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Naturalizing Logico-Mathematical Knowledge: Approaches from
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Bangu Sorin. [Book review]

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2019

Pantsar , M 2019 , ' Naturalizing Logico-Mathematical Knowledge: Approaches from
Philosophy, Psychology and Cognitive Science. Edited by Bangu Sorin. [Book review] ' ,
Philosophical Quarterly , vol. 69 , no. 275 , pp. 432-435 . <https://doi.org/10.1093/pq/pqy051>

<http://hdl.handle.net/10138/325182>

<https://doi.org/10.1093/pq/pqy051>

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Naturalizing Logico-Mathematical Knowledge: Approaches from Philosophy, Psychology and Cognitive Science. By SORIN BANGU (ed.). (New York: Routledge, 2018. Pp. 305. Price: £115.00, eBook £20.00)

There is no questioning the importance of a book like *Naturalizing Logico-Mathematical Knowledge*. With volumes like *The Oxford Handbook of Numerical Cognition* coming out in recent years, it is clear that there is a wealth of empirical data available for philosophical considerations on the cognitive foundations of mathematical knowledge. There have also been important monographs, such as Stanislas Dehaene's *The Number Sense* and Susan Carey's *The Origin of Concepts*, that have acquired the status of standard reference works both for empirical scientists and philosophers. What has been lacking, however, is a distinctively philosophical volume that aims to connect the empirical developments to fundamental issues in the philosophy of mathematics. This is what *Naturalizing Logico-Mathematical Knowledge* sets out to do, and based on a conference held at the University of Bergen in 2015, Sorin Bangu has gathered together a fine group of authors to contribute.

As the subtitle of the volume suggests, the authors are not limited to philosophers. The psychologists Kristy vanMarle and Karen Wynn provide overviews of the empirical data on early numerical cognition, and the cognitive scientists Josephine Relaford-Doyle and Rafael Núñez provide interesting empirical data on the actual number concepts used by mathematically untrained adults. In addition, the psychologist Rolf Reber writes about the connection between beauty and truth in mathematics.

However, the overarching theme of the book is philosophical. It consists of Bangu's short but informative introduction and fourteen contributed chapters. Wynn's chapter, "Origins of Numerical Knowledge", was originally published already in 1995 and it is the only contribution not written especially for this volume. Bangu (p. 7) explains that Wynn's contribution is included because "the philosophical significance of this kind of work has not received enough philosophical attention". It is easy to agree with that and Wynn has undoubtedly had a crucial role as a pioneer in the study of infant numerical cognition. Nevertheless, the inclusion of a 23-year-old paper is somewhat baffling. A great deal of progress has been made recently in this area, as becomes clear from vanMarle's highly informative chapter. One wishes that Wynn's paper could at least have been updated for this volume.

The contributions are generally of high quality, but the range of topics leaves something to be desired. The most glaring omission is that of geometrical knowledge, which is only given the occasional passing reference. Given the rising amount of empirical work devoted to the cognitive basis of geometry, this is unfortunate. Indeed, with quite few exceptions, the mathematical knowledge discussed in the book refers to *arithmetical* knowledge. While this emphasis is understandable – after all, numerical cognition is the most developed area of empirical research when it comes to naturalizing mathematical knowledge – one would have hoped for more variety in the topics. In particular worth noting is that for a book called *Naturalizing Logico-Mathematical Knowledge*, there is conspicuously little about *logical* knowledge in it. Aside from part of the chapter by Penelope Maddy, logic is not given explicit treatment. In addition, Paul J. Robinson and Richard Samuels provide an excellent chapter debunking regress arguments against inferential rule-following, which is highly relevant for all areas of mathematics, including logic. However, the vast majority of the papers in the volume discuss numerical cognition and arithmetical knowledge, making the title of the book somewhat misleading.

Although there is variance in their novelty and relevance, most contributions succeed in tackling a pertinent philosophical problem. In some chapters, most notably Mark Fedyk's chapter on intuition and Fabio Sterpetti's defence of the "analytic" rather than axiomatic view of mathematical knowledge, the discussion of naturalization is conducted on a philosophical basis without making substantial connections to empirical data. Some of the empirical papers, most notably Reber's chapter on beauty and truth, do not engage sufficiently in the philosophical literature. Most chapters, however, follow the main theme of the book by making a philosophical contribution that in some way applies empirical results. Byeong-uk Yi's chapter provides a good example of the great usefulness of this kind of approach. Yi repeats his earlier argument for the plural property view of numbers, but this time with parts devoted to the empirical data on numerical estimation without counting. The result is an interesting new take on an old debate (whether numbers are properties and if so, what are they properties of). Similarly, Max Jones in his chapter discusses the possibility of updating perhaps the most famous pre-90's account of naturalizing mathematical knowledge – that of Philip Kitcher – and manages to shed new light by evoking modern empirical data concerning numerical cognition.

One topic that could have received more attention in the book is to what extent mathematical knowledge is culturally determined. Starting from such early origins as acquiring first numeral words, the social character of mathematical practice and learning is a central topic in the modern literature. The two

contributions that discuss it explicitly (Helen de Cruz's chapter on learning through testimony and the chapter by Karim Zahidi and Erik Myin stressing the sociocultural character of mathematical knowledge) are thus important additions to the collection, as is Dirk Schlimm's well-researched chapter on the importance of particular external representations of numbers to arithmetical thinking. Equally important is the chapter by Jean-Charles Pelland, in which he points out a seemingly simple, but important flaw in the standard way of explaining the ontogeny of number concepts by acquiring a numeral list: at least in some cases, people must have been able to acquire number concepts without being given ready-made numeral lists.

With several excellent chapters discussing the philosophical problems involving the connections between numbers, numerals and early numerical cognition, Bangu's book is successful in many ways. Despite not quite delivering what it says on the cover, this book provides something that has been badly missed in the philosophy of mathematics: a single volume that serves as a good introduction to many of the issues at the center of naturalizing mathematical knowledge. Due to its limited scope, however, equally many important issues are not included. Nevertheless, it is a collection of high-quality contributions which works both as an introduction to the subject and as a highly useful review of the kind of philosophical issues involved. In this function, one can only hope that it will reach a wide audience among researchers and students interested in the cognitive foundations of mathematics. Equally importantly, one hopes that it will also be read by philosophers of mathematics skeptical of the naturalization project.

Word count: 1064

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