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Structure formation with two periods of inflation: beyond PLIn Λ CDM

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Abstract. We discuss structure formation in models with a spectator field in small-field inflation which accommodate a secondary period of inflation. In such models, subgalactic scale primordial fluctuations can be much suppressed in comparison to the usual power-law Λ CDM model while the large scale fluctuations remain consistent with current observations. We discuss how a secondary inflationary epoch may give rise to observable features in the small scale power spectrum and hence be tested by the structures in the Local Universe.

Keywords: inflation, physics of the early universe, power spectrum

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1 Introduction

The standard “Lambda Cold Dark Matter” (hereafter *plain* Λ CDM, or Λ CDM with *POwer Law Inflation*¹) model of cosmology is founded on three key assumptions: firstly, that the Universe contains, in addition to baryonic matter, a collisionless, “cold” dark matter component that accounts for $\sim 5/6$ of the matter density; secondly, that the expansion of the Universe is described by the Friedmann equations with a cosmological constant term, Λ , and, thirdly, the implicit assumption that structure formation has its origin in the primordial perturbations seeded by inflation.

The plain Λ CDM model has successfully predicted structure formation over many epochs and orders of magnitude, from the structures observed in the Cosmic Microwave Background (CMB) at $z = 1100$ to the sizes and distributions of clusters and galaxies at much later time. However, tensions between the CMB and measurements on small scales have also been reported. These have either been attributed to poorly understood systematic effects in various data sets, or interpreted as an indication of new physics.

On small scales, predictions for structure formation concern the abundance and internal structure of low mass dark matter halos in the Local Universe. Here, observations of the Milky Way and Andromeda satellite populations in particular, appear to be in disagreement with N-body simulations based on the plain Λ CDM model.

This tension is twofold. The “missing satellites” problem [1, 2] refers to the apparent paucity of luminous satellite galaxies compared to the large number of dark matter substructures predicted in plain Λ CDM. While processes including supernova feedback (e.g. [3]), and cosmic reionisation (e.g. [4]) are expected to reduce the number of observed satellite galaxies, the apparent excess of substructures in the plain Λ CDM is not limited to the lowest mass scales: simulations also predict the presence of subhalos so massive that they should not be affected by reionisation (and hence deemed “too big to fail”, [5]), but whose internal structure seems incompatible with that of the brightest observed satellites.

Recent hydrodynamic simulations have shown that, when baryonic effects are included, the observed Local Group satellite populations can be reproduced in Λ CDM (e.g. [6]), but only if the Local Group mass is $\sim 2 - 3 \times 10^{12} M_{\odot}$, at the lower end of the observational

¹Here “power law” indicates that primordial power spectrum is of a “power-law” form. Do not confuse with power law inflation model in which the inflaton potential is given by an exponential function.

limits [7]. It has also been shown that, if the CDM assumption of the plain Λ CDM model is relaxed, simulations of warm dark matter [8, 9] and photon-coupled dark matter [10], which both reduce the small-scale power after decoupling, give a better agreement to satellite kinematics than the equivalent plain Λ CDM models. Conversely, observed structures in the Lyman- α forest [11, 12], and the abundance of Milky Way satellites [13] also provide upper limits for any reduction of small scale power relative to plain Λ CDM.

Warm dark matter models prevent the hierarchical formation of structures below the free-streaming scale (e.g. [15]), but above that, the abundance of structures relative to CDM differs significantly over only a narrow range. Consequently, within current limits, WDM effects are testable only at or below the scale of ultra-faint dwarf galaxies [8]. This gives WDM models limited predictive power. Furthermore, the motivation for the required WDM particle appears somewhat ad-hoc. Another, perhaps more attractive possibility, is to modify the power spectrum of primordial perturbations. The modification should be such that it persists over ~ 4 orders of magnitude; hence e.g. local features in the inflation potential are unlikely to do the job.

Instead, here we propose a unified description of structure formation that relies on two periods of inflation and accounts both for the power on CMB scales and for the apparent suppression of power on subgalactic scales.

There exist models for two periods of inflation making use e.g. of a suitably arranged inflaton potential with an intervening phase transition [16, 17].²

In this paper we consider a model where the inflaton field, giving rise to a slow roll inflation, is complemented by another scalar field, which is dynamically irrelevant during the first period of inflation. Such a scalar is called a spectator field. We know that spectator fields exist: the Higgs field is one example (see, e.g., [19–21] for fluctuations of the mean Higgs field during inflation), barring the possibility that the Higgs field itself is the inflaton. Another, much studied spectator field is the curvaton [22–24] which imprints the perturbations it receives during inflation on radiation and matter after inflation.³ Rather than the inflaton, the curvaton could then be the origin of the whole of the observed spectrum of perturbations.

Here we do not assume that the spectator field contributes to the primordial perturbation in a significant manner. Instead, we assume that it gives rise to second period of inflation, which happens under the conditions to be discussed below. Such a second period of inflation affects the power spectrum both on the CMB scales as well as on subgalactic scales. In this paper we shall discuss a model with two periods of inflation that yields the desired suppression at small scales while remaining in agreement with the observed properties of the CMB power spectrum, making it possible to address the apparent shortfalls of the plain Λ CDM model at small scales in completely different, and astrophysically decoupled regimes. Such agreement, however, makes certain demands on the inflaton model; not all inflationary potentials are consistent with two inflationary periods. We will also demonstrate that for our working example, the fact that some subgalactic structure has actually been observed gives rise to a lower limit on the number of e -folds of the first inflationary period.

This paper is organized as follows. In section 2, we briefly introduce the general concept of a spectator field, and in sections 2.1 and 2.2 we describe the background evolution and

²It has been argued that the CMB spectral μ distortion would be useful to differentiate models with a small-scale suppression of the matter power spectrum due to late-time effects (such as different dark matter properties) from those caused by a modification of the primordial power spectrum [18].

³Another example would be the modulated reheating model [25, 26].

perturbation, respectively. In section 3, we introduce a concrete inflation model. In section 4 we discuss the predictions for small scale power. We conclude with a summary in section 5.

2 Spectator fields

Fields whose energy densities during inflation are irrelevant for the expansion rate of the Universe are called spectators. If they are light, both their mean field values and the field perturbations will be subject to inflaton-driven inflationary expansion. At the onset of inflation the spectator energy density is subdominant. The spectator field σ is assumed to begin to oscillate during the radiation-dominated period after the inflaton field ϕ has decayed (or during the period dominated by the oscillations of the inflaton). Its energy density depends on the initial spectator field value σ_* , as well as on the exact form of the spectator potential. During inflation, the mean spectator field is subject to fluctuations and in a given inflationary patch is one realization of the probability distribution, which is determined by the Fokker-Planck equation⁴ as was first pointed out by Starobinsky [27] (for a discussion in the context of spectators, see e.g. [28, 29]).

At the end of inflation, the spectator square-mean-field has a value σ_* , which serves as the initial spectator field value for its subsequent evolution. As a consequence of its stochastic evolution during inflation, it may have a value greater than the Planck mass M_{pl} . If inflation lasts long enough, the probability distribution for the initial curvaton field equilibrates; otherwise there will be a dependence on the curvaton field value before the onset of inflation (however, equilibration is not automatic; for a recent discussion on the fluctuations of spectator fields, see [29]). In any case, if $\sigma_* > M_{\text{pl}}$, the spectator may end up dominating the Universe even before it starts to oscillate. This happens, provided that the spectator is still slowly rolling down its potential, whence a period of secondary inflation can ensue [30–33]. Thus, very crudely, first there is a period of inflation driven by the “usual” inflaton field; then the inflaton decays; after a while, the energy density of the slowly rolling spectator becomes dominant and generates a second period of inflation, which ends when the spectator decays. This scenario, which we investigate in the present paper, has some very interesting consequences, in particular for the spectrum of perturbations at small scales. As we will discuss, these consequences will also depend on the details of the slow roll inflation model.

Depending on the duration of the secondary inflation driven by the spectator, the current observable scales (such as CMB) may have exited the horizon either during the primary slow roll inflationary phase or during the secondary, spectator-driven phase. In the standard case with no inflating spectator, the required number of e -folds is usually taken to be $N \sim 50$ – 60 e -folds. However, depending on the duration of the secondary inflationary phase, we may tolerate primary e -folds as low as $N \sim 10$. In such a case, the predictions for the primordial power spectrum can be drastically modified.

The spectator modifies the spectrum of perturbations already at the CMB scales. Because the first phase of inflation ends early, the modes corresponding to the CMB scales which exited the horizon closer to the end of the inflaton period, at which the inflaton starts to move faster, even if it is still slowly rolling. As a result, there will be a large running of the spectral index, which may also significantly modify the predictions for astrophysical scales.

⁴Assuming slow-roll inflation, as will be done here.

2.1 Background evolution

Before describing primordial density fluctuations, let us first look at the background evolution when a primary inflationary period is followed by a secondary period driven by a spectator.

If the spectator is to drive a secondary phase of inflation, it has to dominate the Universe before it begins to oscillate. This condition is given by

$$U(\sigma_*) \geq \rho_r(t_{\text{osc}}), \quad (2.1)$$

where σ_* is the spectator field value set during the first inflationary phase, $\rho_r(t_{\text{osc}})$ is the radiation energy density at the beginning of the spectator oscillation $t = t_{\text{osc}}$ and $U(\sigma)$ is the potential of σ . The spectator oscillation starts when $H \sim m_\sigma$, where m_σ is the mass of the spectator. Throughout this paper we will assume that the spectator potential reads simply

$$U(\sigma) = \frac{1}{2}m_\sigma^2\sigma^2, \quad (2.2)$$

whence the above condition can be rewritten as

$$\frac{1}{2}m_\sigma^2\sigma_*^2 \geq 3M_{\text{pl}}^2m_\sigma^2, \quad (2.3)$$

where M_{pl} is the reduced Planck mass. From (2.3) we obtain the condition for the spectator-driven secondary inflation as

$$\sigma_* \gtrsim \sqrt{6}M_{\text{pl}}. \quad (2.4)$$

The mass and the initial field value of the spectator are generally assumed to be model parameters. If one has a long period of initial inflaton-driven inflation so that the curvaton reaches the Fokker-Planck equilibrium distribution, a typical value of the amplitude of the spectator field is given by [28, 29]

$$\sigma_* \simeq \frac{H_*^2}{m_\sigma}, \quad (2.5)$$

with H_* being the Hubble rate during inflation, in which de-Sitter (a constant H_*) background is assumed. Although whether this distribution can be reached or not depends on inflation models, when an inflation model with a potential of the plateau type is adopted, the spectator field can obtain the de-Sitter equilibrium [29]. On the other hand, for the case of large-field inflation models, the equilibrium solution would not be reached although the spectator can still acquire a super-Planckian amplitude $\sigma_* > M_{\text{pl}}$ in the regime of an eternal inflation [29].

Once the equilibrium value is reached, by using the fact that the inflationary Hubble scale is directly related to the slow-roll parameter ϵ as $H_*^2/M_{\text{pl}}^2 \simeq 1.6 \times 10^{-7}\epsilon$,⁵ the above expression can be recast as

$$\sigma_* \sim 8M_{\text{pl}} \left(\frac{10^6 \text{ GeV}}{m_\sigma} \right) \left(\frac{\epsilon}{2 \times 10^{-5}} \right), \quad (2.6)$$

from which one can see that the amplitude of a spectator field can be as large as $\sigma_* > \mathcal{O}(M_{\text{pl}})$ for some set of parameters. As noted before, when $\sigma_* > M_{\text{pl}}$, the spectator can give rise to the secondary inflation. From eq. (2.6) we can see that such a scenario may in fact be generic, given the stochastic behaviour of σ during the primary inflation and the relation (2.5). Even if the equilibrium value is not reached, the initial value of the spectator can be regarded as

⁵Here we assume that primordial perturbations are generated only from an inflaton.

Figure 1. Schematic picture of evolutions of energy densities of the inflaton field ρ_ϕ , spectator field ρ_σ and radiation ρ_{rad} in models where the secondary inflation is driven by a spectator. The number of e -folds N is normalized to 0 at the end of the first inflation in this figure. Depending on the decay rate of the inflaton, the Universe may have been dominated by oscillating ϕ from the end of the first inflation to the beginning of the second inflation. In this case, there is no intermediate radiation-dominated phase.

a model parameter, which does not prohibit σ_* of having a value as large as $\sigma_* > \mathcal{O}(M_{\text{pl}})$. Therefore a scenario with a second period of inflation driven by a spectator could be realized in a broad class of model.

In figure 1, we show a typical thermal history in the scenario with two periods of inflation, the second driven by a spectator. We display the evolution of energy densities of the inflaton, the spectator and radiation, denoted as ρ_ϕ , ρ_σ and ρ_{rad} , respectively. In the following, we focus on the case where the spectator gives a second inflationary period after the CMB scales have already exited the horizon close to the end of the first period of inflation so that the small scale fluctuations are suppressed.

2.2 Density perturbation

In a scenario where a spectator field is present, both the inflaton and the spectator may contribute to the primordial fluctuations (in which case the spectator is a curvaton) for the mode which exited the horizon during the first inflationary epoch driven by the inflaton. For the modes which exited the horizon during the second inflationary epoch driven by a spectator, primordial fluctuations are sourced from the spectator alone. Although we consider a case where fluctuations from a spectator field can be neglected in the next section, here we also discuss the curvature perturbation generated by a spectator in order to clarify in what case spectator fluctuations can be neglected. Here we consider the curvaton type model.

As mentioned above, for modes which exited the horizon during the first inflationary period generated by the inflaton field, the curvature perturbation ζ is in general given by the

sum of two contributions:⁶

$$\zeta_I = \zeta_I^{(\phi)} + \zeta_I^{(\sigma)}, \quad (2.7)$$

where $\zeta_I^{(\phi)}$ and $\zeta_I^{(\sigma)}$ are respectively the curvature perturbations generated from the inflaton and the spectator (which in this case should be called the curvaton). A subscript I indicates that the perturbations correspond to modes which exited the horizon during the first inflation. The inflaton part can be written as

$$\zeta_I^{(\phi)} = \frac{1}{M_{\text{pl}}^2} \frac{V(\phi)}{V_\phi(\phi)} \delta\phi_* = \frac{1}{\sqrt{2\epsilon} M_{\text{pl}}^2} \delta\phi_*, \quad (2.8)$$

in which $V(\phi)$ and $V_\phi(\phi)$ are the potential for the inflaton and its derivative with respect to the inflaton field ϕ and the fluctuation of the inflaton is given by $\delta\phi_* = H_*/(2\pi)$. Therefore the primordial power spectrum sourced by inflaton fluctuations is given by

$$\mathcal{P}_\zeta^{(\phi)}(k) = \frac{1}{12\pi^2 M_{\text{pl}}^6} \frac{V(\phi)^3}{V_\phi(\phi)^2}. \quad (2.9)$$

For the curvaton part, here we give the expression for the case where the curvaton drives the secondary inflation since we mainly consider this kind of scenario in this paper. By adopting the δN formalism [42–44], the curvature perturbation generated from the spectator is given by $\zeta^{(\sigma)} = (\partial N / \partial \sigma_*) \delta\sigma_*$ where N is the number of e -folds from the epoch when the mode exited the horizon to that at the decay of the curvaton. However, in the case where the second inflation is driven by a spectator, the σ_* -dependence of N comes from the period during the second inflation. The number of e -folds during such a secondary inflation is given by [32]

$$N_2 = -\frac{1}{M_{\text{pl}}^2} \int_{\sigma_*}^{\sigma_{\text{end}}} \frac{U(\sigma)}{U_\sigma(\sigma)} d\sigma, \quad (2.10)$$

where $U(\sigma)$ and $U_\sigma(\sigma)$ are the potential of the curvaton and the derivative with respect to σ , respectively. Here σ_{end} is the value of σ at the end of the second inflation. Here we adopt the potential (2.2) and hence N_2 is given by

$$N_2 = \frac{1}{4M_{\text{pl}}^2} (\sigma_*^2 - \sigma_{\text{end}}^2), \quad (2.11)$$

from which we can obtain [32]

$$\zeta_I^{(\sigma)} = \frac{\sigma_*}{2M_{\text{pl}}^2} \delta\sigma_*, \quad (2.12)$$

with $\delta\sigma_* = H_*/(2\pi)$. The condition where fluctuations from the curvaton are negligible can be written as

$$\frac{\zeta_I^{(\sigma)}}{\zeta_I^{(\phi)}} = \sqrt{2\epsilon} \frac{\sigma_*}{M_{\text{pl}}} \ll 1, \quad (2.13)$$

from which one can see that, even when $\sigma_* > M_{\text{pl}}$, fluctuations from a spectator can be negligible if small-field inflation models with very small ϵ are assumed.

⁶For a general discussion on a scenario where the inflaton and the spectator can both contribute to the curvature perturbation, we refer the readers to [30–32, 34–40] for the curvaton model and [41] for modulated reheating scenario.

On small scales where modes exited the horizon during the second inflation driven by a spectator, the curvature perturbation is given by the same expression as for the ones generated from the inflaton during the first inflationary period. Hence one can write

$$\zeta_{II}^{(\sigma)} = \frac{1}{M_{\text{pl}}^2} \frac{U}{U_\sigma} \delta\sigma_{*II} \simeq 3 \frac{m_\sigma}{M_{\text{pl}}} \left(\frac{\sigma_*}{10M_{\text{pl}}} \right)^2, \quad (2.14)$$

with $\delta\sigma_{*II} = H_{*II}/2\pi$ being determined by the Hubble parameter during the second inflation H_{*II} . The mass for the spectator should be much smaller than $H_*(< M_{\text{pl}})$ since otherwise the spectator field plays the role of the inflaton, and hence $\zeta_{II}^{(\sigma)} \ll 10^{-5}$. Therefore primordial fluctuations on small scales would be much smaller than those generated on large scales from the inflaton fluctuations. This gives an effective cutoff in the power spectrum at some scale, which may have interesting implications for small scale structure while CMB scale is not affected. In the next section, we assume the above kind of scenario in an inflaton model with very small ϵ .

3 A concrete model and power spectrum

To compute the power spectrum in this kind of scenario explicitly, we also need to specify the inflaton model. Although many inflationary potentials would be admissible to have a model with a consistent n_s and r with current observations, here let us consider the following inflaton model, which is a hybrid inflation with a fractional power whose potential is given by

$$V(\phi) = V_0 \left(1 + \left(\frac{\phi}{\mu} \right)^p \right), \quad (3.1)$$

where V_0 represents the scale of inflation, which is determined by the normalization condition. Here μ and p are model parameters which will be chosen to give the spectral index n_s consistent with current observations. This type of inflation model is also called “valley hybrid inflation (VHI)” in [45].

The slow-roll parameters in this model are given by

$$\epsilon = \frac{p^2}{2} \left(\frac{M_{\text{pl}}}{\mu} \right)^2 \frac{x^{2p-2}}{(1+x^p)^2}, \quad \eta = p(p-1) \left(\frac{M_{\text{pl}}}{\mu} \right)^2 \frac{x^{p-2}}{1+x^p}, \quad (3.2)$$

where we have defined

$$x \equiv \frac{\phi}{\mu}. \quad (3.3)$$

The number of e -folds counted from the end of the first inflationary period is given by

$$N = \frac{1}{p} \left(\frac{\mu}{M_{\text{pl}}} \right)^2 \left[\frac{1}{2-p} (x_*^{2-p} - x_{\text{end}}^{2-p}) + \frac{1}{2} (x_*^2 - x_{\text{end}}^2) \right]. \quad (3.4)$$

Now we consider the case of $p < 1$, which corresponds to $n_s < 1$, however when we assume $N = 50\text{--}60$, n_s is very close to unity. As described below, if the secondary inflation is driven by a spectator field, N can be much reduced, which can lead to a value consistent with current observations ($n_s \sim 0.96$). To consider the situation where the spectator does not contribute to primordial fluctuations but only affects the background dynamics, we require a very small value for ϵ as shown in the previous section. To realize this, here we assume a

Figure 2. Primordial power spectrum (left) and linear matter power spectrum (right) for a spectator model with VHI model whose potential is given by eq. (3.1). Here we assume $p = 0.84$. For the primordial power spectrum, there are two characteristic damping scales: k_2 and k_{end} , which correspond to the modes that crossed the horizon at the time of the start of the second inflation and the end of the first inflation, respectively. In the matter power spectrum, the additional effect of oscillatory damping of the transfer function is visible.

large value for μ i.e. $\mu \gg M_{\text{pl}}$ (for concreteness, we assume $\mu = 10^3 M_{\text{pl}}$ in the following). We also need a very small $x = \phi/\mu \ll 1$. In this case, the approximate number of e -folds is

$$N = \frac{1}{p(2-p)} \left(\frac{\mu}{M_{\text{pl}}} \right)^2 x_*^{2-p}. \quad (3.5)$$

When $x_* \ll 1$ and $p < 1$, we have $\epsilon \ll |\eta|$,⁷ which, with the help of eq. (3.2) gives the spectral index n_s as

$$n_s \simeq 1 + 2\eta \simeq 1 - \frac{2(1-p)}{(2-p)N}. \quad (3.6)$$

If we take $p = 0.8$, the spectral index is $n_s \simeq 0.993$ for $N = 50$, which is outside the region allowed by Planck. However, when the curvaton generates a secondary inflation to make N much smaller, say $N = 10$, the spectral index becomes $n_s \simeq 0.966$, which gives a good fit to the current Planck data.

In figure 2, we show the primordial power spectrum (left) and matter power spectrum (right) for the case of $p = 0.84$ in the VHI inflation model, in which the CMB scale corresponds to the modes exited the horizon at $N \sim 8$ during the first inflationary period. For comparison, the power spectrum is also shown for the case where the power-law form $P_\zeta \propto k^{n_s-1}$ is assumed. As can be understood from eq. (2.8) in the previous section, the curvature perturbations are generally suppressed on small scales where the modes exit the horizon close to the end of inflation, since V_ϕ is getting larger. As discussed in the previous section, in our scenario, small scale fluctuations of the modes which exited the horizon during the second inflationary period are much smaller than those on larger scales generated during the first inflation. Therefore we can see the cutoff of the primordial power spectrum at around $k_{\text{end}} \sim 10^2 \text{ Mpc}^{-1}$ which corresponds to the mode which exited the horizon at the end of the first inflationary period. This can be clearly seen in the left panel of figure 2, which gives interesting implications to the tension between CMB and subgalactic scales. We

⁷With the value of the model parameters assumed here, we have very small ϵ as $\epsilon = 10^{-6}-10^{-5}$.

also note that the power spectrum deviates from the power-law form and is more suppressed around the suppression scale, which is due to the fact that the slow-roll gradually violates towards the end of inflation. This feature would be particular to models in which the end of inflation is realized by the violation of slow-roll. The cutoff would be more sharp in the case where the end of inflation is triggered by some other mechanism such as the one with a “water-fall” field stopping the inflation like in hybrid double inflation model discussed in [17].

To discuss the implication of our scenario for the small scale structure, we calculate the matter power spectrum in the present Universe. In the right panel of figure 2, we plot the matter power spectrum at $z = 0$ for the same model. To calculate the matter power spectrum, we need to incorporate the effects of the evolution of fluctuations after the modes crossed the horizon, which is quantified by the transfer function. In particular, there are two periods of inflation in our model, which is quite similar to the case of thermal inflation where a mini-inflation occurs after the first inflation driven by the inflaton. To obtain the transfer function in our scenario, we follow the formalism adopted in [46] where the transfer function in the thermal inflation case is discussed, which we describe below using the notation of [46].

We have numerically solved a set of equations to obtain the evolutions of the (gauge-invariant) curvature perturbations

$$R_{\delta\rho_i} \equiv R - \frac{H}{\dot{\rho}_i} \delta\rho_i, \quad (3.7)$$

for $i = \text{“m”}$ and “rad” , which represent matter and radiation, respectively. Here “matter” corresponds to an oscillating inflaton field. As mentioned above, we consider the case where fluctuations of the spectator field are negligible, and hence we do not consider those for the spectator. The total curvature perturbation is given by

$$R_{\delta\rho} = \frac{1}{\dot{\rho}_{\text{tot}}} \sum_{i=\text{m,rad}} \dot{\rho}_i R_{\delta\rho_i}, \quad (3.8)$$

where ρ_{tot} is energy density for the total component. The evolutions for $R_{\delta\rho_m}$ and $R_{\delta\rho_{\text{rad}}}$ are respectively given by

$$\begin{aligned} & \frac{d^2 R_{\delta\rho_m}}{dN^2} + \frac{3}{2} \frac{\rho_{\text{tot}} + p_{\text{tot}}}{\rho_{\text{tot}}} \frac{dR_{\delta\rho_m}}{dN} + \left(2 + \frac{\rho_m}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \right) \frac{dR_{\delta\rho_m}}{dN} \\ & - \frac{1}{3} \frac{q^2}{H^2} \left(\frac{\rho_m}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \right) R_{\delta\rho_m} = - \frac{\frac{4}{3}\rho_{\text{rad}}}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \left(\frac{dR_{\delta\rho_{\text{rad}}}}{dN} - \frac{1}{3} \frac{q^2}{H^2} R_{\delta\rho_{\text{rad}}} \right), \\ & \frac{d^2 R_{\delta\rho_{\text{rad}}}}{dN^2} + \frac{3}{2} \frac{\rho_{\text{tot}} + p_{\text{tot}}}{\rho_{\text{tot}}} \frac{dR_{\delta\rho_{\text{rad}}}}{dN} + \left(1 + \frac{\frac{8}{3}\rho_{\text{rad}}}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \right) \frac{dR_{\delta\rho_{\text{rad}}}}{dN} \\ & + \frac{1}{3} \frac{q^2}{H^2} \left(1 - \frac{\frac{8}{3}\rho_{\text{rad}}}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \right) R_{\delta\rho_{\text{rad}}} = - \frac{2\rho_m}{\rho_{\text{tot}} + p_{\text{tot}} + \frac{2}{3}q^2} \left(\frac{dR_{\delta\rho_m}}{dN} - \frac{1}{3} \frac{q^2}{H^2} R_{\delta\rho_m} \right) + \Delta_{\text{ET}}, \end{aligned} \quad (3.9)$$

where $q = k/a$ with k being the wave number of the mode and a being the scale factor. N is the number of e -folds and p_{tot} is pressure for the total component. We have also taken account of the energy transfer terms, which is represented by Δ_{ET} in the above equation, since the spectator field is assumed to decay into radiation in our scenario. The energy transfer terms

Figure 3. Thermal history (left) and the energy transfer factor Q_{ET} for the case we consider in this paper as a representative example. In both panels, the horizontal axis is the number of e -folds normalized to 0 at some initial time after the first inflation ends.

are derived from the energy-momentum tensor ($T_i^{\mu\nu}$) conservation equation for radiation: $T_{\text{rad};\mu}^{\mu 0} = \Gamma \rho_\sigma A$ and $T_{\text{rad};\mu}^{\mu i} = \Gamma \rho_\sigma (v^i + B^i)$ with Γ the decay rate of the spectator, v and B respectively the total velocity and the metric perturbation for g_{0i} component, from which we can explicitly write down Δ_{ET} as

$$\Delta_{\text{ET}} = \frac{1}{H} \left(\frac{dY_{\text{rad}}}{dN} + 2Y_{\text{rad}} + \frac{3}{4\rho_{\text{rad}}} \frac{d\rho_{\text{rad}}}{dN} Y_{\text{rad}} \right) - \frac{1}{3} \frac{q^2}{H^2} F_{\text{rad}}, \quad (3.10)$$

where $Y_{\text{rad}} = \rho_\sigma \Gamma A / (4\rho_{\text{rad}})$ and $F_{\text{rad}} = 3\rho_\sigma \Gamma (v + B) / (4\rho_{\text{rad}})$.

Actually, the energy transfer terms can be neglected for the case we consider here, which we are going to show. To evaluate the size of the energy transfer terms, we adopt the Newtonian gauge and rewrite eq. (3.10) by using Einstein equations and the energy-momentum conservations with energy transfer terms, which leads to the expression below:

$$\Delta_{\text{ET}} = \frac{\rho_\sigma}{4\rho_{\text{rad}}} \frac{\Gamma}{H} \left(-\frac{dR}{dN} + \frac{\rho_{\text{tot}}}{\rho_\sigma} R - 3R \right) + \frac{\rho_\sigma}{4\rho_{\text{rad}}} \frac{\Gamma}{H} \frac{3dR_{\delta\rho_m}}{dN}. \quad (3.11)$$

From this equation, one can see that the factor $Q_{\text{ET}} \equiv \frac{\rho_\sigma}{4\rho_{\text{rad}}} \frac{\Gamma}{H} R_{\delta\rho}$ can roughly characterize the size of the energy transfer terms. In figure 3, we show the evolution of Q_{ET} as a function of the number of e -folds (right panel) and the thermal history for the case we consider in the following as a representative example (left panel) where the evolutions of energy densities of matter (oscillating inflaton), radiation and the spectator, denoted respectively as ρ_m , ρ_{rad} and ρ_σ , are depicted. In the figure, the number of e -folds is normalized to 0 at some epoch after the first inflation ends. We take the initial conditions for the curvature perturbations as $\mathcal{R}_{\delta\rho_m} = \mathcal{R}_{\delta\rho_{\text{rad}}} = 1$ since the amplitudes are irrelevant to obtain the transfer function. As can be seen from the right panel of the figure, the value of Q_{ET} is very small during the whole evolution, which indicates that we can safely neglect the energy transfer terms in eq. (3.9).

By solving eq. (3.9) without the energy transfer terms along with the background equations of motion for ρ_m , ρ_{rad} and ρ_σ , we can follow the evolutions of the perturbations. The transfer function can be obtained by

$$T(k) \equiv \frac{R_{\delta\rho}(k, t_f)}{R_{\delta\rho}(k, t_i)} \quad (3.12)$$

Figure 4. Transfer function for the case we consider in this paper as a representative example, whose thermal history is shown in the left panel of figure 3.

where t_i and t_f are the initial and final times. The initial time is taken to be some time after the first inflation ends, and the final time is set to be some time after the spectator field decays and the Universe becomes radiation dominated.

In figure 4, the transfer function for the case with thermal history depicted in the left panel of figure 3 is shown as a function of $\kappa = k/k_2$ where k_2 is the wave number corresponding to the mode which “touched” the horizon at the start of the second inflation. The above transfer function exhibits oscillatory damping at scales smaller than k_2 which can be related to k_{end} , the mode which exited the horizon at the end of the first inflation once we fix the background evolution. Since $k = aH$ holds at the time of the horizon crossing, one has

$$\frac{k_2}{k_{\text{end}}} = \frac{a_2}{a_{\text{end}}} \frac{H_2}{H_{\text{end}}}, \quad (3.13)$$

where a subscript “2” denotes that the quantity is evaluated at the beginning of the second inflation. In figure 5, a schematic figure of the horizon crossings for two characteristic scales k_2 and k_{end} is shown. Assuming that the Universe is ϕ oscillation-dominated,⁸ in which $H \propto a^{-3/2}$, we obtain

$$k_2 = k_{\text{end}} \left(\frac{H_2}{H_{\text{end}}} \right)^{1/3}. \quad (3.14)$$

Assuming that the Hubble parameter during inflation does not change much, we can approximate the Hubble parameter at the end of the first inflation as $H_{\text{end}}^2 \simeq H_*^2 \simeq 1.6 \times 10^{-7} \epsilon M_{\text{pl}}^2$ where the 2nd equality comes from the argument just above eq. (2.6). The Hubble rate at the beginning of the second inflation can be written as $H_2^2 \simeq m_\sigma^2 \sigma_*^2 / (3M_{\text{pl}}^2)$, from which one has

$$k_2 \sim 0.01 \, k_{\text{end}} \left(\frac{m_\sigma}{10^6 \text{ GeV}} \right)^{1/3} \left(\frac{\sigma_*}{10 M_{\text{pl}}} \right)^{1/3} \left(\frac{10^{-5}}{\epsilon} \right)^{1/6}. \quad (3.15)$$

⁸In the thermal history we consider in the following, a short period of radiation-dominated Universe appears as shown in the left panel of figure 3, however, as far as such RD period is short, the following estimate should give a good approximation.

Figure 5. Schematic figure of corresponding scales during the two periods of inflation, the final radiation dominated era, and the (possible) intermediate radiation (or matter-like ϕ oscillation) dominated phase.

For the case of $p = 0.84$ depicted in figure 2, the damping scale corresponding to the end of the first inflation is given by $k_{\text{end}} \sim 10^2 \text{ Mpc}^{-1}$, and hence the damping scale in the transfer function is estimated as $k_2 \sim \mathcal{O}(1) \text{ Mpc}^{-1}$.

In the large and small scale limits ($k \ll k_2$ and $k \gg k_2$, respectively), the transfer function can be approximated as

$$T(\kappa) \rightarrow \begin{cases} 1 & (k \ll k_2), \\ 0.16 \cos(2.72(\kappa - 3.5)) & (k \gg k_2), \end{cases} \quad (3.16)$$

where the case of $k \gg k_2$ is obtained by fitting a cosine function to $T(\kappa)$ calculated numerically.

Cosmological N-body simulations with the thermal inflation case with the transfer function given in [46] have been studied by [47]. The transfer function for the thermal inflation is similar to ours, however, it should be noted that, in our model, the primordial power spectrum also gives the suppression on small scales due to the mechanism discussed in the previous section, and hence the power spectrum after the second inflation is given by

$$\mathcal{P}(k)|_{\text{after 2nd inf.}} = T^2(k) \mathcal{P}_\zeta(k), \quad (3.17)$$

where $T(k)$ is given in figure 4 and $\mathcal{P}_\zeta(k)$ can be calculated by eq. (2.9) when only the inflaton contributes to the primordial curvature perturbation.

We input $\mathcal{P}(k)|_{\text{after 2nd inf.}}$ to the public code **CAMB** [48] to obtain the matter power spectrum at late time, which is shown in the right panel of figure 2. As discussed above, there are two damping scales in the model, k_2 and k_{end} , corresponding to the mode which exited the horizon at the end of the first inflation, and the one which “touched” the horizon at the start of the second inflation. This may give interesting consequences for structure formation on subgalactic scales.

4 The power spectrum at small scales

Alternatives to, or extensions of, the plain Λ CDM model are most easily distinguished at small scales. Whereas plain Λ CDM predicts bottom-up hierarchical structure formation after

decoupling on all observable scales, alternatives deviate in several characteristic ways. For example, the free-streaming motions of relativistic “warm” dark matter particles, such as sterile neutrinos, dampen and erase initial perturbations, greatly reducing the number of structures below a certain “free streaming mass” [15]. They also require that lower-mass halos and the galaxies within them only form “top down” from the fragmentation of more massive objects. Models where dark matter is coupled to photons in the early Universe erase structures similar to warm dark matter, but with some resonances at particular scales (e.g. [10, 49]). Models where dark matter is self-interacting can change the inner structure of dark matter halos, as well as lower the abundance of satellites [50, 51].

The most sensitive observations of small scale structures are the abundance of present-day Local Group dwarf galaxies in halos of $\sim 10^8$ – $10^{9.5}M_\odot$, and structures observed in the Lyman- α forest at redshifts $z = 3$ – 4 . Based on Lyman- α forest measurements, warm dark matter models with thermal relics of less than 3.5 keV can be excluded at 99% confidence level [11], but in the presence of Lepton asymmetry, the correspondence between particle mass and structure damping differs [52]. However, the effects of WDM on structure formation are similar in all models, and it is convenient to parameterise them by the equivalent thermal relic mass.

For Local Group dwarf galaxies, several caveats exist: while a simple comparison between the number of observed satellite galaxies, and the number of halos appears to strongly disfavour plain Λ CDM, astrophysical processes are understood to prevent the formation of dwarf galaxies in low-mass halos, and also reduced the abundance of low mass halos compared to collisionless simulations (e.g. [53]). Furthermore, the uncertain mass of the Milky Way and of the Local Group has a significant effect on the expected number of satellite halos. Indeed, for a massive Milky Way, plain Λ CDM is strongly disfavoured, while for a low mass Milky Way, many WDM models can be excluded [13]. In simulations with Local Group analogues in the allowed mass range, it appears that moderate WDM models are slightly favoured over CDM, but observations of dwarf galaxies are not discriminant enough to distinguish them [8, 54]. A clearer distinction may be possible based on the discovery or non-detection of even lower mass dark matter halos, e.g. through their perturbations of stellar streams, Milky Way halo stars, or via gravitational lensing. While CDM predicts thousands of subhalos in the 10^6 – $10^8M_\odot$ range surrounding the MW, most WDM models predict very few [55, 56].

A modification of the initial power spectrum could act similar to warm dark matter at the dwarf galaxy scale in the present Universe. In figure 6, the ratio of the linear matter power spectra at $z = 0$ between the spectator VHI model and the plain Λ CDM model, $P(k)/P(k)_{\text{CDM}}$ is shown. In both models, the spectral index at the reference scale of $k = 0.05 \text{ Mpc}^{-1}$ is taken as $n_s = 0.9645$ and other cosmological parameters are assumed as $\Omega_b h^2 = 0.02225$, $\Omega_c h^2 = 0.1198$, $h = 0.6727$ and $A_s = 2.2 \times 10^{-9}$ [57], where Ω_b and Ω_c are density parameters for baryon and CDM, h is the Hubble parameter in units of 100 km/sec/Mpc and A_s is the amplitude of primordial power spectrum at the reference scale. For the spectator VHI model, we have chosen the model parameters, μ and V_0 , for a fixed value of p in such a way that we obtain the above values of n_s and A_s at the reference scale. By fixing the model parameters in this way, the number of e -folds corresponding to the mode $k = 0.05 \text{ Mpc}^{-1}$ are $N = 10.2, 9.4, 7.6$ and 7.8 for $p = 0.78, 0.8, 0.82$ and 0.84 , respectively. For guidance, as dashed lines, we also include several WDM models, assuming thermal relics of masses in the range 1–10 keV. It can be seen that, compared to WDM models, the power spectrum in the spectator model begins to deviate at significantly larger scales.

Figure 6. Ratio of the linear matter power spectra at $z = 0$ between either spectator VHI models (solid lines) or warm dark matter models (dashed lines) relative to the plain Λ CDM model. The top x -axis shows the equivalent mass, M of a sharp k -space filter, indicative of the halo mass corresponding to a given length scale k .

Depending on the model parameters, the reduction of the primordial power, and the modification of the transfer function, discussed in section 3, can result in a reduction of the linear matter power spectrum on galactic and supergalactic scales. Below the cut-off scale k_{end} , there is a sharp suppression, more rapid than in the WDM case. Defining the “filtering mass” as the scale where the abundance of halos is suppressed by half relative to the standard model [15], we see that the spectator-VHI model with $p \sim 0.82$ has a similar filtering scale to a WDM model with $m_{\text{WDM}} \sim 2 \text{ keV}$, near the lower limits allowed by Lyman- α observations [11].

The scale-dependence of the WDM or VHI models relative to the standard model, however, is quite different: we see that the WDM models change the power by less than 1% only half an order of magnitude above the filtering scale, while in the spectator model, there is an even weaker suppression on large scales. Models based on observed stellar kinematics, abundance matching, and direct hydrodynamic cosmological simulations, place the lowest-mass dwarf galaxies in dark matter halos around 10^8 – $10^9 M_{\odot}$. While WDM solutions to the postulated plain Λ CDM failures in this regime would affect only a very narrow mass range, the signature of the spectator model would lead to a weak change in galaxy abundance over a range of masses and affect different scales in the Lyman- α forest. At the very low mass end, just below the filtering scale, the spectator model predicts a very sharp decline in power. For a filtering mass of $\sim 10^9 M_{\odot}$ (corresponding to the peak mass of typical Milky Way dwarf spheroidals), the abundance of substructures below $10^7 M_{\odot}$ is greatly reduced in the WDM model, but for the spectator model, such substructures are non-existent. In this regime,

where structures can only be detected indirectly, even a very small number of detections could therefore place significant constraints on spectator models.

In combination, this makes the spectator model clearly falsifiable. Quantitative predictions will require full, cosmological and hydrodynamic simulations, and will be the subject of future work.

5 Conclusion and discussion

We have argued that, in models with a spectator field in the framework of a small-field inflation, after the inflaton-driven expansion there generically arises a second inflationary epoch which is driven by the spectator field. Depending on model parameters, the second inflation can generate the large number of e -folds, possibly even $N \sim 40$ – 50 . In this case, CMB scale fluctuations correspond to the modes which exited the horizon at $N \sim 10$ when counted from the end of the first inflation. Let us recall that this is a very small number compared to the standard scenario where $N \sim 50$ – 60 . Galactic scales correspond to the perturbations exiting around the very end of the first inflation.

As we discussed in section 2, in the case of small-field inflation, the spectator fluctuations tend to give a negligible contribution. As a result, primordial power spectrum on small scales can be much suppressed compared to the standard plain Λ CDM model while on large scales, the prediction can be consistent with observations of CMB. This is demonstrated in figure 6. Hence a model of the type with two periods of inflation would give interesting implications for the tension between small and large scale structure such as is manifest in “missing satellites” and “too big to fail” problems. As baryonic physics strongly regulate galaxy formation on these scales, a quantitative investigation of this issue will require cosmological hydrodynamic simulations, which are left for future work.

Recent weak lensing results from KiDS [58] continue to yield σ_8 smaller than the value implied by the Planck data by 2 to 3σ . There are extensions of the plain Λ CDM model devised to alleviate the apparent tension; these include allowing for the curvature of the Universe, adopting dark energy models with a time-varying equation of state, modifying general relativity, assuming decaying dark matter (see e.g., [58] for a recent discussion).

While the linear matter power spectrum shown in figure 6 also suggests a large-scale effect of the modified transfer function, at low redshifts, this is likely to be washed out due to mode coupling in the full, non-linear evolution. More promising are observations of the 21 cm forest around the time of reionisation, which could be able to either detect or rule out the characteristic bump in the power spectrum resulting from our model. With SKA promising to make these measurements during the next decade [14], more detailed numerical studies of the non-linear evolution in different inflation models seem to be particularly timely.

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