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Modelling sociocognitive aspects of students' learning

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Abstract

We present a computational model of sociocognitive aspects of learning. The model takes into account a student's individual cognition and sociodynamics of learning. We describe cognitive aspects of learning as foraging for explanations in the epistemic landscape, the structure (set by instructional design) of which guides the cognitive development through success or failure in foraging. We describe sociodynamic aspects as an agent-based model, where agents (learners) compare and adjust their conceptions of their own proficiency (self-proficiency) and that of their peers (peer-proficiency) in using explanatory schemes of different levels. We apply the model here in a case involving a three-tiered system of explanatory schemes, which can serve as a generic description of some well-known cases studied in empirical research on learning. The cognitive dynamics lead to the formation of dynamically robust outcomes of learning, seen as a strong preference for a certain explanatory schemes. The effects of social learning, however, can account for half of one's success in adopting higher-level schemes and greater proficiency. The model also predicts a correlation of dynamically emergent interaction patterns between agents and the learning outcomes.

Keywords: Sociocognitive learning, Cognitive dynamics, Opinion dynamics, Computational model

1. Introduction

The computational modelling of complex social phenomena such as opinion formation [1, 2], the formation of collaborative groups [3, 4, 5], and even progress in science [6] has proved its usefulness as a new tool for conceptualisation and reasoning in fields traditionally found in the human sciences. In the cognitive science and psychology of learning computational and complex systems modelling has advanced our understanding of learning processes, the learning of causal knowledge and categorisation, all fundamental processes behind higher cognitive activities [7, 8, 9, 10]. Already a decade ago a few seminal studies began using computational modelling to study the sociodynamics of collaborative learning in science education [3, 11]. The educational sciences, however, need to address both cognitive and social aspects at the same time, which requires the computational model of an educational phenomenon to integrate cognitive and social factors. This may explain why in the field of educational research computational approaches have advanced slowly, and why we have yet to see similar progress to that in the social and cognitive sciences. In fact, educational researchers have questioned the value of the computational studies, claiming that computational models are inappropriate for educational research and could distort the picture of what the experts believed were core components of the educational sciences [12]. It is certainly true that bringing the complex phenomena of learning and instruction within the scope of modelling requires many simplifications and idealisations. However, contrary to fears that modelling may distort how researchers conceive the complex phenomena in education, the modelling may instead clarify ways to conceptualise those phenomena and help one to see how phenomena traditionally studied separately, such as cognitive and social aspects, are actually intertwined and how one should actually conceptualise them as connected phenomena.

Another reason for slower progress in adopting computational modelling in educational research might be that educational research is fragmented on several disconnected sub-fields based on different and even conflicting research

paradigms which only weakly discuss each other (for a partial list of different research paradigms, see e.g. [13]). This fragmentation into specialised and isolated schools with different paradigms may hinder educational researchers to explore uses of computational approaches, not because of the inappropriateness
35 of the computational methods, but due to conceptual stagnation within the isolated schools, with their fixed and competing paradigms (c.f. [6]).

The present study seeks further advancement in modelling the social and cognitive aspects of teaching-learning processes in science education, referred here briefly as *sociocognitive* aspects of learning. The viewpoint here is that
40 cognitive and social aspects must be treated as interconnected sociocognitive components of teaching-learning process. Moreover, the notion of "teaching" must be explicitly taken into account through the structured training sequence that guides learning (i.e. teaching should be seen as designed to reach a certain target). The basic assumptions in modelling such a teaching-learning process
45 are that the process is affected by: 1) the context of learning and its design (i.e. teaching is structured and supervised), 2) students' cognitive abilities and proficiencies, and 3) social interactions. The basic idealisations of the model, with regard to each issue, are that: 1) the teaching-learning sequence can be described by overlapping, but separable, abstract schemes; 2) the relevant cog-
50 nitive ability is the proficiency to use the given scheme, enhanced/weakened by success/failure; and 3) social interaction either increases or decreases proficiency independently of cognitive abilities. These three *sociocognitive* aspects and how they are idealised are discussed in more detail in what follows.

The context of learning addressed here involves a three-person group's learn-
55 ing task to explain a set of empirical observations[14, 15], for which only a few possible explanatory schemes of different levels of sophistication are available. Some well-known and extensively studied cases of learning scientific knowledge, in particular in the context of learning concepts of electric current and voltage, consist of two closely related concepts, one central relational scheme, one or two
60 constraining schemes (conservation laws), and a few explanatory models making use of the relational and constraining schemes to variable extent (see [14, 15] and

references therein). An example of this kind of situation is described in more
 details in Appendix A. Despite the apparent simplicity of the situation, there
 are several interesting and non-trivial aspects to be modelled, such as how do
 65 learners' cognitive states evolve as a result of their interaction and in response
 to unfolding evidence (events), and how do the interaction patterns affect the
 learning outcomes? Here we model the teaching-learning task (see Appendix A
 for more details) and the corresponding explanatory schemes as an epistemic
 landscape [16, 17], which is an abstract representation of the knowledge required
 70 to solve the task and which provide the background for designing the task. We
 describe the cognitive dynamics as the agent's exploration of the cognitive land-
 scape as a probabilistic selection of an explanatory scheme [11, 18] guided by
 the agent's proficiency in using explanatory models as well as their memory of
 success and failure in providing explanations [14, 15]. Social interactions, on
 75 the other hand, are described by an agent-based model of how agents' proficien-
 cies develop solely through their mutual comparisons of their proficiencies and
 mutual appraisals. In the computational model presented here, these patterns
 of mutual appraisals of proficiencies are not predetermined, but emerge dynam-
 ically as in the bounded confidence models of opinion formation [1, 19]. The
 80 present study is a first step in a direction which attempts to combine cognitive
 and sociodynamics aspects of learning. We hope that the results presented here
 encourage the use of computational approaches in theoretical descriptions of
 learning and in guiding empirical research settings. We believe that this will
 provide new viewpoints on the complexities of learning dynamics.

85 **2. Theory: Sociocognitive aspects of learning**

The context of learning we address here is a task that requires students to
 explain experimental observations of a simple physical system (DC circuits),
 which is designed so that only a few possible explanatory schemes of different
 levels of sophistication are available [15]. The task belongs to a class of cases
 90 consisting of two closely related concepts, one relational scheme relating the

concepts, one or two constraining schemes (conservation laws), and a few explanatory models which use the relational and constraining schemes to variable extents (see [14, 15] and Appendix A for more details). The task we discuss here involves three explanatory schemes with ascending complexity and can thus be
95 represented as a three-tiered structure. A specific example of such a scheme and its detailed description is given elsewhere [14, 15]. In what follows we describe briefly how the present sociocognitive model of learning in this case takes into account: 1) the structure of the explanatory schemes as an epistemic landscape, 2) learners’ cognitive dynamics, and 3) the sociodynamics of learning.

100 2.1. *Epistemic landscapes in learning*

The three-tiered system of explanatory schemes can be represented as an epistemic landscape, which is an abstract representation of the explanatory power of explanatory schemes. Such descriptions have been previously used in studies describing the cognitive and social effects of discovery and knowledge
105 foraging [16, 17]. Here we use the notion of an epistemic landscape to describe the explanatory schemes relevant to the design of the training sequence (e.g. in the learning task).

A three-tiered system of explanatory schemes consists of schemes S_k , $k=1,2,3$, in which the hierarchical level k is defined according to the complexity of the
110 scheme. A level 1 scheme, for example, operates with only a single concept, a level 2 scheme with two concepts and at least one relational or constraining condition, and a level 3 scheme includes essentially all components of a level 2 and one additional relational condition [14, 15]. More complex schemes require greater proficiency from the user of the scheme, such as mathematical proficiency
115 in deriving predictions from the scheme or making deductions based on it. The utility of a given scheme can be seen as a trade-off measure between the scheme’s complexity and the amount of evidence (events) which the learner needs to explain. The scheme S_1 is simple and, thus, its utility for a simple set of evidence (events) is high, but decreases when evidence consists of many
120 events, which are usually correlated. The scheme S_3 is the most complex one

and requires great proficiency. Because it is complex to use, it has low utility in simple cases, but its utility increases with accumulation of evidence. The scheme S_2 falls between these extremes.

2.2. Cognitive dynamics of learning

Recent research has approached conceptual learning from the viewpoint of dynamic systems theory, in which emergent, robust cognitive states are associated with stable learning outcomes [14, 15, 20, 21]. The assumption of a cognitive system as a dynamic system and cognitive states as dynamic emergent states is well established in developmental psychology and the cognitive sciences [7, 8, 9]. A dynamic systems viewpoint motivates us to describe the cognitive process as a dynamic change in the epistemic landscape, which guides the evolution of the cognitive state. In this case, learning is seen as a developmental process, where by foraging the epistemic landscape, learners improve their proficiency in future performances with similar types of tasks (e.g. explaining or problem solving). The learning process then resembles the growth of skill or cognitive competence, which often follows a sigmoidal learning curve. The sigmoidal learning curve, on the other hand, is thought to result from the logistic accumulation of stimulus due to feedback from success or failure [8, 9]. From this viewpoint, learning an explanatory scheme can be seen as a search for the scheme of the highest utility in the epistemic landscape, constrained only by the learner's proficiency in exploring that landscape. Each event, where a student explains something, affects proficiency through a logistic growth process; the outcome of the search dynamically affects the evolution of the learner's proficiency. Learners' proficiency thus depends on the success or failure of the learning. Success supports and encourages the use of a successful strategy and increases students' proficiency. Failure, on the other hand, may significantly decrease the learner's willingness to try again, and as a result may eventually lead to lower proficiency.

2.3. Sociodynamics of learning

150 Proficiency, as informed by the broader notion of self-efficacy [22, 23], is not only a personal cognitive feature, but largely depends on peer-comparison and appraisal between peers [22, 23, 24]. Learners’ constant comparison of their own proficiency to their peers’ proficiencies affects learners’ self-conception of their own proficiency as well as their conceptions of their peers’ proficiencies [22, 23].

155 Peer influence is assumed to function through constant social comparisons and validations of one’s efficacy in comparison to that of one’s peers. As Bandura notes: “shared social appraisals serve as persuasory modes of influence on beliefs of personal efficacy” ([22], p 234). Peer comparison and appraisal is also one of the most elementary forms of conceptualising reciprocal social relationships on which interaction is based and should be part of any model of social interactions

160 [4, 8]. Several studies have examined the interaction and discourse patterns in learning and have discerned the different roles of the dyadic and triadic interaction patterns on successful learning [8, 24]. Here the self-proficiency of the individual learner is assumed to arise from networked learners’ self-proficiencies and their conceptions of each others’ proficiencies (peer-proficiencies) with no

165 independent emergence of a more holistic group efficacy (cf. [22, 23]). Consequently, sociodynamics in this study is assumed to be driven by mutual comparisons between students, who continuously update their conceptions of their own proficiency (self-proficiency) and that of their peers (peer-proficiency). The

170 key element assumed to regulate the effects of comparison is *appraisal*, which in a positive case leads to the enhancement of proficiency, and in a negative case, to its decline. Which effect accrues depends on the agent’s self-proficiency in relation to its conception of peer-proficiencies.

3. Computational model

175 The computational model of sociocognitive learning presented here is an agent based multi-optional model for the probabilistic selection of different explanatory schemes (options of states) guided by the *utility* of these states (com-

pare to e.g. [18, 25]). We first describe the multi-state probabilistic model of explanatory scheme selection and then the sub-models of the epistemic landscapes, cognitive dynamics and social dynamics needed in the complete model of sociocognitive learning.

3.1. Selection of explanatory scheme

The sociocognitive learning model comprises N agents and M explanatory schemes S , from which the agents can select the preferred scheme, one at a time. The possible state of the agent is denoted by vector $\bar{S}_k$ of M dimensions, where the preferred scheme k (only one) is indicated by a value of 1 and unselected schemes by values of 0. States are mutually exclusive so that $\bar{S}_i \cdot \bar{S}_j = 0$. A given particular state of the agent γ is denoted as $\bar{s}_\gamma = \bar{S}_k$. The probability $P(\bar{s}_\gamma = \bar{S}_k)$ that agent γ selects scheme S_k is assumed to follow the canonical probability distribution [18, 25]

$$P(\bar{s}_\gamma = \bar{S}_k) = \frac{\exp [\beta \bar{m}_\xi \cdot \bar{S}_k]}{\sum_{j=1}^M \exp [\beta \bar{m}_\xi \cdot \bar{S}_j]} \quad (1)$$

where β is parameter related to the *confidence* of choice, $\beta \ll 1$ indicating low confidence (i.e. high noise or randomness) and $\beta \gg 1$ high confidence (i.e. low noise or randomness). The vector $\bar{m}$ contains the cognitive and social effects on the choice of the state, and it is given by [18, 25]

$$\bar{m}_\xi = \frac{1}{v_\xi} \sum_{\gamma=1}^N \kappa_{\xi\gamma} \bar{s}_\gamma + \bar{u} \quad (2)$$

where v_ξ is the number of contacts of agent ξ to other agents. The coefficients $\kappa_{\xi\gamma} \in [0, 1]$ define the strength of the directed contact between agent ξ and agent γ . The strength of link $\kappa_{\xi\gamma}$ describes here the agent ξ 's estimate of agent γ 's proficiency (competence) and is thus a natural candidate influencing the state selection. The strength of link $\kappa_{\xi\gamma}$ is termed peer-proficiency in what follows, and the model of social dynamics described later on determines how it develops. The utility vector is composed of utility functions of explanatory schemes. The utility functions depend on the exogenous variables ϵ and κ , where $\kappa \equiv \kappa_{\xi\xi}$

is the self-proficiency of agent ξ (in what follows κ refers self-proficiency if no explicit reference to the agent is necessary). The utility vector is thus as follows

$$\bar{u} = (u(\bar{S}_1; \epsilon, \kappa_{\xi\xi}), \dots, u(\bar{S}_k; \epsilon, \kappa_{\xi\xi}), \dots, u(\bar{S}_M; \epsilon, \kappa_{\xi\xi})) . \quad (3)$$

The utilities enter in the state selection probability in Eq. (1) only through scalar product $\bar{u} \cdot \bar{S}_k$, which reduces to [18]

$$\bar{u} \cdot \bar{S}_k = u_k(\epsilon, \kappa_{\xi\xi}) \quad (4)$$

The probability of state selection can be now written in a simpler form [18]

$$P(\bar{s}_\xi = \bar{S}_k) = \frac{1}{1 + \sum_{j \neq k} \exp[-\beta \Delta_{kj}]} \quad (5)$$

where the factor Δ_{kj} is given by

$$\Delta_{kj} = \omega \frac{1}{v_\xi} \sum_{\gamma=1}^N \kappa_{\xi\gamma} \bar{s}_\gamma \cdot (\bar{S}_k - \bar{S}_j) + (1 - \omega) [u_k(\epsilon, \kappa_{\xi\xi}) - u_j(\epsilon, \kappa_{\xi\xi})] . \quad (6)$$

The first term of Eq. (6) represents the effect of social interaction and its influence on model selection based on how other agents have made selections.

185 The strength of this *conformity* effect is tuned by parameter ω . The second term, consisting of utilities, is the individual agent's own choice, but this choice is also affected indirectly by social dynamics through (self-) proficiency $\kappa \equiv \kappa_{\xi\xi}$.

3.2. The epistemic landscape

The epistemic landscape consists of utility functions $u_k(\epsilon, \kappa)$, which describe
 190 the epistemic utility of schemes S_k . The detailed forms of the functions are, fortunately, not important here; it is enough that they can serve to describe the assumed generic features of the three-tiered system. Therefore, the mathematical description of the epistemic landscape adopted here is based on a set of suitably flexible functions, which are given in Table 1. Convenient mathematical
 195 forms are provided by q-exponential distributions [26, 27] for S_1 and S_3 , and beta-distribution [28] for S_2 . Apart from being sufficiently flexible, these distributions have the advantage that they can be derived with the Maximum

Table 1: The utility functions $u(\epsilon, \kappa)$ forming the epistemic landscape. The normalization factors N_1 , N_2 and N_3 are defined so that maximum value of each utility function is 1. The utility functions for S_1 and S_3 are bivariate q-exponential-distributions and S_2 is bivariate beta-distribution. The parameters chosen here are $q = 1.5$, $\lambda = 0.3$ and $p = 2.5$

State	Utility function
S_1	$N_1 \left[\left(1 - (1 - q)(\epsilon/\lambda)^2\right) \left(1 - (1 - q)(\kappa/\lambda)^2\right) \right]^{1/(1-q)}$
S_2	$N_2 \left[\epsilon^{p-1}(1 - \epsilon)^{p-1} \right] \left[\kappa^{p-1}(1 - \kappa)^{p-1} \right]$
S_3	$N_3 \left[\left(1 - (1 - q)((1 - \epsilon)/\lambda)^2\right) \left(1 - (1 - q)((1 - \kappa)/\lambda)^2\right) \right]^{1/(1-q)}$

Entropy method under given constraints [27, 28], a property which we will not, however, pursue further here. The epistemic landscape and the corresponding selection probability P_k of a state S_k based only on utility differences with no social effects $\omega = 0$ are shown in Fig. 1 for two different parameters β . The epistemic landscape thus consists of three manifolds of which Fig. 1 show the schemes with the greatest utility in a given region. However, the schemes overlap little, and different schemes are clearly separable in Fig. 1. In what follows, we use only the epistemic landscape with $\beta = 2.0$, which is high enough confidence that the structure of the epistemic landscape clearly determines the outcome of state selections, and further increasing the value of β does not significantly affect the results.

3.3. Cognitive dynamics

The model of sociocognitive learning introduced here assumes that learning takes place as foraging for explanation schemes across the epistemic landscape. We assume that foraging is guided simply by the topography of the epistemic landscape, as a "hill climbing" (HS) in the direction of the steepest change of the gradient of the landscape (c.f. refs. [6, 17, 16]). When exogenous parameter ϵ increases by $\delta\epsilon$ (a new event or cue becomes available), the agent selects the most probable explanatory scheme from the neighborhood of its current state either by switching the state, "uphilling" by increasing the proficiency or,

Figure 1: The epistemic landscape (a) corresponding to explanatory schemes S_1 (lower left corner), S_2 (middle) and S_3 (upper right corner) in space spanned by evidence consisting of events ϵ and learner’s proficiency κ . The gray (lemon shaped) line denotes the borders between different states in the landscape. The probability $P(\bar{s}_\xi = \bar{S}_k)$ of finding an agent in state S_k , $k = 1, 2, 3$ are shown for $\beta = 0.5$ (b) and $\beta = 2.0$ (c). The contours are shown for $P = 0.95, 0.90, 0.86, 0.82, 0.78, 0.74, 0.70, 0.67, 0.60, 0.55, 0.50$.

if more advantageous, ”downhilling” by decreasing the proficiency. The state selection rule from state characterized by $\{S_k, \epsilon, \kappa\}$ to the new state $\{S'_k, \epsilon', \kappa'\}$ with $\epsilon' = \epsilon + \delta\epsilon$ and $\kappa' = \kappa \pm \delta\kappa$ is based on making stochastic selection of possibilities based on the set $\{P_k(\epsilon, \kappa), P_{k\pm 1}(\epsilon', \kappa), P_k(\epsilon', \kappa')\}$, where for brevity $P_k \equiv P(\bar{s}_\xi = \bar{S}_k)$ if no reference to any specific agent is needed. Parameter $\delta\kappa$ defines how far the epistemic space is explored in the dimension κ . The stochastic selection of an event based on these probabilities is realised by the ”roulette wheel” method [29, 30], which we will explain later on.

Proficiency, however, is a skill-like property, and a response to success and failure must be described while taking into account the dependence of that response on the current state and cognitive limits to learning a skill. These constraints are assumed to be the reason for sigmoidal-type learning trajectories in skill-learning, which are therefore often modelled as a logistic growth (see e.g. [8, 9] and references therein). Consequently, dynamic model of logistic growth describes the development of agent’s proficiency as

$$\kappa \leftarrow \kappa \pm \mu_{\pm} \kappa (1 - \kappa) \quad (7)$$

where μ_+ is used if the selected step in the exploration of the epistemic landscape increases proficiency and μ_- is used for decreasing proficiency. The effect of memory can be either symmetric ($\mu_+ = \mu_-$) or asymmetric ($\mu_+ \neq \mu_-$), taking into account the agent's different responses to success and failure.

230 3.4. Sociodynamics

The sociodynamics is described here with a variant of the bounded confidence model, which takes into account agents' mutual comparisons of their proficiencies [1, 11, 19]. The self-proficiency $\kappa_{\xi\xi}$ of agent ξ is affected by his or her peers via (appraisal-based) comparison, wherein peer γ evaluates agent ξ 's proficiency as $\kappa_{\gamma\xi}$ while ξ evaluates agent γ 's proficiency as $\kappa_{\xi\gamma}$. The self- and peer-proficiencies are altered as a consequence of this comparison by *appraisal* between the agents (described by parameter α) and if the self-evaluation is sufficiently similar to the peer-evaluation. The probability that an effect takes place is given by [11, 19]

$$p_{\xi\gamma} = \frac{p_0}{1 + \exp[-\delta_{\xi\gamma}/\sigma]} \quad (8)$$

where $\delta_{\xi\gamma} = \kappa_{\xi\gamma} - \kappa_{\xi\xi}$ is the difference between self- and peer-proficiencies; the higher is the agent ξ 's conception of agent γ 's proficiency in relation to agent's own proficiency the higher is the probability that effect takes place (is propagated). The parameter $\sigma = 2\sigma'$ controls how large the difference between
235 proficiencies can be in order for the appraisal to affect the proficiencies (i.e. propagation of the effect) [11, 19]. Here we use the roulette wheel method (see explanation in section 4.2) to determine whether the proficiencies change (i.e. the effect propagates).

If the effect propagates, the changes in proficiencies $\kappa_{\xi\xi}$ and $\kappa_{\xi\gamma}$ are given
240 by [19]

$$\kappa_{\xi\xi} \leftarrow \kappa_{\xi\xi} + p_{\xi\gamma} (\kappa_{\gamma\xi} - \kappa_{\xi\xi}) \kappa_{\xi\xi} (1 - \kappa_{\xi\xi}) \quad (9)$$

$$\kappa_{\xi\gamma} \leftarrow \kappa_{\xi\gamma} + p_{\xi\gamma} [\alpha (\kappa_{\gamma\xi} - \kappa_{\xi\xi}) + (1 - \alpha) (\kappa_{\gamma\gamma} - \kappa_{\xi\gamma})] \kappa_{\xi\gamma} (1 - \kappa_{\xi\gamma}) \quad (10)$$

where the strength of the effect of appraisal is governed by parameter α . The model of social interactions in Eqs. (9)-(10) is dyadic (denoted as d) when it

involves only a pair of agents. This is the most common interaction studied in the context of learning (see e.g. ref. [24] and references therein). For completeness, however, indirect triadic interaction (denoted as t) is studied by including
245 a third agent ζ so that peer-proficiency $\kappa_{\xi\zeta}$ is updated according to the rule in Eq. (10) when ξ and γ interact with probability $p_{\xi\gamma}$.

In the present model appraisal affects peer- and self-proficiencies through the formation of strongly connected and mutually supporting dyads and triads.
250 The effect of these patterns on the dynamic evolution of learning is monitored by counting the number N and intensities W of four central patterns: dyad, augmented dyad, spoke and triad. A *dyad* is a strongly reciprocally connected pair of agents ξ and γ with $\kappa_{\xi\gamma} \approx \kappa_{\gamma\xi} \approx 1$ and no other connections. An *augmented dyad* (dyad+) is a dyad in which both agents in a strong dyad are connected
255 to the same third agent. A *spoke* is a pair of strong dyads in which both dyads share a common agent. Finally, a *triad* is a strongly and reciprocally connected triadic pattern in which all connections are roughly of the same strength (i.e. an egalitarian peer-proficiency pattern). The number of these different patterns can be counted from the adjacency matrix K' , which has elements $[K']_{\xi\gamma} = \kappa_{\xi\gamma}$
260 and $[K']_{\xi\xi} = 0$ (excluding self-proficiencies), as shown in Table 2 (for details and proofs, see e.g. [31, 32]). The intensities W of these patterns [33] are also obtained from matrix K' as geometric means of the links (peer-proficiencies) that constitute the pattern (see Table 2).

4. Simulations

265 4.1. Variables and parameters

The control (exogenous) variable is event ϵ . The output (endogenous) variables are hierarchy index k of the selected explanatory scheme S_k and the learner's proficiency κ , which changes dynamically as a part of the learning process. The output variables depend on the parameters, which are *memory* μ_+
270 and μ_- , *appraisal* α and *conformity* ω .

Table 2: The number N and intensities W of peer-proficiency patterns based on peer-proficiency strengths $\kappa_{\xi\gamma}$. The adjacency matrix K' has elements $[K']_{\xi\gamma} = \kappa_{\xi\gamma}$ and $[K']_{\xi\xi} = 0$ (excluding self-proficiencies). Five other matrices derived from K' are introduced to count the number and intensities of relevant peer-efficacy patterns: the symmetric part of K' , denoted by S' and the asymmetric part $A' = K' - S'$; matrices S and A , where entries are 1 for all non-zero entries in S' and A' , respectively; the symmetric matrix E , constructed so that if $K_{\xi\gamma} \neq 0$ or $K_{\gamma\xi} \neq 0$, then $E_{\xi\gamma} = E_{\gamma\xi} = 1$. Standard matrix operations are used so that T stands for transpose, Tr trace and $\circ$ indicates the element-wise multiplication (Hadamard product). The logical inverse (complement) of the matrix is denoted by $\sim$ (e.g. $\tilde{K}$ as complement of K). Details and proofs of the formulas are given in refs. [31, 32]. The index refers to the standard indexing of the triadic and dyadic patterns [31]. Intensities W are based on the definition in ref. [33].

Pattern	Index		Number		Intensity
dyad	102	N_d	$\sum(\tilde{E}^2 \circ S)/2$	W_d	$N_d^{-1} \sum(\tilde{E}^2 \circ S')^{\frac{1}{2}}/2$
dyad ⁺	120U	N_{d+}	$\sum(AA^T \circ S)/2$	W_{d+}	$N_{d+}^{-1} \sum(A'A'^T \circ S')^{\frac{1}{4}}/2$
spoke	201	N_s	$\sum(S^2 \circ \tilde{E})/2$	W_s	$N_s^{-1} \sum(S'^2 \circ \tilde{E})^{\frac{1}{4}}/2$
triad	300	N_t	$\sum(S^3)/6$	W_t	$N_t^{-1} \sum(S'^3)^{\frac{1}{6}}/6$

4.2. Simulation method

The learning process as foraging across the epistemic landscape is simulated based on the probability of model selection P_k in Eq. (5). At each instant when the value of ϵ increases by $\delta\epsilon$ (here $\delta\epsilon = 0.01$), it is decided whether: 1) the model switch happens; 2) proficiency increases, decreases or remains unchanged; and 275 3) whether the chosen explanatory scheme S_k explains the event or not. Each of these three steps is characterised by a set of probabilities, and event selection is carried out by the roulette wheel -method [30]. Such method of selection has been used also in stochastic modelling of emergence social structures and 280 interaction patterns due to preferential selection [29].

In the roulette wheel -method a discrete set of N possible events k with probabilities p_k are arranged with cumulative probability $\Phi_k = \sum_{i=1}^k p_i / \sum_{i=1}^N p_i$. The event k is selected if random number $0 < r < 1$ falls in the slot $\Phi_{k-1} < r < \Phi_k$. In case 1) the probabilities p_k are given by Eq. (5) and $p_k = P(\bar{s}_\xi = \bar{S}_k)$ with $k = 1, 2, 3$. In case 2) one has three probabilities $p_1 = P_{k'}(\epsilon + \delta\epsilon, \kappa)$, 285 $p_2 = P_{k'}(\epsilon + \delta\epsilon, \kappa + \delta\kappa)$ and $p_3 = P_{k'}(\epsilon + \delta\epsilon, \kappa - \delta\kappa)$ for any given scheme $S_{k'}$. In case 3) the probabilities to explain or not are simply related to the utilities as $p_1 = u_k$ and $p_2 = 1 - u_k$, respectively, for any given scheme S_k . For the sociodynamics in a group of three the interaction event of agent ξ interacting 290 with agent γ with a probability $p_k = p_{\xi\gamma}$ given in Eq. (8) is selected based on the six possible values $p_k \in \{p_{12}, p_{13}, p_{21}, p_{23}, p_{31}, p_{32}\}$. All simulations are carried out for equally distributed set of all initial values of κ , for 100 steps with $\delta\epsilon = 0.01$ and $\delta\kappa = 0.01$ in a mesh of 100x100 points and for 3000 or 9000 repetitions, mentioned separately for each case studied in what follows.

4.3. Representation of data

The outcome of the simulations is the (number density) distribution $n_k(\epsilon, \kappa)$ of the choice of each explanatory scheme S_k given the variable combination (ϵ, κ) . Cumulative changes in κ during the simulation lead eventually to the accumulation of explanatory scheme choices at certain regions in (ϵ, κ) -space, seen as peaked values of $n_k(\epsilon, \kappa)$. However, distribution $n_k(\epsilon, \kappa)$ may not be the

best candidate for direct comparisons of empirical results of learning. Although $n_k(\epsilon, \kappa)$ provides essential information about the explanatory scheme choice and what scheme the learner may hold, this information is in most cases clearly beyond direct empirical access (see e.g. ref. [20]). In an empirical setting, the possibility to explicate the chosen scheme requires that explanation is verbalised or expressed in some way, but low utility schemes do not explain and are thus not easily expressed (see section 2.1. for a discussion of utility as a trade-off between complexity of explanation and evidence explained). One then faces a situation in which a learner may hold an explanatory scheme but is unable to explicate it in a given situation. In this case we expect explanatory strength $\psi_k(\epsilon, \kappa)$, which takes into account only those selections of S_k which explain the evidence, is the quantity to compare to the empirical results. Such a utility-weighted probability is also used in probabilistic decision theory, where it is called the utility factor [25]. The explanatory strength is obtained directly from simulations (first selecting the scheme, then deciding whether it explains), but in practice it is close to utility-weighted density,

$$\psi_k(\epsilon, \kappa) \approx u_k(\epsilon, \kappa)n_k(\epsilon, \kappa). \quad (11)$$

Explanatory strengths are represented in the form of a landscape (similarly to utility) showing only states S_k with the highest probability density P_k .

The output data from simulations depend on three central parameters which are memory μ_+ and μ_- , appraisal α and conformity ω . In order to represent
300 relevant dependencies on these parameters we define two integral quantities: learning gain Γ and similarity (of state selections) Ω .

The *learning gain* Γ_k for each scheme S_k is defined as

$$\Gamma_k = \frac{1}{1 - \langle \kappa \rangle_o} \left(\frac{\langle \kappa \rangle}{\langle \kappa \rangle_o} - 1 \right) \frac{\langle \epsilon \rangle}{\langle \epsilon \rangle_o} \quad (12)$$

where brackets $\langle \dots \rangle$ denotes average over ψ_k with given values of parameters and $\langle \dots \rangle_o$ with all memory, appraisal and conformity set to zero to give a basis of comparisons. Note that learning gain is defined so that negative values occur
305 only if κ decreases in the learning sequence. The factor $1 - \langle \kappa \rangle_o$ in the denomina-

tor takes into account the maximum change in κ in the normalisation. Positive gains mean that explanatory strength increases in high proficiency regions of the landscape and explains more (ψ increases at regions of higher than initial values of ϵ), while negative values indicate a decrease in explanatory strength
310 at high values of κ and ϵ .

The *similarity* Ω_{pq} of states S_p and S_q is defined as (with arguments dropped)

$$\Omega_{pq} = (\delta_{pq}/\delta_{pq}^{(o)}) - 1, \text{ where} \quad (13)$$

$$\delta_{pq} = \iint \Theta[n_p - n_q] (n_p - n_q) d\epsilon d\kappa \quad (14)$$

The step-function $\Theta[\dots]$ introduces an asymmetry $\Omega_{pq} \neq \Omega_{qp}$ so that negative and positive contributions to similarity can be discerned. The quantity δ_{pq}^o is calculated for the reference state, which is set to the state of vanishing conformity
315 $\omega = 0$. The similarity is introduced here in order to monitor how conformity affects the model choices and preferences. Positive values of similarity between states S_p and S_q , where $p > q$ (higher complexity), indicates that conformity drives learning and favours selection of higher level schemes.

5. Results

320 The simulations are carried out for a fixed epistemic landscape, which is the abstract representation of the design of the teaching sequence (see Fig. 1). In the epistemic landscape, with unfolding events ϵ , the learner selects a best explaining explanatory scheme, based on the utility differences of the schemes. The selection probability depends on the learners confidence β and
325 proficiency κ . Here we study an intermediate case $\beta = 2$ only. Proficiency is a dynamic parameter which depends on memory μ_+ and μ_- . This study explores intermediate memory-parameter values with asymmetry $r = \mu_-/\mu_+$. We also vary parameters appraisal α and conformity ω to model the sociodynamics. Here we show the results for explanatory strength $\psi(\epsilon, \kappa)$, learning gain Γ_k ,
330 and selection similarity Ω_k .

5.1. Effects of cognitive dynamics: memory

The memory μ_+ of success or μ_- of failure of a given scheme S_k in explaining events ϵ obviously plays a significant role in shaping the evolution of explanatory strength $\psi(\epsilon, \kappa)$. The effect of memory is shown in Fig. 2 for symmetric cases $\mu_+ = \mu_-$ (upper row) and for asymmetric cases $\mu_+ \neq \mu_-$ (lower row). In the epistemic landscape consisting of three regions (see Fig. 1), symmetric memory leads to the formation of a high density of states S_2 near the peak of utility function u_2 . This is expected and parallels empirical findings that, in a three-tiered case, the explanatory schemes of high utility in the middle-part of a training sequence are abundant and survive even to the end of the training sequence [15]. In the end of the training sequence, when ϵ and κ have values close to 1, the probability of selecting the highest hierarchy scheme S_3 is high, indicated as white regions in upper right corner of figures in Fig. 2. However, at the same time, the number density corresponding to low-hierarchy scheme S_1 (normally associated with low-proficiency regions of the landscape) increases slightly, thereby segregating the landscape into low- and high- hierarchy regions. The segregation is recognised as the growth of high-density regions (white regions, see the left panel in Fig. 2 and the simultaneous formation of black void regions where density ψ is very low. In the asymmetric case, where the memory of failure is suppressed ($r = \mu_-/\mu_+ < 1$), the evolution of selected states favours high-hierarchy, high-proficiency states 2 and 3. Low-hierarchy state 1 with low proficiency disappears altogether (a black void region forms in the lower right corner); as expected, learning becomes enhanced.

The effects of memory and asymmetry of memory is also evident in learning gains Γ_k , shown in Fig. 3. The increase in symmetry decreases the learning gain of scheme S_1 of nearly half its initial value, while the learning gain of scheme S_2 remains intact, and the gains of scheme S_3 increase. Interestingly, increased symmetry also has a negative effect on scheme S_3 for $\mu < 0.04$, and for $\mu = 0.03$ the symmetric case produces the most moderate learning gain. In what follows, we focus on cases of symmetric memory only, and especially on the case $\mu = 0.03$, where cognitive gains are lowest and beneficial effects of social

learning are best discernible. For comparison, we have included the memoryless case $\mu = 0.0$ even though it has no relevance to learning as such.

5.2. Effects of social dynamics: appraisal and conformity

365 *The effect of appraisal α on explanatory strength $\psi(\epsilon, \kappa)$ is shown in Fig. 4*
for memoryless case $\mu = 0$ (upper row) and for symmetric memory $\mu=0.03$ (two
lower rows) as well as for three different appraisals $\alpha = 0.3, 0.6$ and 0.9 , for both
dyadic (d) and triadic (t) interactions. The initial state of agents is sampled
randomly from a homogeneous distribution of proficiencies $0.33 < \kappa < 0.66$.
370 In what follows, the set of agents with a initial proficiencies is referred to as a
cohort. Set of agents with initial values sampled from range $0.33 < \kappa < 0.66$
are thus referred as middle cohort (mid), while the set of agents with initial
proficiencies sampled from $0.0 < \kappa < 1.0$ is a total (tot) cohort. The effect
of appraisal is moderate up to $\alpha \approx 0.5$ and leads only to a slight diffusive
375 broadening of the initial cohort. The effects of appraisal become clear when $\alpha \approx$
 0.6 , as shown in Fig. 4 in the middle row. Then a number of agents prefers high-
hierarchy high-proficiency scheme S_3 and, with increased success, the ψ peaks
at very high proficiencies in the end of the training sequence. Consequently,
learning is enhanced due to social appraisal effects. When appraisal increases to
380 $\alpha = 0.9$ the final situation becomes quite extreme; the majority of agents holds
the highest hierarchy scheme S_3 , but many agents also prefer low-hierarchy
scheme S_1 with low proficiency. Learning is then polarised: many agents benefit
greatly from the appraisal, but others suffer from the indirect side-effects of
segregation. Such a polarising effect of appraisal is most evident in the dyadic
385 model of interaction.

The effect of agents' interaction patterns on learning dynamics and learning
gains is shown in Fig. 5. The basic interaction patterns are dyadic (d) and
triadic (t), where a third agent is also present in appraisal comparisons (see Eqs.
(9)-(10) for details), as well as a combination of these (d+t). This affect the
390 formation of patterns based on peer-proficiencies, of which the dyads, augmented
dyads, spokes and triads (see Table 2) are of particular interest. The number N

Figure 2: The effect of memory μ on explanatory strength ψ when evidence unfolds (described as an increasing number of events ϵ). In the upper row, the memory of failure and success is equal (symmetric memory) $\mu_+ = \mu_- = \mu$ ($r = 1$), with $\mu = 0.01, 0.03$ and 0.05 . The cases with asymmetric memory $r = \mu_-/\mu_+$ with $\mu=0.03$ and $r = 0.1, 0.3$ and 0.5 are shown in the lower row. The full range $\kappa \in [0, 1]$ of initial proficiencies are considered (total cohort) because agents are not interacting ($\alpha = \omega = 0$). The contours are shown for $P = 0.80, 0.70, 0.50, 0.25, 0.15, 0.10, 0.05, 0.02, 0.01, 0.0050, 0.0025$. The number of repetitions for each of 100×100 data points is 9000.

Figure 3: The effect of memory μ on learning gains Γ_k for a given scheme S_k . The results for asymmetric memory with $r = 0.0, 0.2, \dots, 1.0$ are also shown as indicated by the legend. Values are normalised by using reference values $\langle \kappa \rangle_0$ and $\langle \epsilon \rangle_0$ corresponding $\mu_+ = \mu_- = 0.00$. The full range of initial proficiencies are considered (total cohort) because agents are not interacting ($\alpha = \omega = 0$). The number of repetitions is 3000 for each data point.

Figure 4: The effect of appraisal α on explanatory strength ψ in the absence of memory $\mu = 0.00$ (upper row) on and with memory $\mu = 0.03$ (two lower rows). The results are shown for $\alpha = 0.3, 0.6$ and 0.9 and for dyadic (d) and triadic (t) interaction of agents. Otherwise as in Fig. 2

Figure 5: The number N_{pats} of peer-proficiency patterns as a function of appraisal α for dyadic (d), dyadic-triadic (d+t) and triadic (t) agent interactions (upper row). The average intensities W , as calculated from formulas in Table 2, are nearly constant in regions of maximum counts and they are: for (d+t) and (t) $W_t \approx 0.5$, $W_d = W_{d+} \approx 1.0$; for (d) $W_s \approx 0.4$ and $W_{d+} \approx 0.7$. The results of pattern counts are averages of 2000 repetitions for each data point. The learning gains Γ_k for schemes S_k , $k = 1, 2, 3$ are shown for the middle (mid) cohort in the middle row, and for the total (tot) cohort in the lower row. The results are given for symmetric memory $\mu = 0.03, 0.06$ and 0.09 and for dyadic (d) and triadic (t) agent interactions, as indicated by the legend. The results are averages of 3000 repetitions for each data point.

of these patterns are shown in Fig. 5 as a function of appraisal α . The results show that at values $0.3 < \alpha < 0.7$ there is a transition from egalitarian triads with an average intensity of link (peer-proficiency) $W \approx 0.5$ to strong dyads or augmented dyads with an average intensity of link $W \approx 1.0$. The range of values of α where the transition happens depends somewhat on the model d, d+t or t but is generally similar in all cases. Only in the case of model d spokes and single dyads appear in substantial numbers. The results in Fig. 5 show that strong dyads and augmented dyads appear when the appraisal reaches sufficiently high values $\alpha > 0.5$. The intensities correspond to the average peer-proficiency that constitute the patterns and is close to 1 for dyadic patterns and 0.5 (average of the mid-cohort) for egalitarian triadic pattern. This corresponds to the cases where self-proficiencies of the agents constituting the strong dyads are also high and where learning gains Γ_3 for scheme S_3 begin to increase rapidly, whereas gains Γ_1 for S_1 decrease, as expected. The effect is larger with the middle cohort than with the total cohort, indicating that learning gains benefit from group heterogeneity. Comparing these results with the results in Figs. 4 and 3, one can conclude that the polarisation of learning to the high-achieving group with high self-proficiency and to the low achieving group with low self-proficiency is linked to the emergence of strong dyads when $\alpha > 0.5$. Strong dyads, however, increase their learning at the cost of a left-out third partner. When a strong dyad forms, one agent of the group of three is left out and suffers from a decrease of appraisal, eventually diminishing the agent's self-proficiency. Nevertheless, the left-out agent can still hold high peer-proficiency towards the agents in the strong dyad (the case of augmented dyad, dyad+). In the egalitarian case, which dominates at low appraisal region $\alpha < 0.5$, the peer-proficiency patterns are triads with average proficiencies (0.5 in present case). In this case, all agents moderately increase their self-proficiencies, but the increase is not comparable to strong dyads. The current model features no parameter combinations leading to substantial learning gains in egalitarian groups and no combinations which could prevent polarisation and segregation in high-appraisal groups. In short, the model corresponds in one region to supportive, egalitarian learning with a

moderate increase in learning gains, but on the other end, elite group -based learning at the cost of marginalising some of the agents. In the elite groups, the
425 social effects are comparable to the high cognitive learning gains.

The cohort effect, which refers to the effect of diversity in the initial distribution of proficiencies, is demonstrated by comparing the results in Fig 5 for the middle (mid) cohort with initial proficiencies distributed equally in range $\kappa \in [0.33, 0.66]$, and for the total cohort with $\kappa \in [0.0, 1.0]$. The relative learn-
430 ing gains for the highest hierarchy scheme S_3 for the high-appraisal region are higher for the mid-cohort than for total cohort. On the other hand, a decrease of gains in the mid-cohort for high appraisal is stronger than in the total cohort, due to the relatively stronger polarisation effect in the mid-cohort than in the total cohort. The total cohort seems to suppress these differences, apparently
435 owing to greater heterogeneity in the initial proficiencies, but at the cost of preventing similarly effective growth of gains as for more homogeneous groups. In summary, the most egalitarian learning groups are those with moderate appraisal, a heterogeneous initial distribution of agent proficiencies, and triadic social interaction patterns, but these groups do not have the highest learning
440 gains. The highest learning gains, on the other hand, occur in the most non-egalitarian elite groups where proficiencies are initially more homogeneous than in the total cohort, but the high gains comes at the cost of decreasing gains for about one third of the agents.

The effect of conformity of selections is shown in Fig. 6. The conformity
445 has little effect on the form of distribution of explanatory strength $\psi(\epsilon, \kappa)$ and the form remains as it is for $\omega = 0$. However, the absolute values of densities $n_k(\epsilon, \kappa)$ change, even though the form remains nearly intact. This effect is most conveniently studied in terms change in similarity Ω_{pq} between schemes S_p and S_q . For greater discernibility of the changes Fig. 6 shows the similarity so that:
450 1) the positive value indicates increased similarity if $p > q$ and initially $n_p > n_q$ (the high-hierarchy scheme dominates), and if $p < q$ with $n_p < n_q$ (the low-hierarchy scheme dominates); 2) the negative value, in all other cases, indicates increased similarity and positive decreased similarity. The results in Fig. 6

Figure 6: The effect of conformity ω on selection differences Δ for increasing appraisal $\alpha = 0.3, 0.6, 0.9$. Only the dyadic (d) interaction is considered. Results are for a total cohort. Results are averages of 2000 repetitions for each data point. Note that negative values should be multiplied by a factor of three.

show that increase in conformity ω leads to increased similarity in most cases,
as expected. Curiously, only similarity Q_{23} behaves differently, and scheme 2
decreases in the region where it initially dominates, since scheme S_2 essentially
feeds the growth of scheme S_3 .

6. Discussion and conclusions

We have presented a model of sociocognitive learning, which takes into ac-
count some very primary features of a student's learning process on the levels
of individual cognition and of the sociodynamics of learning. The cognitive
aspects of learning the model describes as foraging for explanations on the epis-
temic landscape, guided by the structure (set by the instructional design) of that
landscape through success or failure in foraging. The sociodynamics of learn-
ing, on the other hand, is described as an agent-based model, where agents (i.e.
learners) compare and appraise their own proficiency (self-proficiency) against
their peers' proficiency (peer-proficiency) in using the explanatory models. The
learning outcome is then simultaneously affected by: 1) the epistemic demands
of the task and the likelihood of available explanatory schemes to make sense
of the evidence, 2) the learner's cognitive proficiency in using the explanatory
schemes that the task demands, and finally, 3) the interaction between different

learners and how they affect each other's proficiency through mutual comparisons and appraisals.

The model is an idealised description of the teaching-learning sequence in its
 475 assumptions that: 1) the teaching-learning sequence can be described as abstract
 schemes; 2) the only relevant cognitive property that characterises the learner
 and changes during the teaching-learning sequence is the learners' proficiency
 in using the given scheme, enhanced/weakened by success/failure; and 3) social
 480 interaction either increases or decreases proficiency independently of cognitive
 abilities. Traditional empirical research also focuses on these three aspects, but
 mostly separately and as disconnected areas of learning (see refs. [14, 15, 20, 21]
 and references therein). In this sense, the restrictions on these properties only is
 no more limiting than limitations on focusing on traditional empirical research.
 The advantage of the idealised modelling approach is that interdependencies, at
 485 least on the level of how the generic outcomes are affected, are accessible.

The model is applied here in the case of a three-tiered system of explanatory
 schemes which can serve as a generic description of certain well-known empiri-
 cally studied learning situations (see e.g. [15] and references therein). Cognitive
 dynamics, as it is implemented in the model, leads to the formation of dynam-
 490 ically robust outcomes of learning, seen as densities of preferred explanatory
 schemes for certain events ϵ and proficiencies κ . These states are outcomes of
 the interplay between the design of the learning task and a few basic aspects of
 learners' cognitive and social dynamics, described here as learners' proficiency
 and appraisal. As expected, the memory of success enhances learning by increas-
 495 ing proficiencies, while the memory of failure decreases learning by decreasing
 proficiencies.

The origin of these robust states is in the learning dynamics and how it inter-
 acts with the context (the structure of the learning task). Such robust learning
 states are usually claimed to be found in research of students conceptions, espe-
 500 cially in conceptual change research (see [14, 15, 20, 21] and references therein).
 Here, however, we propose a different viewpoint, where learning outcomes are
 seen as outgrowths of learning dynamics and task designs. Nevertheless, the

learning outcomes bear a close resemblance to the target knowledge (scientific knowledge), but this is because learning tasks, represented here as epistemic
505 landscapes, are constructed to reflect that knowledge, thus guiding learners and giving rise to robust states that resemble the target states. In some cases, depending on the learners' proficiency and the development of the proficiency, learning outcomes may match the target knowledge, but in other cases, may fall short of targeted outcomes. However, even the states that fail to match
510 the targeted states, are robust, thus giving the impression of pre-existing conceptual states of the learner. Consequently, the assumption of static, context-independent students' misconceptions that are directly accessible in empirical settings needs to be reconsidered and one needs a richer picture emphasising the dynamic nature of the robust states (compare to ref. [20, 21]).

515 In the present model, the effects of social learning, as they are monitored through learning gains, can account for a considerable part of successful learning even in case the memory effects (i.e. cognitive effect) in learning are high. For example, in acquiring the high-level schemes S_3 in the middle (mid) cohort (see Fig. 5, middle row) for $\mu = 0.09$ with low $\alpha = 0.3$ gains are approximately
520 0.9 while for high $\alpha = 0.9$ gains are approximately 1.8. In this case about 50 % of the learning gains in acquiring the high-level schemes S_3 with high proficiency ($\kappa > 0.8$) can be attributed to appraisal effects that increase the learners' proficiency. Similar increase is found also for total (tot) cohort (lowest row in Fig. 5). For weaker memory effect the role of social learning may be
525 substantially higher, accounting for nearly all gains in learning for example in case of $\mu = 0.03$. Although it is not possible to make direct comparison with real learning situations, this is in rough agreement with empirical findings where increased self-efficacy (of which proficiency is a part) can account for 50%-60% of the success in problem solving and the effect is larger the lower is the student's
530 level of cognitive performance [22, 23].

In the present model, however, the appraisal effects also lead to polarisation in learning, and the model predicts a strong correlation between the dynamically emergent interaction patterns and the strength of that polarisation. The

most important conclusions in regard to these correlations are: 1) appraisal sup-
535 ports the formation of a high-achieving group of learners due to the formation of
strong, mutually supportive dyads; 2) strong appraisal leads to the polarisation
of learner population to high and low achievers. Triadic instead of dyadic inter-
action ameliorates polarisation, but does not suppress it; 3) egalitarian groups
are obtained for moderate or and low appraisals. In egalitarian groups, learning
540 is only moderately enhanced.

The results presented here demonstrate the intrinsically complex nature of
learning phenomena in which cognitive and social factors intermix. They also
underscore the importance of conceptualising and approaching learning from an
integrated viewpoint of the sociodynamics of learning. The model is a highly
545 simplified and idealised, but nevertheless produces results, which resemble em-
pirical observations of how cognitive and social dynamics of learning affect learn-
ing outcomes. This supports the view that learning processes can be modelled
and that modeling provides insight into the dynamics of learning and how dif-
ferent cognitive and social aspects are connected. We believe that model studies
550 as presented here help to find more integrated ways to conceptualise learning
processes and demonstrate that even the idealised and generic results of com-
putational models can provide insights into complex sociocognitive interdepen-
dencies. Realising the complex nature of the teaching-learning process may also
eventually help science education as a research field to develop new paradigms
555 of research and thus help to prevent science education research fragmenting into
isolated and specialised research areas that focus on cognitive and social factors
separately without attempting to connect them.

Appendix A

The design of the learning task and its organization as a teaching sequence
560 can be considered as a kind of pre-defined epistemic landscape, where learning
is assumed to take place. The design of the teaching sequence is namely based
on the idea of what kind of knowledge and what kinds of explanatory schemes
are expected to be learned at different stages during the learning sequence, and
what kind of competence or proficiency these stages require from a learner. For
565 example in case of DC circuits many of the suggestions to teach and design
teaching sequences explicitly address these questions [14, 15].

In case of learning DC-circuits it is possible to discern three levels of theoretical
sophistication, few explanatory schemes typical to each level and a limited
set of basic concepts and relational and constraining schemes governing the use
570 of these concepts. The target knowledge is to learn and to differentiate concepts
electric current, voltage and resistance, and to learn the basic relational schemes
(laws), Kirchhoffs first and second law, and Ohms law. To achieve this, first
tasks (tasks in set I) are circuits, where components are in series (current and
KI assumed to be learned), second tasks (set II) are circuits with components in
575 parallel (voltage and KII assumed to be learned), and the third and final tasks
(set III) are more complicated circuits with series and parallel connected components
(Ohm's law and differentiation of concept assumed to be learned). The
learners competence in using these ascending levels of abstraction is referred
in what follows as proficiency in constructing and using explanatory schemes
580 [14, 15]. The detailed tasks are as follows:

I Light bulbs in series. The participants compared two variants (a single
light bulb and two light bulbs) in terms of the brightness of the bulbs. This
comparison produces events $\epsilon < 0.33$ to be explained.

II Light bulbs in parallel. The first variant again involves a single light bulb.
585 The second variant involves two light bulbs in parallel. Comparing the two
variants yields events $0.333 < \epsilon < 0.667$ to be explained .

III Comparison of the brightness of light bulbs in series (I) and in parallel (II).

In the first variant, participants compare the brightness of light bulbs in series, and parallel circuits to the one-bulb case only. In the second variant,
590 participants compare series and parallel cases to each other. This produces events $\epsilon > 0.667$ to be explained.

The structure of targeted explanatory schemes characterising the above described situations can be schematically represented the generic three-tiered system, where level of explanatory scheme is defined according to constraining and
595 relational schemes involved in the construction of the scheme, i.e. according to the complexity of the explanatory scheme. The sets I-III of tasks are designed so that they start from task requiring low proficiency to be explained correctly (i.e. explanations that agree with observations) but increased proficiency is required to give correct explanations in subsequent tasks. In our previous study exploring
600 students learning process of DC-circuits we presented on basis of empirical results five different types (excluding the simplest self-explanatory schemes) of explanatory schemes: two simple schemes based on one concept (S_1 here); two schemes containing two concepts, both associated with different type of constraining schemes (S_2 here); and finally, a scientific scheme (S_3 here) which
605 involves constraining schemes and relational scheme relating the two concepts [14, 15].

The utility u_k of an explanatory scheme S_k can be regarded as a balance between theoretical complexity of the explanatory scheme and evidence that it explains. In simple situations (little evidence) simple schemes with limited ex-
610 planatory power are favoured, but in complex situations (broader range possibly structured evidence) complex schemes with high explanatory power gain utility. In a three-tiered theory-structure this kind of utility is rational in the sense that it depends on possibility to deduce the lower level schemes from higher level schemes. Such a utility u_k of the scheme k depends on external (exogeneous)
615 variable: the events ϵ , which is assumed to be cumulative so that $\epsilon > \epsilon'$ is larger evidence set containing all the events contained in ϵ' . In the present case the evidence is assumed to form a set of ascending complexity, so that tasks in set

I correspond values $0 < \epsilon < 0.33$, tasks in set II values $0.33 < \epsilon < 0.66$ and in set III $0.66 < \epsilon < 1.00$. In practice, each task in case of DC circuits consists of
620 about nine features to be explained (the brightness of bulbs, effect of order and configurations of bulbs, and comparisons to previously encountered situations). The discrete evidence space is thus idealised as a continuous one in the present study.

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