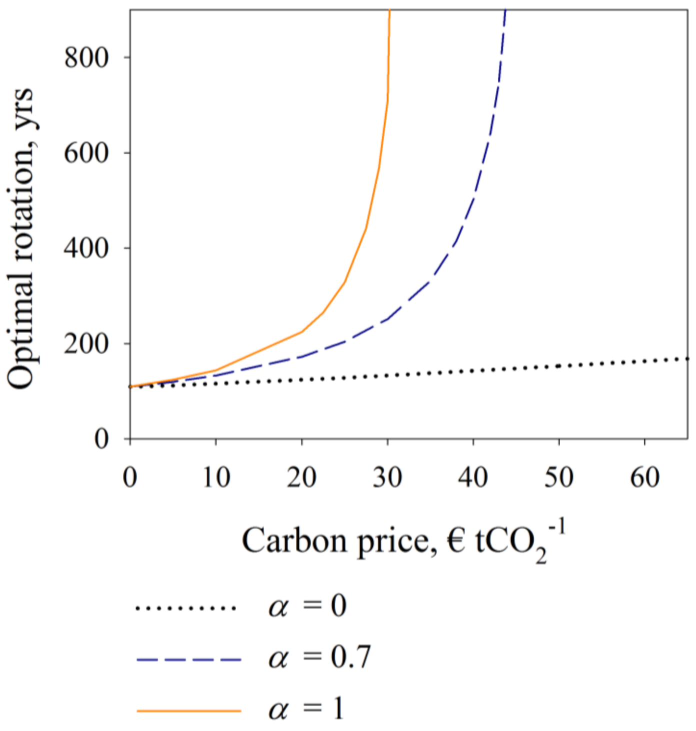

Figure 3. The dependence of optimal rotation on carbon price with different values for carbon release from harvested wood. Note: $\delta = 0.03$, p = €40 m^{-3}, w = €1000 ha^{-1}.

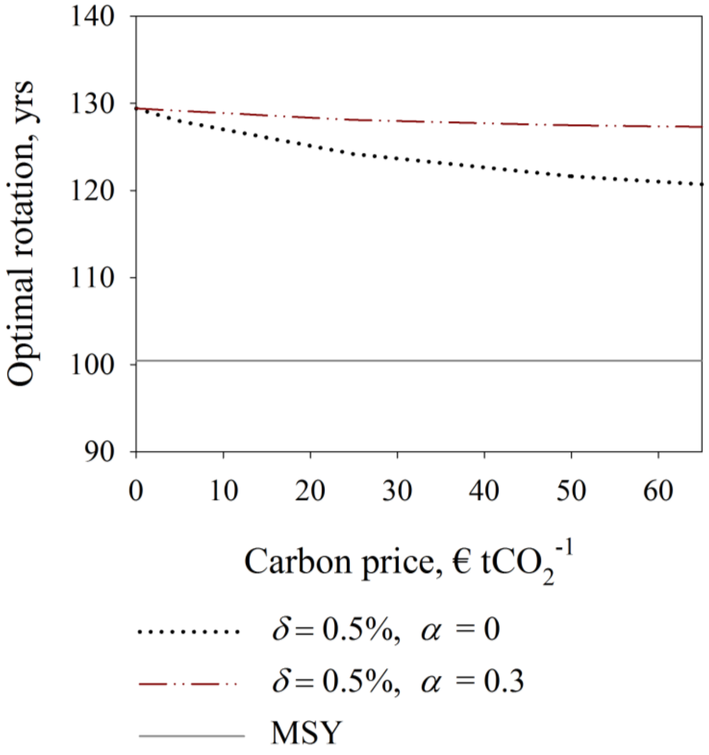

Figure 4. The dependence of optimal rotation on carbon price, given low carbon release from harvested wood and a 0.5% interest rate. Note: p = €40 m^{-3}, w = €1000 ha^{-1}.

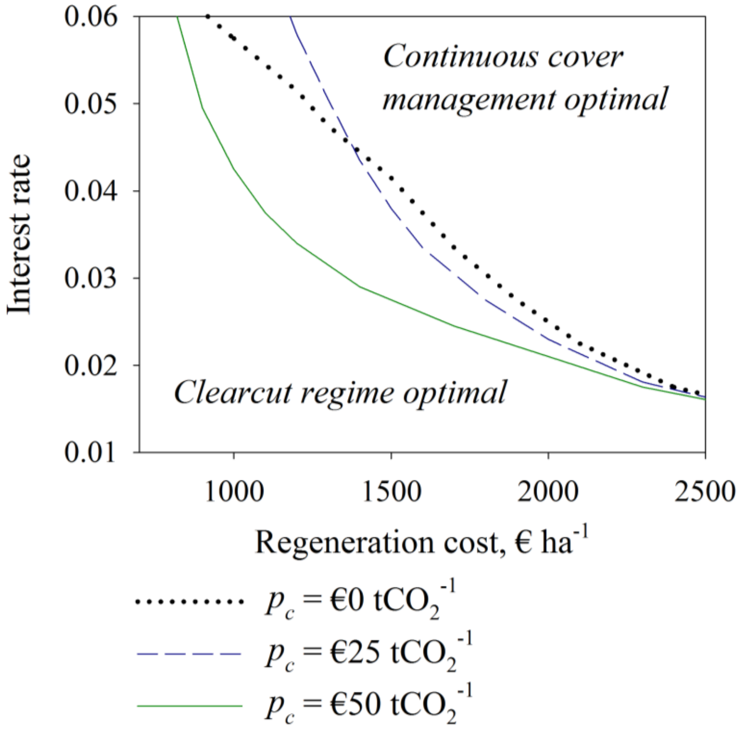

Figure 5. The optimality of continuous cover forestry *vs*. clearcuts, with carbon prices €0–€50 tCO_2^{-1}. Note: $p = €40\ \text{m}^{-3}$, $\alpha = 0.7$.