<https://helda.helsinki.fi>

Optimal carbon storage in even-and uneven-aged forestry

Assmuth, Aino

2018-02

Assmuth , A & Tahvonen , O 2018 , ' Optimal carbon storage in even-and uneven-aged forestry ' , Forest Policy and Economics , vol. 87 , pp. 93-100 . <https://doi.org/10.1016/j.forpol.2017.09.004>

<http://hdl.handle.net/10138/310805>

<https://doi.org/10.1016/j.forpol.2017.09.004>

cc_by_nc_nd

acceptedVersion

Downloaded from Helda, University of Helsinki institutional repository.

This is an electronic reprint of the original article.

This reprint may differ from the original in pagination and typographic detail.

Please cite the original version.

Optimal carbon storage in even- and uneven-aged forestry

Abstract

We study the effects of forest carbon storage on optimal stand management by applying a model where optimal harvests are partial cuttings, implying uneven-aged forestry, or both partial cuttings and clearcuts, implying even-aged forestry. Optimal carbon storage postpones partial cuttings and increases stand volume along the rotation. Carbon pricing may shorten or lengthen the rotation period depending on interest rate and speed of carbon release from wood products. If the carbon price is high, the shadow value of forest biomass is negative, implying that a higher interest rate leads to higher stand density. In empirically realistic examples, carbon pricing causes a switch from clearcuts to continuous cover management rather than *vice versa*.

Keywords: carbon sequestration, continuous cover forestry, Faustmann model, optimal rotation, uneven-aged forestry

16 1. Introduction

17 Forest ecosystems hold more than double the amount of carbon in the atmosphere (FAO 2006).
18 Carbon storage can be enhanced by reducing deforestation and increasing afforestation, but also by
19 changing forest management in existing forests. The latter option may help prevent price increases of
20 agricultural land and products. As the present stand-level literature mostly deals with changing the
21 rotation period, a wider set of management adaptation options has remained unexplored. Our study
22 applies a generalized stand-level model, where optimal harvests are solely partial cuttings, or both
23 partial cuttings and clearcuts. This model allows the joint optimization of wood production and carbon
24 storage in uneven-aged and even-aged forestry. We show that, in addition to the rotation period,
25 economically efficient carbon storage also changes stand density along the rotation and, in many
26 cases, the optimal management regime from even-aged to uneven-aged management.

27 An extensive body of literature exists on carbon storage and afforestation potential on a national
28 level (e.g. Lubowski et al. 2006, Mason and Plantinga 2013). At the stand level, a seminal paper by
29 van Kooten et al. (1995) examines the effect of carbon taxes and subsidies on optimal rotation age.
30 Their numerical results suggest that internalizing carbon benefits tends to increase rotation ages only
31 moderately. The numerical study by Stainback and Alavalapati (2002) on slash pine forests in the
32 southern U.S. suggests that carbon storage increases sawtimber yields but decreases pulpwood yields,
33 and considerably increases the value of forestland. Guthrie and Kumareswaran (2009) study the effect
34 of carbon credit schemes on the length of rotation period under stochastic timber prices. Olschewski
35 and Benítez (2010) show that carbon storage increases optimal rotation length considerably in tropical
36 fast-growing stands. Akao (2011) shows using an extended Faustmann model that carbon storage
37 may shorten or lengthen the optimal rotation. Hoel et al. (2014) develop the approach in van Kooten
38 et al. (1995) by including forests' multiple carbon pools and the use of wood for bioenergy.

39 These studies (as well as many others) apply the generic Faustmann rotation model (Samuelson
40 1976), where forests can be harvested by clearcutting only. As such, this model is best suited for

41 forest plantations, which, however, account for only 7% of global forest area (Payn et al. 2015). How
42 well this model is suited for more natural forests is questionable. Moreover, risks induced by climate
43 change may favor more diverse management practices in semi-natural forests and even in areas
44 currently dominated by the rotation regime (Gauthier et al. 2015). The main alternative is uneven-
45 aged or continuous cover forestry, which applies partial cuttings (i.e. thinnings) and relies on
46 continuous natural regeneration. Compared to even-aged forestry, this regime is likely to be more
47 favorable to many forest-dwelling species (Calladine et al. 2015) and more resilient against the many
48 threats of climate change (Thompson et al. 2009). Thus, clear interest exists in exploring whether
49 carbon storage favors uneven-aged forestry compared to even-aged forestry or *vice versa*.

50 Goetz et al. (2010) study uneven-aged forestry and carbon storage, but do not analyze the choice
51 between management regimes. Pukkala et al. (2011) study the regime choice, but apply a model
52 without sound economic basis. Gutrich and Howarth (2007) raise the question of management
53 regimes with carbon storage, but the choice is analyzed without optimization. Thus, the question of
54 whether carbon storage favors continuous cover or clearcut forestry is completely open. Our objective
55 is to answer this question analytically using a model with sound economic basis and that covers both
56 management alternatives simultaneously.

57 The economics of uneven-aged management have been studied since Adams and Ek (1974).
58 While most models attempt to circumvent the dynamic complexities of optimizing uneven-aged
59 management¹, Haight (1985) and Haight and Getz (1987) specify and numerically compute a
60 theoretically sound dynamic optimization model for uneven-aged management. In another line of
61 research, Chang (1981) and Chang and Gadow (2010) study optimal partial cutting cycles and
62 growing stocks in uneven-aged stands, while Parajuli and Chang (2012) extend the model with carbon
63 sequestration. Recently it has been shown that both clearcut and continuous cover management can

¹ Various lines of research have been discussed in e.g. Getz and Haight (1989) and Rämö and Tahvonen (2014).

64 be covered by a single framework (Tahvonen 2015, 2016). In Tahvonen (2016), the Clark (1976, p.
65 263–269)² assumptions on forest aging are revised to account for natural regeneration. This leads to
66 an analytically solvable model that includes both forest management regimes and their optimal
67 choice. In the present paper, the model is further extended to include carbon storage.

68 Unlike all previous studies, our study analyzes optimal carbon storage without restrictions on
69 the management system. This allows us to show that the set of economically efficient methods for
70 enhancing carbon storage in forests is much wider than previously thought. We show analytically that
71 carbon pricing may either increase or decrease the optimal rotation age, depending on assumptions
72 on interest rate and carbon release from wood products. When carbon pricing increases rotation
73 length, it tends to cause a switch from clearcuts to continuous cover forestry. This regime shift follows
74 when the model is computed using empirically realistic parameter values. Additionally, we show that
75 carbon storage postpones the beginning of thinning and increases stand volume before the possible
76 clearcut. If the carbon price is very high relative to wood price, the shadow price of stand volume is
77 negative, as the scarce resource is not wood but the remaining capacity for carbon sequestration.
78 These effects have remained unnoticed in both studies based on the generic Faustmann model and
79 studies that include thinning in numerically computed frameworks. All these results are new and
80 reveal that carbon storage implies major changes in the established understanding of managing forest
81 resources.

82 We continue by introducing the model and deriving the optimality conditions. This is followed
83 by an analysis of optimal thinning, after which we present results on optimal rotation and management
84 regime choice. Empirical and numerical examples are given alongside with analytical results. The
85 proofs can be found in the appendix.

² Recall that Faustmann (1849) included thinning in his bare land formula, but remained silent about the possibilities of harvesting trees by thinnings only. A similar route is followed by Clark (1976), while Samuelson (1976) neglects thinning altogether. Halbritter and Deegen (2015) extend the continuous-time thinning and rotation model with a focus on optimized artificial regeneration.

86 2. An economic model for wood production and carbon storage with endogenous management
87 regimes

88 Let $x(t)$ denote the stand volume ($\text{m}^3 \text{ha}^{-1}$) and $h(t)$ the rate of harvested volume ($\text{m}^3 \text{a}^{-1} \text{ha}^{-1}$) in
89 thinning. Regeneration cost is w , annual interest rate δ , and stumpage price p . At the initial moment
90 t_0 the stand volume is x_0 . Stand volume develops as a product of aging $g(t)$ and density-dependent
91 growth $f(x)$:

$$92 \quad \dot{x} = g(t)f(x(t)) - h(t), \quad x(t_0) = x_0. \quad (1)$$

93 Clark (1976, p. 264) assumes that f is single-peaked and that growth of old stands finally ceases
94 independently of volume, i.e. $g'(t) < 0$ and $g(t) \rightarrow 0$ as $t \rightarrow \infty$. These assumptions on aging may
95 be suitable if the model describes the growth of trees planted at t_0 and no natural regeneration occurs,
96 i.e. a pure planted forest. Given this assumption, the outcome is a finite optimal rotation. However, if
97 new saplings can emerge into the stand without planting, density-dependent growth may occur even
98 in an “old” stand. To include natural regeneration, assume that $g(t)f(x)$ may remain strictly
99 positive as $t \rightarrow \infty$. Suppose further that the aging function g and the growth function f are
100 continuous and twice differentiable and

$$101 \quad f(0) \geq 0, \quad f(\underline{x}) = 0, \quad f''(x) < 0, \quad f'(\hat{x}) = 0, \quad 0 < \hat{x} < \underline{x} \quad (A1)$$

$$102 \quad g(0) > 0, \quad g'(t) < 0, \quad g''(t) > 0, \quad \lim_{t \rightarrow \infty} g(t) = \tilde{g} > 0, \quad (A2)$$

$$103 \quad \tilde{g} f'(0) > \delta, \quad (A3)$$

104 where $\underline{x}$ is the carrying capacity of the site, and $\hat{x}$ denotes the growth-maximizing stand volume.
105 Assumption (A3) restricts the analysis to outcomes where continuous cover solutions are not *a priori*
106 ruled out. Figure 1a) shows the original Clark (1976) growth function and 1b) a modified function
107 that satisfies assumptions (A1) and (A2).

108 [Figure 1 here]

109

110 Let $p_c \geq 0$ denote the economic value of one CO₂ unit, $\mu > 0$ the amount of CO₂ per one unit
111 of wood, and $p_c \mu \alpha$ the value of released CO₂ per one unit of harvested wood. If wood is burned
112 immediately we set $\alpha = 1$, as implicitly assumed in the current New Zealand carbon credit system
113 (Manley and Maclaren 2010, Adams and Turner 2012). If carbon storage in wood products is
114 permanent, it may be possible to assume $\alpha = 0$. If CO₂ is instead gradually released as each wood
115 product decomposes according to its specific qualities and usage, we set $0 < \alpha < 1$ (see Appendix 1).

116 The optimization problem takes the form

$$117 \quad \max_{\{h(t), T, x(T) \geq 0\}} V = \frac{-w + \int_0^T e^{-\delta t} [(p - p_c \mu \alpha) h(t) + p_c \mu g(t) f(x(t))] dt + e^{-\delta T} [(p - p_c \mu \alpha) x(T)]}{1 - e^{-\delta T}} \quad (2)$$

118 subject to (1) and

$$119 \quad h \in [0, h_{\max}], \quad (3)$$

120 and where $T \in [0, \infty)$. Notice that the integral term in (2) yields the thinning revenues net of the value
121 of released carbon. Choosing a finite rotation period implies a clearcut and even-aged forestry, while
122 infinite rotation allows maintaining continuous forest cover and thinning without clearcut. Problem
123 (2) can be solved in two steps (cf. Clark 1976, p. 265–269). Thus, we first solve optimal thinning
124 while taking the rotation period as fixed, and given optimal thinning we solve for the optimal rotation
125 period T .

126 The Hamiltonian function for the problem of optimizing thinning reads as

$$127 \quad H = (p - p_c \mu \alpha) h(t) + p_c \mu g(t) f(x) + \varphi(t) [g(t) f(x) - h(t)]. \quad (4)$$

128 The Hamiltonian is linear in $h(t)$ and the necessary optimality conditions take the form (Seierstad
129 and Sydsæter 1987, p. 178–182: theorem 1 and 3)

$$130 \quad \text{if } p - p_c \mu \alpha - \varphi < 0, \quad h = 0, \quad (5a)$$

$$131 \quad \text{if } p - p_c \mu \alpha - \varphi = 0, \quad h \in [0, h_{max}] \quad (5b)$$

$$132 \quad \text{if } p - p_c \mu \alpha - \varphi > 0, \quad h = h_{max}, \quad (5c)$$

$$133 \quad \dot{\varphi} = \varphi \delta - H_x = \varphi \delta - g(t) f'(x) (p_c \mu + \varphi), \quad (6)$$

$$134 \quad \varphi(T) - (p - p_c \mu \alpha) \geq 0, \quad x(T) \geq 0, \quad [\varphi(T) - (p - p_c \mu \alpha)] x(T) = 0. \quad (7)$$

135 To analyze the sufficiency of the necessary conditions, note that

$$136 \quad H_{hh} = 0, \quad H_{xx} = g(t) f''(x) (p_c \mu + \varphi), \quad H_{xh} = 0, \quad H_{xx} H_{hh} - H_{xh}^2 = 0.$$

137 It is not possible to rule out $p_c \mu + \varphi < 0$ and $H_{xx} > 0$ *a priori*. Thus, Hamiltonian may not be concave
 138 in h and x . However, by the sufficiency theorem of Arrow (Sydsæter et al. 2008, p. 332), the necessary
 139 conditions are sufficient if maximized Hamiltonian is concave in x . By (5a, b) it follows that if optimal
 140 h remains either in the singular solution or $h = 0$ regime, the condition $0 < p \leq p_c \mu \alpha + \varphi \leq p_c \mu + \varphi$
 141 holds true ($\alpha \leq 1$), implying that necessary optimality conditions are sufficient when T is taken as
 142 fixed.

143 To obtain the optimality condition for the rotation period we differentiate (2) w.r.t. to T .

144 Rearranging and utilizing (1) yields

$$145 \quad \frac{\partial V}{\partial T} = \frac{e^{-\delta T}}{1 - e^{-\delta T}} \left\{ [p + (1 - \alpha) p_c \mu] g(T) f(x(T)) - \delta [(p - p_c \mu \alpha) x(T) + V] \right\}. \quad (8a)$$

146 Since $e^{-\delta T} / (1 - e^{-\delta T}) > 0$, for any finite T and $\delta > 0$ we obtain the necessary optimality condition
 147 for finite rotations as

$$148 \quad y(T) \equiv [p + (1 - \alpha) p_c \mu] g(T) f(x(T)) - \delta [(p - p_c \mu \alpha) x(T) + V] = 0. \quad (8b)$$

149

150

151 3. Results and discussion

152 3.1 Optimal thinning

153 The switching function for (4) is $\sigma \equiv p - p_c \mu \alpha - \varphi$, and $\dot{\sigma} = -\dot{\varphi} = 0$. By (6) and (5b),

$$154 (p - p_c \mu \alpha) \delta - g(t) f'(x) [p + (1 - \alpha) p_c \mu] = 0. \quad (9)$$

155 Rearrange (9) into

$$156 f'(x) g(t) = \frac{p - p_c \mu \alpha}{p + (1 - \alpha) p_c \mu} \delta. \quad (10)$$

157 By (10), $f'' < 0$ and $f'(\hat{x}) = 0$ in (A1), the singular solution satisfies the properties

$$158 \text{ if } p > p_c \mu \alpha, f'(x) > 0 \Rightarrow x < \hat{x}, \quad (11a)$$

$$159 \text{ if } p = p_c \mu \alpha, f'(x) = 0 \Rightarrow x = \hat{x}, \quad (11b)$$

$$160 \text{ if } p < p_c \mu \alpha, f'(x) < 0 \Rightarrow x > \hat{x}. \quad (11c)$$

161 Hence the optimal stand volume on the singular path will be below (above) the growth-maximizing
162 stand density if stumpage price exceeds (is smaller than) the value of released CO₂. Differentiating
163 (9) with respect to time and utilizing (1) yields:

$$164 h(t) = g(t) f(x) + (p - p_c \mu \alpha) \frac{g'(t) \delta}{[p + (1 - \alpha) p_c \mu] g^2(t) f''(x)}, \quad (12)$$

165 where the sign of the quotient is positive. Thus, if $p > p_c \mu \alpha$, the rate of thinning exceeds stand
166 growth, and stand volume decreases on the singular path (Figure 2). If $p < p_c \mu \alpha$, the harvest rate is
167 lower than stand growth, and stand volume increases, but harvest rate decreases on the singular path.
168 The special case $p = p_c \mu \alpha$ implies $g(t) f'(\hat{x}) = 0$, i.e. thinning maximizes stand growth.

169 Given $x_0 = 0$, there exists an initial time interval $t \in [0, t_1]$ and a regime where $h(t) = 0$, i.e.
170 the stand is initially left to grow undisturbed. The correct choice of $\varphi(0)$ implies that

171 $\varphi(t_1) = p - p_c \mu \alpha$ at the moment when the solution for $\dot{x} = g(t)f(x)$, $x_0 = 0$ intersects the singular
 172 solution in x, t plane, and h jumps to the singular-solution level defined by (12).

173 If carbon storage is omitted by setting $p_c = 0$, equation (10) reduces to $f'(x)g(t) = \delta$ as in
 174 Clark (1976, p. 265). Note that in (10), $p_c > 0$ implies $(p - p_c \mu \alpha) / [p + (1 - \alpha)p_c \mu] < 1$. Hence it
 175 follows by the concavity of f that given a positive interest rate, the higher the carbon price, the
 176 higher the stand density at any stand age along the singular solution. Additionally, this implies that
 177 thinning must begin later under optimal carbon storage, i.e.

$$178 \quad \frac{\partial t_1}{\partial p_c} > 0. \quad (13)$$

179 Independently of carbon price, zero interest rate implies that optimal thinning maintains the
 180 stand density at the growth-maximizing level (Equation 10). The same result is obtained in the
 181 theoretical case with no CO₂ release from harvested trees ($\alpha = 0$) and given $p_c \rightarrow \infty$. This is a natural
 182 outcome when only the maximization of carbon sequestration matters. If $0 < \alpha < 1$, the RHS of (10)
 183 approaches $-\alpha\delta / (1 - \alpha) < 0$ from above as $p_c \rightarrow \infty$. Given $\alpha < 1$ this value is finite, implying that
 184 thinning may be optimal even with very high carbon price. If all CO₂ is released immediately ($\alpha = 1$),
 185 we obtain $f'(x)g(t) = (p - p_c \mu)\delta / p \rightarrow -\infty$ as $p_c \rightarrow \infty$, implying that thinning becomes
 186 suboptimal.

187 By implicit differentiation, $\partial x / \partial \alpha > 0$ showing that the faster CO₂ is released from harvested
 188 wood, the higher is the optimal stand volume along the singular solution. If $p > p_c \mu \alpha$, it follows by
 189 (10) that $\partial x / \partial \delta < 0$, i.e. with a higher interest rate it is optimal to begin thinning earlier and keep the
 190 biomass level lower. However, given $p < p_c \mu \alpha$, we obtain $\partial x / \partial \delta > 0$ and that a higher interest rate
 191 postpones thinning and implies a higher biomass level along the singular solution. It becomes optimal
 192 to utilize the initial high-growth regime and carbon sequestration benefits longer, and to begin

193 thinning only once the high stand volume begins decreasing stand growth and the immediate benefits
 194 from carbon sequestration.

195 When $p < p_c \mu \alpha$, we obtain by (5b) that $\varphi < 0$, i.e. if the value of released CO₂ per harvested
 196 unit exceeds the stumpage price, the shadow price of the stand volume is negative. This is because
 197 any increase in stand volume decreases stand growth and thus the valuable sequestration of carbon.
 198 Further, an increase in stand volume provides no additional benefit, as the direct net revenues from
 199 harvesting are negative. In this case, the scarce resource is not wood but the remaining capacity for
 200 carbon sequestration. The negative shadow price explains the seemingly counterintuitive finding that
 201 the optimal harvest rate can be positive even when the direct net revenues from thinning are negative:
 202 stand volume is controlled to maintain a sufficient rate of stand growth and carbon sequestration. As
 203 stand growth exceeds the harvest rate, and the value of released CO₂ cannot be larger than the value
 204 of sequestered CO₂ ($0 \leq \alpha \leq 1$ implies $p_c \mu \alpha \leq p_c \mu$), the combined net revenues from harvesting and
 205 carbon storage will actually be positive.

206 To present empirical examples, we apply a growth function specification calibrated using the
 207 growth model in Bollandsås et al. (2008), estimated for Norway spruce (average productivity site)³:

$$208 \quad g(t)f[x(t)] = \left(\frac{1.6}{1 + 0.04t^{1.2}} + 1 \right) 0.065[x(t) + 8] \left[1 - \frac{x(t) + 8}{378} \right], \quad x_0 = 0. \quad (14)$$

209 Given (14), the differential equation $\dot{x} = g(t)f[x(t)]$, $x_0 = 0$ can be solved analytically, and the
 210 solution shows the development of stand volume without harvesting (solid line in Figure 2). If left
 211 undisturbed, the stand will approach a volume of 370 m³ ha⁻¹ ($= \underline{x}$). Stand growth is maximized at a
 212 volume of 181 m³ ha⁻¹ ($= \hat{x}$, dash-dotted line in Figure 2). The long-run maximum sustained thinning
 213 yield is approximately 6.2 m³ a⁻¹ ha⁻¹ ($= \tilde{g} f(\hat{x})$). Singular solutions with various carbon prices are

³ Our growth function specification yields similar long-run maximum sustained thinning yield, maximum volume, and maximum growth as the Bollandsås et al. (2008) model.

shown in Figure 2 (dotted line and dashed lines). We set the CO₂ mass per wood volume unit as $\mu = 0.7 \text{ tCO}_2 \text{ m}^{-3}$ following Niinimäki et al. (2013). Additionally, we set $\alpha = 0.7$, reflecting an assumption that half of the carbon content of harvested wood is released back to the atmosphere after 10 years (see Appendix 1).

[Figure 2 here]

The interceptions of the undisturbed growth path and the singular solutions show the switching moment where it is optimal to begin thinning (circle symbols in Figure 2). As shown analytically, a higher carbon price postpones the switch to thinning and increases the stand volume along the solution for optimal thinning. If the carbon price is sufficiently large to imply that $\varphi = p - p_c \mu \alpha$ is negative, stand volume exceeds the growth-maximizing level and continues to increase while the stand is thinned.

The effect of carbon storage on optimal thinnings has not been studied analytically in the previous literature. Thinnings have been included in the numerical analysis of the even-aged problem by e.g. Pohjola and Valsta (2007), Daigneault et al. (2010), and Niinimäki et al. (2013). According to these studies, carbon pricing postpones thinnings and increases stand volume along the rotation. We have derived this result analytically in addition to showing how optimal thinning depends on model parameters.

Numerical studies have not explicitly analyzed the situation where stumpage price is lower than the value of released CO₂. Our analysis reveals that in this case, optimal stand density increases with interest rate, and the shadow value of stand volume is negative. Moreover, while thinnings cause a direct cost, they are optimal as this optimizes the development of forest biomass as a carbon sink and a source of income from carbon storage. Given average stumpage prices in Nordic countries (around €40m⁻³) and assuming instant release of CO₂ at harvest, such a situation occurs if the carbon price

exceeds €60 tCO₂⁻¹. However, stumpage price approaches zero when the distance from the site to the nearest road and wood-processing plant increases toward the extensive margin. In contrast, the economic value of the carbon storage service is independent of site location. Hence, the value of released CO₂ may dominate the value of wood even with realistic carbon prices.

243

3.2 Optimal rotation age

The question of whether a clearcut regime is economically preferable to continuous cover management is ultimately a question of optimal rotation age. Finite optimal rotation implies clearcuts, while infinite rotation implies the optimality of continuous forest cover.

The necessary condition for a finite optimal rotation period is given by Equation (8b). To exclude the case where the stand is clearcut before the singular path is reached (cf. Clark 1976, p. 267), assume that $y(t_1) > 0$. Differentiating $y(T)$ and applying (10) and $V'(T) = 0$ yields

$$y'(T)_{|y(T)=0} = [p + (1 - \alpha)p_c\mu]g'f < 0. \quad (15)$$

Thus, given any finite T satisfying (8b), the necessary conditions are sufficient for an optimal unique rotation age. The simplicity of this uniqueness result follows from the properties of the singular solution and it can be contrasted with the uniqueness result for the rotation model without thinning (Akao 2011, Gong and Löfgren 2016).

If $\lim_{t \rightarrow \infty} g(t) = \tilde{g}$ is very low and $(p - p_c\mu\alpha)x(T) + V > 0$, then $y(T)$ will become negative when T is high enough. Thus, the optimal rotation is finite if the long-term yield from thinning is low enough and the sum of clearcut net revenues and bare land value is positive. However, in (8b) $[p + (1 - \alpha)p_c\mu]g(T)f(x(T)) - \delta[(p - p_c\mu\alpha)x(T)] > 0$ by (10) and the concavity of f . Thus, if the bare land value is negative or sufficiently small albeit positive, then $y(T) > 0$ for $T \in [t_1, \infty)$ and the optimal rotation will be infinite. In these cases, it is optimal to continue thinning forever without clearcuts, i.e. to apply continuous cover forestry.

For interpretation, write (8b) as

$$\left[p + (1 - \alpha) p_c \mu \right] [h(T) + \dot{x}(T)] = \delta \left[(p - p_c \mu \alpha) x(T) + V \right]. \quad (16)$$

Thus, at the moment of clearcut, the rate of wood and carbon revenues net of their decrease equals the interest on the sum of clearcut net revenues and the value of bare land. Conversely, an infinite rotation and continuous cover management is optimal if $y(T) \geq 0$ for all $T \in [0, \infty)$ in (8b). Let us denote thinning satisfying (12) by $\lim_{t \rightarrow \infty} h(t) = \tilde{g}f(\tilde{x}) = \tilde{h}$. By the uniqueness result (6), the condition

$$\left[p + (1 - \alpha) p_c \mu \right] \tilde{h} \geq \delta (p - p_c \mu \alpha) \tilde{x} + V \quad (17)$$

implies that no solutions for finite rotation periods for (8b) exist, implying that the remaining optimal solution is to apply continuous cover forestry instead of clearcuts. Thus, continuous cover management is optimal, if the steady-state wood and net carbon storage revenues from thinning exceed the interest earnings for the values of clearcut net revenues and bare land.

3.3 The effect of carbon price on rotation age and choice of management regime

Factors implying a longer optimal rotation period favor continuous cover forestry and *vice versa*. Taking into account that the solution satisfies the singular condition (7), the derivative of (8b) with respect to carbon price can be given in the form

$$\frac{\partial y(T)}{\partial p_c} = (1 - \alpha) \mu g(T) f(x(T)) + \delta \mu \alpha x(T) - \delta \frac{\partial V}{\partial p_c}. \quad (18)$$

The first term $(1 - \alpha) \mu g(T) f(x(T)) \geq 0$ is the increase in net carbon storage revenues at T as carbon price increases. The second term $\delta \mu \alpha x(T) \geq 0$ is the decrease in the interest cost from postponing the clearcut, because a higher carbon price translates to a higher value of released CO₂ when the stand is clearcut. The third term $-\delta \partial V / \partial p_c < 0$ reflects the increase in interest cost through increased bare land value. As carbon storage revenues are an additional source of income, they can only increase the bare land value.

286 *Proposition 1. Given that $|p - p_c \mu \alpha|$ is low enough, α close enough to one, and $\delta > 0$, an increase*
287 *in carbon price lengthens optimal finite rotation periods. Proof, Appendix 2.*

288 The conditions specified in Proposition 1 are sufficient, but not necessary for the outcome that the
289 rotation period increases with carbon price. Thus, at least under the given conditions, carbon storage
290 and a higher carbon price support longer rotations and continuous cover forestry. Numerical
291 examples, assuming the growth specification (14) and a 3% interest rate, show that optimal rotation
292 length increases with carbon price regardless of the speed of CO₂ release from harvested wood (Figure
293 3). However, the magnitude of the effect varies substantially. Given $\alpha = 1$, i.e. assuming that all of
294 the CO₂ is released immediately, a carbon price above €30 tCO₂⁻¹ leads to a shift from clearcuts to
295 continuous cover management. With slower CO₂ release from wood products ($\alpha = 0.7$), carbon
296 pricing implies continuous cover management when carbon price exceeds €45 tCO₂⁻¹. Given the case
297 of no CO₂ release ($\alpha = 0$), carbon pricing does not reduce clearcut net revenues and increases bare
298 land value considerably, implying that the incentive to postpone clearcutting is weak.

299

300 [Figure 3 here]

301

302 However, a higher carbon price does not always imply longer rotation:

303 *Proposition 2: Given $\alpha = \delta = 0$ and $w > 0$, an increase in carbon price shortens optimal finite*
304 *rotation periods. Proof, Appendix 3.*

305 Under the given assumptions, rotation age decreases with carbon price. This can be understood by
306 comparing economically optimal solutions to the solution that maximizes sustained yield (MSY).
307 Given positive regeneration cost, zero interest rate, and no carbon pricing, the optimal rotation period
308 is longer than the MSY rotation. When no carbon is released from harvested trees ($\alpha = 0$), a higher
309 carbon price favors the volume-maximizing solution and thus shortens the rotation period. Numerical
310 examples suggest that the result in Proposition 2 may also hold when interest rate is positive but low.

311 Given $\delta = 0.5\%$ and $\alpha = 0$, rotation age decreases from 129 to 120 years as carbon price increases
312 from zero to $\text{€}70 \text{ tCO}_2^{-1}$ (Figure 4), while the MSY rotation is 100 years.

313

314 [Figure 4 here]

315

316 According to most of the literature (e.g. van Kooten et al. 1995, Gutrich and Howarth 2007,
317 Pohjola and Valsta 2007, Olschewski and Benítez 2010, Pihlainen et al. 2014), valuing carbon storage
318 increases optimal rotation age. However, Akao (2011) shows that within the generic Faustmann
319 framework this is not always the case: carbon pricing may shorten finite optimal rotation periods if
320 no carbon is released after harvesting. Hoel et al. (2014) obtain a similar result using a model without
321 regeneration cost. Using a generalized framework with optimized thinning and the option of
322 continuous cover forestry, we show that carbon pricing may indeed either increase or decrease
323 optimal rotation age. However, we find that the decreasing effect requires strong assumptions: the
324 interest rate must be close to zero, and carbon release after harvesting must be minimal.

325 Given more plausible assumptions, carbon pricing lengthens rotations and in many cases leads
326 to infinite rotations, implying continuous cover forestry. Such regime shifts are beyond the scope of
327 the existing studies that are restricted to the clearcut regime only. Both van Kooten et al. (1995) and
328 Hoel et al. (2014) find that with a sufficiently high carbon price the clearcut is postponed indefinitely,
329 i.e. harvesting becomes suboptimal. We show that when restrictions on the management system are
330 relaxed, high carbon prices tend to imply continuous cover forestry in the form of partial cuttings,
331 instead of total abandonment of wood production.

332 We showed analytically that continuous cover management becomes optimal when bare land
333 value is low enough (albeit positive). Increasing the interest rate or the regeneration cost decreases
334 the bare land value toward zero and below. This suggests that given some regeneration cost level,
335 there may exist some interest rate that implies equal profitability of the continuous cover and

clearcutting regimes. Moreover, such interest rate is the higher, the lower is the regeneration cost. This yields break-even curves where the two management regimes are equally profitable (Figure 5). A carbon price of €50 tCO₂⁻¹ widens the optimal application area of continuous cover management compared to the case without carbon pricing. With a carbon price of €25 tCO₂⁻¹ this effect is less clear: given a high interest rate, the continuous cover regime is less competitive than without carbon pricing. Thus, the interplay between the effects of carbon price, interest rate, and other economic parameters is far from straightforward.

Forest carbon storage is currently not included in the Emissions Trading Scheme (ETS) of the European Union (EU). However, New Zealand has integrated forest carbon storage in its ETS (Adams and Turner 2012), and in the future the EU may adopt a similar system to incentivize management adaptation and afforestation by landowners. The EU is committed to a 40% reduction of domestic greenhouse gas emissions from the 1990 level by 2030 (European Commission 2015). Depending on policy scenario, achieving this target would translate to an EU ETS carbon price in the range of €11–€53 tCO₂⁻¹ by 2030, and €85–€264 tCO₂⁻¹ by 2050 (European Commission 2014, p. 80–81). Based on our results, such carbon price levels would imply a switch to continuous cover management with high stand density.

The model used in this study can be augmented to include carbon emissions from the use of harvesting and hauling machinery. Soil carbon dynamics is another possible extension. Given that a clearcut exposes the forestland to sunlight and thus increases topsoil temperatures, it is likely that including carbon fluxes to and from the soil would further improve the relative profitability of continuous cover management under carbon pricing.

[Figure 5 here]

361 4. Conclusions

362 The global temperature target of the Paris agreement (UNFCCC 2015) is ambitious, and may not be
363 attainable without increased carbon storage in forests. To this end, it is highly important to understand
364 how the inclusion of carbon storage changes optimal stand management. Existing forest economic
365 models typically apply the generic Faustmann model, which is limited to harvesting forest by
366 clearcuts only, and thus excludes thinning and the possibility of continuous cover forestry. To adapt
367 management of forest resources to the threat of climate change, it is necessary to generalize the classic
368 forest economic models to incorporate more diverse management options. We apply a generalized
369 model that can still be studied analytically.

370 Our analysis shows that optimal carbon storage postpones thinnings and increases stand volume
371 along the rotation. Further, we show that carbon pricing may shorten or lengthen the rotation period,
372 depending on interest rate and the speed of carbon release from wood products. Our empirical
373 examples suggest that carbon pricing may well imply a switch from clearcuts to continuous cover
374 management. We also demonstrate that the shadow value of forest biomass may be negative if carbon
375 price is high. In such cases the economic value of a forest stand is dominated by the remaining
376 capacity for carbon sequestration. This may have strong effects on the value of forest stands along
377 the rotation, but we leave this subject to future studies. Further, as forest heterogeneity is vital for
378 maintaining resilience under climate change, future research should develop understanding of optimal
379 carbon storage in size-structured mixed-species forest stands.

380

381 Appendix 1. Emissions from wood products:

382 Denote the decay rate of wood products by $g\kappa$ and the harvested volume by Y . At moment τ , the
383 increase in atmospheric carbon stock equals the initial stock minus what is left at moment τ :

384
$$A(\tau) = \mu Y - \mu Y e^{-g\kappa\tau},$$

385 implying that the emission level at moment τ is

$$\frac{dA(\tau)}{d\tau} = g\mu Y e^{-g\tau}.$$

Denote the social discount rate by r . As the economic present value of emissions occurring at τ equals $p_c g\mu Y e^{-g\tau} e^{-r\tau}$, the present value of emissions over an infinite time horizon is

$$\int_0^{\infty} p_c \mu Y g e^{-(g+r)\tau} d\tau = p_c \mu Y \frac{g}{g+r}.$$

Hence, per unit of wood product, the present value of emissions due to decay can be given as

$$\alpha = \frac{g}{g+r}.$$

This approach has previously been used in Pihlainen et al. (2014).

Appendix 2. Proof of proposition 1:

Write (8b) in the form

$$y(T) \equiv [p + (1-\alpha)p_c\mu]g(T)f[x(T)] - \frac{\delta}{1-e^{-\delta T}} \left\{ -w + p_c\mu \int_0^{t_1} g(t)f[x(t)]e^{-\delta t} dt + \int_{t_1}^T e^{-\delta t} \left\{ (p-p_c\mu\alpha)h(t) + p_c\mu g(t)f[x(t)] \right\} dt + (p-p_c\mu\alpha)x(T) \right\} = 0.$$

Thus,

$$\frac{\partial y(T)}{\partial p_c} = (1-\alpha)\mu g(T)f[x(T)] + [p + (1-\alpha)\mu]g(T)f'[x(T)] \frac{\partial x(T)}{\partial p_c} - \frac{\delta}{1-e^{-\delta T}} \left\{ \begin{aligned} & \mu \int_0^{t_1} g(t)f[x(t)]e^{-\delta t} dt + \frac{\partial t_1}{\partial p_c} e^{-\delta t_1} p_c\mu g(t_1)f[x(t_1)] \\ & - \frac{\partial t_1}{\partial p_c} e^{-\delta t_1} \left\{ (p-p_c\mu\alpha)h(t_1) + p_c\mu g(t_1)f[x(t_1)] \right\} - \mu\alpha x(T) + (p-p_c\mu\alpha) \frac{\partial x(T)}{\partial p_c} \\ & + \int_{t_1}^T e^{-\delta t} \left\{ \mu \left\{ g(t)f[x(t)] - \alpha h(t) \right\} + (p-p_c\mu\alpha) \frac{\partial h(t)}{\partial p_c} + p_c\mu g(t)f'[x(t)] \frac{\partial x(t)}{\partial p_c} \right\} dt \end{aligned} \right\}.$$

Given $\alpha \rightarrow 1$ and $|p - p_c\mu| \rightarrow 0$ and $\delta > 0$ we obtain by (10) and (13) that

$$\frac{\partial y(T)}{\partial p_c} \rightarrow \frac{-\delta}{1-e^{-\delta T}} \left\{ \mu \int_0^{t_1} e^{-\delta t} g(t_1)f[x(t_1)] dt - \mu x(T) \right\} > 0.$$

401 Because $\frac{\partial y(T)}{\partial T} \Big|_{y(T)=0} < 0$ it follows that $\frac{\partial T}{\partial p_c} > 0$. Q.E.D.

402

403

404 Appendix 3. Proof of proposition 2:

405 When $\alpha = \delta = 0$, equation (8b) is written as

$$406 \quad y(T) = (p + p_c \mu) g(T) f[x(T)] - \frac{1}{T} \left\{ -w + \int_0^{t_1} p_c \mu g(t) f[x(t)] dt + \int_{t_1}^T \{ ph(t) + p_c \mu g(t) f[x(t)] \} + px(T) \right\} = 0.$$

407 We obtain

$$408 \quad \frac{\partial y(T)}{\partial p_c} = \mu g(T) f[x(T)] - \frac{1}{T} \left\{ \int_0^{t_1} \mu g(t) f[x(t)] dt + \int_{t_1}^T \mu g(t) f[x(t)] dt \right\}.$$

409 Since $\int_0^{t_1} g(t) f[x(t)] dt = x(T)$ and $\int_{t_1}^T g(t) f[x(t)] dt = \int_{t_1}^T h(t) dt$ by (10), (12) and $\delta = 0$, $y(T)$

410 can be written as

$$411 \quad (p + p_c \mu) \left\{ g(T) f[x(T)] - \frac{1}{T} \left[\int_{t_1}^T h(t) dt + x(T) \right] \right\} = -\frac{w}{T}$$

412 showing that the sign of $\partial y(T) / \partial p_c$ is negative. Thus, $\frac{\partial T}{\partial p_c} < 0$. Q.E.D.

413

414

415 This research did not receive any specific grant from funding agencies in the public, commercial, or
416 not-for-profit sectors.

417

418

419

420

421 References

- 422 Adams, D.M., Ek, A.R., 1974. Optimizing the management of uneven-aged forest stands. Canadian
423 Journal of Forest Research 4(3), 274–287.
- 424 Adams, T., Turner, J.A., 2012. An investigation into the effects of an emissions trading scheme on
425 forest management and land use in New Zealand. Forest Policy and Economics 15, 78–90.
- 426 Akao, K.I., 2011. Optimum forest program when the carbon sequestration service of a forest has
427 value. Environmental Economics and Policy Studies 13(4), 323–343.
- 428 Bollandsås, O.M., Buongiorno, J., Gobakken, T., 2008. Predicting the growth of stands of trees of
429 mixed species and size: A matrix model for Norway. Scandinavian Journal of Forest Research
430 23, 167–178.
- 431 Calladine, J., Bray, J., Broome, A., Fuller, R.J., 2015. Comparison of breeding bird assemblages in
432 conifer plantations managed by continuous cover forestry and clearfelling. Forest Ecology and
433 Management 344, 20–29.
- 434 Chang, S.J., 1981. Determination of the optimal growing stock and cutting cycle for an uneven-aged
435 stand. Forest science 27(4), 739–744.
- 436 Chang, S.J., Gadow, K.V., 2010. Application of the generalized Faustmann model to uneven-aged
437 forest management. Journal of Forest Economics 16(4), 313–325.
- 438 Clark, C.W., 1976. *Mathematical Bioeconomics. The optimal management of renewable resources.*
439 Wiley, New York. 352 p.
- 440 Daigneault, A.J., Miranda, M.J., Sohngen, B., 2010. Optimal Forest Management with Carbon
441 Sequestration Credits and Endogenous Fire Risk. Land Economics 86(1), 155–172.
- 442 European Commission, 2014. Impact Assessment SWD (2014), 15 final. Accompanying the
443 document Communication from the Commission to the European Parliament, the Council, the

European Economic and Social Committee and the Committee of the Regions. A policy framework for climate and energy in the period from 2020 up to 2030. 234 p.

European Commission, 2015. Intended Nationally Determined Contribution of the EU and its Member States. Submission by Latvia and the European Commission on behalf of the European Union and its Member States. https://ec.europa.eu/clima/sites/clima/files/docs/2015030601_eu_indc_en.pdf, accessed 14.8.2017.

FAO, 2006. *Global Forest Resources Assessment 2005. Progress towards sustainable forest management. FAO Forestry paper 147*. Food and Agriculture Organization of the United Nations, Rome. 320 p.

Faustmann, M., 1849. Berechnung des Werthes, welchen Waldboden, sowie noch nicht haubare Holzbestände für die Waldwirthschaft besitzen. Allgemeine Forst- und Jagd-Zeitung 25, 441–455.

Gauthier, S., Bernier, P., Kuuluvainen, T., Shvidenko, A.Z., Schepaschenko, D.G., 2015. Boreal forest health and global change. Science 349 (6250), 819–822.

Getz, W.M., Haight, R.G., 1989. *Population Harvesting: Demographic Models for Fish, Forest and Animal Resources*. Princeton University Press, New Jersey. 408 p.

Goetz, R-U., Hritonenko, N., Mur, R.J., Xabadia, A., Yatsenko, Y., 2010. Forest Management and Carbon Sequestration in Size-Structured Forests: The Case of Pinus sylvestris in Spain. Forest Science 56, 242–256.

Gong, P., Löfgren, K.G., 2016. Could the Faustmann model have an interior minimum solution? Journal of Forest Economics 24, 123–129.

Guthrie, G., Kumareswaran, D., 2009. Carbon subsidies, taxes and optimal forest management. Environmental and Resource Economics 43(2), 275–293.

- 468 Gutrich, J., Howarth, R.B., 2007. Carbon sequestration and the optimal management of New
469 Hampshire timber stands. *Ecological Economics* 62, 441–450.
- 470 Haight, R.G., 1985. Comparison of Dynamic and Static Economic Models of Uneven-aged Stand
471 Management. *Forest Science* 31, 957–974.
- 472 Haight, R.G., Getz, W.M., 1987. Fixed and equilibrium endpoint problems in uneven-aged stand
473 management. *Forest Science* 33, 903–931.
- 474 Halbritter, A., Deegen, P., 2015. A combined economic analysis of optimal planting density, thinning
475 and rotation for an even-aged forest stand. *Forest Policy and Economics* 51, 38–46.
- 476 Hoel, M., Holtsmark, B., Holtsmark, K., 2014. Faustmann and the climate. *Journal of Forest
477 Economics* 20(2), 192–210.
- 478 Lubowski, R.N., Plantinga, A.J., Stavins, R.N., 2006. Land-use change and carbon sinks: econometric
479 estimation of the carbon sequestration supply function. *Journal of Environmental Economics
480 and Management* 51(2), 135–152.
- 481 Manley, B., Maclaren, P., 2010. Harvested Wood Products in the ETS. What would be the Impact?
482 *NZ Journal of Forestry* 55(3), 20–26.
- 483 Mason, C.F., Plantinga, A.J., 2013. The additionality problem with offsets: Optimal contracts for
484 carbon sequestration in forests. *Journal of Environmental Economics and Management* 66(1),
485 1–14.
- 486 Niinimäki, S., Tahvonen, O., Mäkelä, A., Linkosalo, T., 2013. On the economics of Norway spruce
487 stands and carbon storage. *Canadian Journal of Forest Research* 43(7), 637–648.
- 488 Olschewski, R., Benítez, P.C., 2010. Optimizing joint production of timber and carbon sequestration
489 of afforestation projects. *Journal of Forest Economics* 16(1), 1–10.

- 490 Parajuli, R., S.J. Chang, 2012. Carbon sequestration and uneven-aged management of loblolly pine
491 stands in the Southern USA: A joint optimization approach. *Forest Policy and Economics* 22,
492 65–71.
- 493 Payn, T., Carnus, J. M., Freer-Smith, P., Kimberley, M., Kollert, W., Liu, S., Orazio, C., Rodriguez,
494 L., Silva, L.N., Wingfield, M. J., 2015. Changes in planted forests and future global
495 implications. *Forest Ecology and Management* 352, 57–67.
- 496 Pihlainen, S., Tahvonen, O., Niinimäki, S., 2014. The economics of timber and bioenergy production
497 and carbon storage in Scots pine stands. *Canadian Journal of Forest Research* 44(9), 1091–
498 1102.
- 499 Pohjola, J., Valsta, L., 2007. Carbon credits and management of Scots pine and Norway spruce stands
500 in Finland. *Forest Policy and Economics* 9(7), 789–798.
- 501 Pukkala, T., Lähde, E., Laiho, O., Salo, K., Hotanen, J-P., 2011. A multifunctional comparison of
502 even-aged and uneven-aged forest management in a boreal region. *Canadian Journal of Forest*
503 *Research* 41(4), 851–862.
- 504 Rämö, J., Tahvonen, O., 2014. Economics of harvesting uneven-aged forest stands in Fennoscandia.
505 *Scandinavian Journal of Forest Research* 29(8), 777–792.
- 506 Samuelson, P., 1976. Economics of forestry in an evolving society. *Economic Inquiry* 14(4), 466–
507 492.
- 508 Seierstad, A., Sydsæter, K., 1987. *Optimal control theory with economic applications*. Elsevier,
509 Amsterdam. 462 p.
- 510 Stainback, G.A., Alavalapati, J.R.R., 2002. Economic analysis of slash pine forest carbon
511 sequestration in the southern US. *Journal of Forest Economics* 8, 105–117.

512 Sydsæter, K., Hammond, P., Seierstad, A., Strom, A. 2008. *Further Mathematics for Economic*
513 *Analysis*. Prentice Hall, Essex. 632 p.

514 Tahvonen, O., 2015. Economics of Naturally Regenerating, Heterogeneous Forests. *Journal of the*
515 *Association of Environmental and Resource Economists* 2(2), 309–337.

516 Tahvonen, O., 2016. Economics of rotation and thinning revisited: the optimality of clearcuts *versus*
517 *continuous cover forestry*. *Forest Policy and Economics* 62, 88–94.

518 Thompson, I., Mackey, B., McNulty, S., Mosseler, A., 2009. *Forest Resilience, Biodiversity, and*
519 *Climate Change. A synthesis of the Biodiversity / Resilience / Stability Relationship in Forest*
520 *Ecosystems. Technical Series no. 43*. Secretariat of the Convention on Biological Diversity,
521 Montreal. 67 p.

522 UNFCCC, United Nations Framework Convention on Climate Change, 2015. FCCC/CP/2015/L.9:
523 Adoption of the Paris Agreement. <https://unfccc.int/resource/docs/2015/cop21/eng/l09.pdf>,
524 accessed 18.4.2017.

525 van Kooten, G., Binkley, C., Delcourt, G., 1995. Effect of Carbon Taxes and Subsidies on Optimal
526 Forest Rotation Age and Supply of Carbon Services. *American Journal of Agricultural*
527 *Economics* 77(2), 365–374.

528

529

530

531

532

533

534

535

537

538 Figure 1. Stand development a) without natural regeneration and b) with natural regeneration.

539

540

541 Figure 2. The growth function and singular solutions, with a 3% interest rate and carbon prices €0,
 542 €25, and €150 tCO_2^{-1} . Note: $p = \text{€}40 \text{ m}^{-3}$, $\alpha = 0.7$, $w = \text{€}1000 \text{ ha}^{-1}$.

543

544

Figure 3. The dependence of optimal rotation on carbon price with different values for carbon release from harvested wood. Note: $\delta = 0.03$, $p = €40 \text{ m}^{-3}$, $w = €1000 \text{ ha}^{-1}$.

Figure 4. The dependence of optimal rotation on carbon price, given low carbon release from harvested wood and a 0.5% interest rate. Note: $p = €40 \text{ m}^{-3}$, $w = €1000 \text{ ha}^{-1}$.

552

553 Figure 5. The optimality of continuous cover forestry vs. clearcuts, with carbon prices €0–€50 tCO₂⁻¹.

554 Note: $p = €40 \text{ m}^{-3}$, $\alpha = 0.7$.

555