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Key words: Asymptotic local power, Cointegration, Companion matrix, Convergence, Initial condition, Likelihood ratio test, Unit root

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Tests for Cointegration Rank and the Initial Condition*

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Abstract

Many economic events involve initial observations that substantially deviate from long-run steady state. Initial conditions of this type have been found to impact diversely on the power of univariate unit root tests, whereas the impact on multivariate tests is largely unknown. This paper investigates the impact of the initial condition on tests for cointegration rank. We compare the local power of the widely

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used likelihood ratio (LR) test with the local power of a test based on the eigenvalues of the companion matrix. We find that the power of the LR test is increasing in the magnitude of the initial condition, whereas the power of the other test is decreasing. The behaviour of the tests is investigated in an application to price convergence.

Keywords: Asymptotic local power, Cointegration, Companion matrix, Convergence, Initial condition, Likelihood ratio test, Unit root.

1 Introduction

The initial condition may have a large impact on the power of unit root and cointegration tests. Müller and Elliott (2003) find that the power of univariate unit root tests depend on the deviation of the initial observation from its modelled deterministic part. They show that the impact of the initial condition on power varies among commonly used unit root tests, because these put very different weightings on the initial condition. For example, the power of the Dickey–Fuller (DF) τ -test increases in the magnitude of the initial condition, whereas the opposite holds for the DF coefficient test. Thus, the choice between different unit root tests should be carefully considered in applications where the initial observation is expected to be far from its equilibrium value. For instance, in studies on economic convergence where the initial condition is large, we may want to use a test that puts a large weight on the initial condition in order to maximise power. In other cases where we are unsure about the initial condition, it may be advantageous to use tests which are robust with respect to the initial condition. Such tests can be found in Elliott and Müller (2006), and Harvey and Leybourne (2005, 2006).

In many problems it may be preferable to use multivariate unit root or cointegration tests. For instance, richer types of convergence can be studied in a multivariate framework. However, there seems to be no previous work on the impact of the initial condition in the multivariate case. In this paper

we analyse the impact of the initial condition on the power of multivariate unit root and cointegration tests. In particular, we compare the commonly used likelihood ratio (LR) test of cointegration rank in vector autoregressive (VAR) models (Johansen 1996) with a test for the number of unit roots in VAR models based on the eigenvalues of the companion matrix (Ahlgren and Nyblom 2008; henceforth referred to as the Q test). These two tests can be viewed as the multivariate extensions of the DF τ - and coefficient tests, respectively. Other multivariate tests (see e.g. Hubrich et al. 2001 for a review) are not considered here, because they typically depend on nuisance parameters which make the analysis difficult.

The impact of the initial condition is studied through the local power of multivariate tests. The local power function of the LR test has previously been derived by Johansen (1991, 1996) in the VAR model without deterministic terms. Rahbek (1994), and Horvath and Watson (1995) extend Johansen's results to the cases of different local alternatives and known cointegrating vectors. Saikkonen and Lütkepohl (1999) provide a general framework for deriving the local power properties of LR tests of cointegration rank under different assumptions about the deterministic terms. The aforementioned papers assume that the initial condition is zero. We derive the local power under the assumption that the initial condition is not zero and does not vanish asymptotically. This extends the results obtained by Johansen (1991, 1996), and Saikkonen and Lütkepohl (1999) for the likelihood ratio test when the initial condition is zero, as well as Müller and Elliott

(2003) for univariate unit root tests when the initial condition is not zero and matters asymptotically. An alternative approach to the initial condition is taken in Phillips and Magdalinos (2008), who extend the initial condition to the infinite past.

We find that the power of the LR test is increasing in the magnitude of the initial condition, whereas the Q test may have lower power against alternatives close to a unit root when the initial condition is large. In comparison to the univariate case, we find that the impact of the initial condition is similar, but is mitigated in multivariate tests when the dimension of the process is large, or deterministic terms are fitted. The behaviour of the tests is further highlighted by an empirical application to price convergence. We find that large initial conditions in the real exchange rates of transition economies enable us to reject the unit root hypothesis in the real exchange rates using the LR test. This result entails that purchasing power parity holds. However, if deterministic terms are fitted, or if the dimension of the model is increased by including more information, the power gain gradually disappears. In contrast, the Q test does not reject a unit root in the real exchange rates.

The paper is organised as follows. The next section establishes a framework for analysing the initial condition in the cointegrated VAR model. The LR and Q tests of cointegration rank are described in Section 3, and their local power properties are derived in Section 4. Section 5 considers the powers of the tests in finite samples, and Section 6 illustrates the findings in an

application to price convergence. Section 7 concludes.

2 The Initial Condition in the Cointegrated Vector Autoregressive Model

In this section we introduce the cointegrated vector autoregressive (VAR) model and use a new version of the Granger–Johansen representation theorem to define the magnitude of the initial condition.

We consider the p -dimensional VAR model

$$\mathbf{X}_t = \mathbf{\Pi}_1 \mathbf{X}_{t-1} + \cdots + \mathbf{\Pi}_k \mathbf{X}_{t-k} + \mathbf{\Phi} \mathbf{D}_t + \boldsymbol{\varepsilon}_t, \quad t = 1, \dots, T, \quad (1)$$

where $\mathbf{D}_t$ are the deterministic terms, and the errors $\boldsymbol{\varepsilon}_t$ are independent and identically distributed $(\mathbf{0}, \boldsymbol{\Omega})$. We write the model in error correction form:

$$\Delta \mathbf{X}_t = \mathbf{\Pi} \mathbf{X}_{t-1} + \sum_{i=1}^{k-1} \mathbf{\Gamma}_i \Delta \mathbf{X}_{t-i} + \mathbf{\Phi} \mathbf{D}_t + \boldsymbol{\varepsilon}_t, \quad (2)$$

where $\mathbf{\Pi} = \sum_{i=1}^k \mathbf{\Pi}_i - \mathbf{I}$ and $\mathbf{\Gamma}_i = -\sum_{j=i+1}^k \mathbf{\Pi}_j$. For later use we define $\mathbf{\Gamma} = \mathbf{I} - \sum_{i=1}^{k-1} \mathbf{\Gamma}_i$.

The characteristic polynomial of the process is

$$\mathbf{A}(z) = (1 - z)\mathbf{I} - \mathbf{\Pi}z - \sum_{i=1}^{k-1} \mathbf{\Gamma}_i(1 - z)z^i.$$

We assume that $|\mathbf{A}(z)| = 0$ implies that $|z| > 1$ or $z = 1$, and that $\mathbf{\Pi}$ has reduced rank $r < p$. Then $\mathbf{\Pi} = \mathbf{\alpha}\mathbf{\beta}'$, where $\mathbf{\alpha}$ and $\mathbf{\beta}$ are $p \times r$ matrices of full rank. We also assume that $\mathbf{\alpha}'_{\perp}\mathbf{\Gamma}\mathbf{\beta}_{\perp}$ has full rank, which ensures that the process $\mathbf{X}_t$ is integrated of order one, $I(1)$. We use the definition of an integrated process as in Johansen (1996, pp. 34–35). Here $\mathbf{\alpha}_{\perp}$ and $\mathbf{\beta}_{\perp}$ denote the $p \times (p - r)$ orthogonal complements of $\mathbf{\alpha}$ and $\mathbf{\beta}$, respectively, such that $\mathbf{\alpha}'_{\perp}\mathbf{\alpha} = \mathbf{0}$ and $\mathbf{\beta}'_{\perp}\mathbf{\beta} = \mathbf{0}$. In addition, the deterministic terms $\mathbf{D}_t$ are assumed to be bounded by some polynomial in t . Under these conditions, Johansen (1996) shows that $\Delta\mathbf{X}_t - \mathbf{E}(\Delta\mathbf{X}_t)$ and $\mathbf{\beta}'\mathbf{X}_t - \mathbf{E}(\mathbf{\beta}'\mathbf{X}_t)$ can be given initial distributions such that they become $I(0)$.

The moving average representation of the process (2) was derived by Johansen (1996). For our purpose, a more convenient version of the representation theorem is Theorem 1 of Hansen (2005), which under the assumptions above states that the process has the representation

$$\mathbf{X}_t = \mathbf{C} \sum_{i=1}^t \boldsymbol{\varepsilon}_i + \mathbf{C}(L)\boldsymbol{\varepsilon}_t + \boldsymbol{\tau}(t) + \mathbf{C}(\mathbf{X}_0 - \mathbf{\Gamma}_1\mathbf{X}_{-1} - \cdots - \mathbf{\Gamma}_{k-1}\mathbf{X}_{-k+1}), \quad (3)$$

where $\mathbf{C} = \mathbf{\beta}_{\perp}(\mathbf{\alpha}'_{\perp}\mathbf{\Gamma}\mathbf{\beta}_{\perp})^{-1}\mathbf{\alpha}'_{\perp}$. The polynomial $\mathbf{C}(z) = \sum_{i=0}^{\infty} \mathbf{C}_i z^i$ is convergent for $|z| < 1 + \delta$ for some $\delta > 0$. The polynomial $\boldsymbol{\tau}(t)$ is given by $\boldsymbol{\tau}(t) = \mathbf{C}\mathbf{\Phi} \sum_{i=1}^t \mathbf{D}_i + \mathbf{C}(L)\mathbf{\Phi}\mathbf{D}_t$. Hansen gives a recursive formula for calculating the coefficients of the lag polynomial $\mathbf{C}(L)$, and proves in Lemma 2

that the initial value can be expressed as

$$\mathbf{C}(\mathbf{X}_0 - \mathbf{\Gamma}_1 \mathbf{X}_{-1} - \cdots - \mathbf{\Gamma}_{k-1} \mathbf{X}_{-k+1}) = \mathbf{X}_0 - \mathbf{C}(L)(\boldsymbol{\varepsilon}_0 + \mathbf{\Phi} \mathbf{D}_0). \quad (4)$$

Premultiplying (4) by $\boldsymbol{\beta}'$ yields

$$\boldsymbol{\beta}' \mathbf{X}_0 = \boldsymbol{\beta}' \mathbf{C}(L)(\boldsymbol{\varepsilon}_0 + \mathbf{\Phi} \mathbf{D}_0). \quad (5)$$

If the initial value is chosen as in (5), we have that $\boldsymbol{\beta}' \mathbf{X}_t - \mathbf{E}(\boldsymbol{\beta}' \mathbf{X}_t) = \boldsymbol{\beta}' \mathbf{C}(L) \boldsymbol{\varepsilon}_t$, which is $I(0)$.

In practice, we cannot choose the initial value as in (5). In this case $\boldsymbol{\beta}' \mathbf{X}_t - \mathbf{E}(\boldsymbol{\beta}' \mathbf{X}_t)$ is not $I(0)$, although in some sense it is asymptotically stationary (Johansen 1996, p. 15). Since we condition on the initial observations, we may contend ourselves with

$$\boldsymbol{\beta}' \mathbf{X}_0 = \boldsymbol{\beta}' \mathbf{C}(L) \mathbf{\Phi} \mathbf{D}_0,$$

where the right-hand side corresponds to the expected value of $\boldsymbol{\beta}' \mathbf{X}_0$.

We define the initial observation $\boldsymbol{\beta}' \mathbf{X}_0$ to be deviating from the deterministic component of the series, or to be off its equilibrium value, if

$$|\boldsymbol{\beta}'_i(\mathbf{X}_0 - \mathbf{C}(L) \mathbf{\Phi} \mathbf{D}_0)| > c_i$$

for some $i = 1, \dots, r$ and some positive constant c_i . For example, c_i can

be taken to be two standard deviations of the stationary process $\beta'_i \mathbf{X}_t$. In fact, as we shall see later, the initial condition begins to matter when c_i is equal to two standard deviations. Note that application of this condition requires knowledge of the steady-state equilibrium, $\beta' \mathbf{C}(L) \Phi \mathbf{D}_t$, or that we can estimate it consistently.

3 Tests for Cointegration Rank

We consider again the p -dimensional VAR model (1) and (2), but now with $\Phi \mathbf{D}_t$ specialised to $\boldsymbol{\mu}_t = \boldsymbol{\mu}_0 + \boldsymbol{\mu}_1 t$, where $\boldsymbol{\mu}_t$ denotes the deterministic part of the model. Five models for the deterministic terms are considered by Johansen (1996, Section 5.7), and Saikkonen and Lütkepohl (1999). They can succinctly be summarised by the following cases: no deterministic terms, restricted and unrestricted constant, and restricted and unrestricted linear trend. The different models are expressed as restrictions on the parameters $\boldsymbol{\mu}_0$ and $\boldsymbol{\mu}_1$. By exploiting the identity $\mathbf{I} = \boldsymbol{\alpha}(\boldsymbol{\alpha}'\boldsymbol{\alpha})^{-1}\boldsymbol{\alpha}' + \boldsymbol{\alpha}_\perp(\boldsymbol{\alpha}'_\perp\boldsymbol{\alpha}_\perp)^{-1}\boldsymbol{\alpha}'_\perp$, the parameter $\boldsymbol{\mu}_i$ can be decomposed into the directions $\boldsymbol{\alpha}$ and $\boldsymbol{\alpha}_\perp$ as follows: $\boldsymbol{\mu}_i = \boldsymbol{\alpha}(\boldsymbol{\alpha}'\boldsymbol{\alpha})^{-1}\boldsymbol{\alpha}'\boldsymbol{\mu}_i + \boldsymbol{\alpha}_\perp(\boldsymbol{\alpha}'_\perp\boldsymbol{\alpha}_\perp)^{-1}\boldsymbol{\alpha}'_\perp\boldsymbol{\mu}_i$. For example, in the model with a constant restricted to the cointegration space, $\boldsymbol{\alpha}_\perp(\boldsymbol{\alpha}'_\perp\boldsymbol{\alpha}_\perp)^{-1}\boldsymbol{\alpha}'_\perp\boldsymbol{\mu}_0 = \mathbf{0}$ and $\boldsymbol{\mu}_1 = \mathbf{0}$.

The LR statistic for testing the cointegration rank hypotheses

$$H_0 : \text{rank}(\boldsymbol{\Pi}) = r \quad \text{against} \quad H_1 : \text{rank}(\boldsymbol{\Pi}) > r, \quad r = 0, \dots, p-1, \quad (6)$$

in the model (2) is given by

$$LR(r) = -T \sum_{i=r+1}^p \ln(1 - \hat{\lambda}_i), \quad (7)$$

where the eigenvalues $1 > \hat{\lambda}_1 > \dots > \hat{\lambda}_p > 0$ solve

$$|\lambda \mathbf{S}_{11} - \mathbf{S}_{10} \mathbf{S}_{00}^{-1} \mathbf{S}_{01}| = 0,$$

and

$$\mathbf{S}_{ij} = T^{-1} \sum_{t=1}^T \mathbf{R}_{it} \mathbf{R}_{jt}', \quad i, j = 0, 1,$$

are the product moment matrices of the residuals $\mathbf{R}_{0t}$ and $\mathbf{R}_{1t}$ from regressing $\Delta \mathbf{X}_t$ and $\mathbf{X}_{t-1}$ (possibly augmented by a constant and linear trend) on the lagged differences $\Delta \mathbf{X}_{t-1}, \dots, \Delta \mathbf{X}_{t-k+1}$, and possibly a constant and linear trend.

The limiting distribution of $LR(r)$ is given by

$$LR(r) \xrightarrow{D} \text{tr} \left\{ \int_0^1 (d\mathbf{B}) \mathbf{F}' \left[\int_0^1 \mathbf{F} \mathbf{F}' du \right]^{-1} \int_0^1 \mathbf{F} (d\mathbf{B})' \right\}, \quad (8)$$

where $\mathbf{B}$ is a $(p - r)$ -dimensional standard Brownian motion on the unit interval and $\mathbf{F}$ is a Brownian motion which depends on the deterministic terms.

The Q test (Ahlgren and Nyblom 2008) is a test for the same hypotheses as in (6). The approach is to write the general VAR(k) model (1) in

companion form as a large VAR(1) model, and employ the eigenvalues of the companion matrix to test the cointegration rank. Let $\hat{\nu}_1, \dots, \hat{\nu}_p$ denote the sample eigenvalues closest to one measured by the distance $|\hat{\nu}_i - 1|$ and ordered as $\text{Re}(\hat{\nu}_1) \geq \dots \geq \text{Re}(\hat{\nu}_p)$. The test allows for explosive roots, but here we shall assume that there are no explosive roots. The Q statistic is then given by

$$Q(r) = -T \sum_{i=r+1}^p (\text{Re}(\hat{\nu}_i) - 1). \quad (9)$$

The limiting distribution of the $Q(r)$ statistic is given by

$$Q(r) \xrightarrow{D} \text{tr} \left\{ \int_0^1 (d\mathbf{B}) \mathbf{F}' \left[\int_0^1 \mathbf{F} \mathbf{F}' du \right]^{-1} \right\}, \quad (10)$$

where $\mathbf{B}$ and $\mathbf{F}$ are as in (8).

4 Local Power of Tests for Cointegration Rank

For deriving the asymptotic local power, we consider model (2) with $k = 1$:

$$\Delta \mathbf{X}_t = \mathbf{\Pi} \mathbf{X}_{t-1} + \boldsymbol{\varepsilon}_t, \quad t = 1, \dots, T, \quad (11)$$

since the limit results do not depend on the short-term dynamics (see Johansen 1996, pp. 209–210). Here $\mathbf{\Pi}$ is an unrestricted $p \times p$ matrix, and the errors $\boldsymbol{\varepsilon}_t$ are as before independent and identically distributed $(\mathbf{0}, \boldsymbol{\Omega})$. Johansen (1991, 1996) derived the local power function of the LR test in this

model, assuming that the initial condition is zero.

An application of the functional central limit theorem (FCLT) to

$$T^{-1/2}\boldsymbol{\alpha}'_{\perp}\mathbf{X}_{[Tu]} = T^{-1/2}\boldsymbol{\alpha}'_{\perp}\mathbf{X}_0 + T^{-1/2}\sum_{i=1}^{[Tu]}\boldsymbol{\alpha}'_{\perp}\boldsymbol{\varepsilon}_i, \quad (12)$$

with $[\cdot]$ denoting the integer part, shows that the limiting distributions under the null hypothesis (and the local alternatives) do not depend on the initial value $\mathbf{X}_0$, since the first term is $\mathbf{o}(1)$. Following the analysis in Tanaka (1996, Section 3.9), an approximation which retains the initial condition asymptotically is obtained when $\mathbf{X}_0 = \mathbf{O}(T^{1/2})$, implying that the first term in (12) has the same order of magnitude as the second. For simplicity we assume that the initial condition is of the form

$$\mathbf{X}_0 = T^{1/2}\boldsymbol{\gamma}, \quad (13)$$

where $\boldsymbol{\gamma}$ is a constant $p \times 1$ vector. However, the results obtained below also hold if $\mathbf{X}_0$ is assumed to be a random variable of order $\mathbf{O}_P(T^{1/2})$.

We consider local alternatives to the null hypothesis $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, of the form

$$\boldsymbol{\Pi} = \boldsymbol{\Pi}_T = \boldsymbol{\alpha}\boldsymbol{\beta}' + T^{-1}\boldsymbol{\alpha}_1\boldsymbol{\beta}'_1, \quad (14)$$

where $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$ are $p \times s$ matrices of rank s such that $\boldsymbol{\Pi}$ has rank $r+s$, $s > 0$. Under the local alternatives the process has s extra cointegrating vectors $\boldsymbol{\beta}_1$. We make the assumption that the eigenvalues of $\mathbf{I} + \boldsymbol{\alpha}\boldsymbol{\beta}'$ are less than one in

absolute value, so that the process is $I(1)$ under the null hypothesis (6). The derivations of the local power function are based on the results of Johansen (1996, Section 14.2), and Saikkonen and Lütkepohl (1999). We denote by $\mathbf{W}(u)$ a Brownian motion with covariance matrix $\boldsymbol{\Omega}$ and by $\mathbf{B}(u)$ a standard Brownian motion defined by $\mathbf{B}(u) = (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Omega} \boldsymbol{\alpha}_{\perp})^{-1/2} \mathbf{W}(u)$.

Under the local alternatives (14),

$$T^{-1/2} \boldsymbol{\alpha}'_{\perp} \mathbf{X}_{[Tu]} \xrightarrow{D} \mathbf{K}(u), \quad (15)$$

where $\mathbf{K}(u)$ is an Ornstein–Uhlenbeck process defined by the integral equation

$$\mathbf{K}(u) = \boldsymbol{\alpha}'_{\perp} \mathbf{W}(u) + \boldsymbol{\alpha}'_{\perp} \boldsymbol{\alpha}_1 \boldsymbol{\beta}'_1 \boldsymbol{\beta}_{\perp} (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\beta}_{\perp})^{-1} \int_0^u \mathbf{K}(s) ds \quad (16)$$

with $\mathbf{K}(0) = \boldsymbol{\alpha}'_{\perp} \boldsymbol{\gamma}$. If we define the standard Ornstein–Uhlenbeck process $\mathbf{N}(u) = (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Omega} \boldsymbol{\alpha}_{\perp})^{-1/2} \mathbf{K}(u)$, we obtain that

$$\mathbf{N}(u) = \mathbf{B}(u) + \mathbf{a} \mathbf{b}' \int_0^u \mathbf{N}(s) ds, \quad (17)$$

where $\mathbf{N}(0) = (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Omega} \boldsymbol{\alpha}_{\perp})^{-1/2} \boldsymbol{\alpha}'_{\perp} \boldsymbol{\gamma}$, $\mathbf{a} = (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Omega} \boldsymbol{\alpha}_{\perp})^{-1/2} \boldsymbol{\alpha}'_{\perp} \boldsymbol{\alpha}_1$ and $\mathbf{b} = (\boldsymbol{\alpha}'_{\perp} \boldsymbol{\Omega} \boldsymbol{\alpha}_{\perp})^{1/2} (\boldsymbol{\beta}'_{\perp} \boldsymbol{\alpha}_{\perp})^{-1} \boldsymbol{\beta}'_{\perp} \boldsymbol{\beta}_1$.

The limiting distribution of the LR statistic under the local alternatives is obtained by replacing the Brownian motions in (8) by the Ornstein–

Uhlenbeck process in (17):

$$LR(r) \xrightarrow{D} \text{tr} \left\{ \int_0^1 d\mathbf{N} \mathbf{N}' \left[\int_0^1 \mathbf{N} \mathbf{N}' du \right]^{-1} \int_0^1 \mathbf{N} d\mathbf{N}' \right\}. \quad (18)$$

An analogous result holds for the Q test (Ahlgren and Nyblom 2008), yielding

$$Q(r) \xrightarrow{D} \text{tr} \left\{ \int_0^1 d\mathbf{N} \mathbf{N}' \left[\int_0^1 \mathbf{N} \mathbf{N}' du \right]^{-1} \right\}. \quad (19)$$

Since the limiting distributions depend on $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{\Omega}$, $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$ only through $\mathbf{a}$ and $\mathbf{b}$, the results simplify if $s = 1$, i.e. if there is one cointegrating relation under the local alternatives. Then we can find an orthogonal transformation of (17) such that $\mathbf{N}(u)$ is of the form

$$N_1(u) = B_1(u) + f \int_0^u N_1(s) ds, \quad (20)$$

$$N_2(u) = B_2(u) + g \int_0^u N_1(s) ds, \quad (21)$$

$$\mathbf{N}_3(u) = \mathbf{B}_3(u), \quad (22)$$

and the limiting distributions depend only on the scalar parameters $f = \mathbf{b}'\mathbf{a}$ and $g^2 = \mathbf{a}'\mathbf{a}\mathbf{b}'\mathbf{b} - (\mathbf{a}'\mathbf{b})^2$ (Johansen 1996, Corollary 14.5). The first component $N_1(u)$ is an Ornstein–Uhlenbeck process, the second component $N_2(u)$ is a Brownian motion, whereas the third component $\mathbf{N}_3(u)$ is a $(p-r-2)$ -dimensional standard Brownian motion, $p-r > 2$. A further simplification is obtained when $\boldsymbol{\alpha}_1$ and $\boldsymbol{\beta}_1$ are proportional, entailing that $g = 0$. Although

the local powers of the LR and Q tests depend on the parameter g , it turns out to be not of great interest for the impact of the initial condition. This issue is discussed in connection with the simulations below. Consequently, we impose the restriction $g = 0$. In this case the second component $N_2(u)$ becomes a standard Brownian motion, and then the limiting distributions depend only on the parameter f . These results will be convenient for investigating the impact of the initial condition on the local powers of the LR and Q tests. The process (20) has the solution (see e.g. Tanaka 1996, Section 3.9)

$$N_1(u) = e^{fu}\gamma_1 + e^{fu} \int_0^u e^{-fs} dB_1(s), \quad (23)$$

where $N_1(0) = \gamma_1 = \boldsymbol{\alpha}'_{\perp 1} \boldsymbol{\gamma}$ follows from our choice of the initial condition (13). From the expression for $N_1(u)$ in (23), it is seen that the impact on the local power from the initial condition comes from the first term, which is zero when $\gamma_1 = 0$. Since $f < 0$ for stationary alternatives, the effect of the initial condition is to add a geometrically decaying series, $e^{fu}\gamma_1$, to the solution of the Ornstein–Uhlenbeck process. By modifying the Ornstein–Uhlenbeck processes in (18) and (19) as in Saikkonen and Lütkepohl (1999), the results can be extended to models with a constant and linear trend.

From (20)–(22) with $g = 0$, we get

$$dN_1(u) = dB_1(u) + fN_1(u) du,$$

$$dN_2(u) = dB_2(u),$$

$$d\mathbf{N}_3(u) = d\mathbf{B}_3(u).$$

Hence, we simulate the discrete versions of the $(p - r)$ -dimensional Ornstein–Uhlenbeck processes with $g = 0$:

$$N_{1t} = (1 + T^{-1}c)N_{1,t-1} + \varepsilon_{1t}, \quad (24)$$

$$N_{2t} = N_{2,t-1} + \varepsilon_{2t}, \quad (25)$$

$$\mathbf{N}_{3t} = \mathbf{N}_{3,t-1} + \boldsymbol{\varepsilon}_{3t}. \quad (26)$$

We choose $\boldsymbol{\gamma}$ as

$$\boldsymbol{\gamma} = \begin{pmatrix} \gamma\sigma_1(-2c)^{-1/2} \\ 0 \\ \mathbf{0} \end{pmatrix}. \quad (27)$$

Our choice of $\boldsymbol{\gamma}$ implies that $N_{10} = \gamma\sigma_1 T^{1/2}(-2c)^{-1/2}$, which following Müller and Elliott (2003, pp. 1274–1275) measures the initial condition in terms of the unconditional standard deviation of the stationary process (24) with $c < 0$, and γ denotes the standardised initial condition. A value of $\gamma = 1$ corresponds to the initial condition being equal to one standard deviation of the unconditional distribution of N_{1t} when $c < 0$. The process N_{2t} and the

$(p - r - 1)$ -dimensional process $\mathbf{N}_{3t}$, $p - r > 2$, are random walks. We choose the initial conditions of the random walks to be zero, since they do not affect the powers of the tests.

The impact of the initial condition on the local powers does not depend on the value of g . This finding emerged from conducting simulations with different values for g (the results are available upon request). The powers of the LR and Q tests are not directly comparable, though, since their relative powers would also depend on g (Ahlgren and Nyblom 2008).

We have simulated the local powers for different values of $f < 0$, $g = 0$, and for models with no deterministic terms, restricted and unrestricted constant, and restricted and unrestricted linear trend, using the general result of Saikkonen and Lütkepohl (1999). In the simulations we let $t = 1, \dots, 1000$, $p - r = 1, 2, 3$ and $\gamma = 0, 0.5, 1, 2, 4$. The errors ε_{it} are independent and normally distributed $N(0, 1)$, and the number of replications is 100000. The computations and simulations are performed using GAUSS 7.0.17. The function RNDN in GAUSS, which uses the fast acceptance-rejection algorithm of Kinderman and Ramage, is used to generate pseudo-random numbers.

The asymptotic distributions of the LR statistics in the models with an unrestricted constant or linear trend depend on whether $\boldsymbol{\mu}_i = \mathbf{0}$ or $\boldsymbol{\mu}_i \neq \mathbf{0}$, $i = 0, 1$, respectively. In the models with an unrestricted constant or linear trend, we assume that $\boldsymbol{\mu}_i = \mathbf{0}$. Critical values for the case $\boldsymbol{\mu}_0 = \mathbf{0}$ are given by Johansen and Juselius (1990, Table A.2) and for the case $\boldsymbol{\mu}_1 = \mathbf{0}$ by Perron and Campbell (1993). For the other cases, critical values are obtained

from Johansen (1996). For comparison with the results in Saikkonen and Lütkepohl (1999), it is worth noting that they use the notations LR^* and LR^{i0} for the tests with a restricted and an unrestricted constant, respectively, and LR^+ and LR^{PC} for the tests with a restricted and an unrestricted linear trend, respectively. In the Q test, the constant and trend are estimated unrestricted, and critical values are given for the cases $\mu_0 = \mathbf{0}$ and $\mu_1 = \mathbf{0}$ in Ahlgren and Nyblom (2008, Table 1).

The simulated local power functions of the LR and Q tests are plotted in Figures 1–8. It is seen from the figures that the power of the LR test increases in the magnitude of the initial condition γ , whereas the power of the Q test decreases. The power functions of the LR test with $\gamma > 0$ are shifted upwards and become more steep, in particular when $p - r = 1$, and for the Q test they are shifted downwards. The effect of the initial condition on the local power is much more pronounced for the LR test than for the Q test. The local power is decreasing as the dimensions $p - r$ increase, as found by Johansen (1996). In general, with a constant or a constant and linear trend, the impact of the initial condition is mitigated. In the special case $p - r = 1$ and no deterministic terms in the model, the power of the Q test decreases for values of c close to the null hypothesis, whereas the opposite holds for values of c further away from the null hypothesis. A similar result was obtained by Catani (2008) for the local power of the Dickey–Fuller coefficient test (with $p - r = 1$ the test statistics are the same).

Our results may readily be used in applications by noticing that the

coefficient $1 + T^{-1}c = 1 + T^{-1}\alpha_1'\beta_1$ can be interpreted as the autoregressive parameter in a near-integrated relation. For example, assume that we have $T = 100$ observations and an autoregressive coefficient of 0.91 (choose $c = -9$). Then in the model with no deterministic terms, if $p - r = 2$, the power of the LR test is 0.166 when $\gamma = 0$, 0.230 when $\gamma = 2$, and 0.502 when $\gamma = 4$. The corresponding powers of the Q test are 0.269 when $\gamma = 0$, 0.225 when $\gamma = 2$, and 0.143 when $\gamma = 4$. In the model with an unrestricted constant, the power gains of the LR test are smaller but still substantial. Then if $p - r = 2$, the power of the LR test is 0.128 when $\gamma = 0$, 0.160 when $\gamma = 2$, and 0.285 when $\gamma = 4$. The corresponding powers of the Q test are 0.204 when $\gamma = 0$, 0.175 when $\gamma = 2$, and 0.121 when $\gamma = 4$. The value of $\gamma = 2$ is found to be the point when the initial condition begins to matter for the power of the tests.

5 Power in Finite Samples

The previous section demonstrated that the asymptotic local power of the LR test increases in the magnitude of the initial condition and the local power of the Q test decreases in the magnitude of the initial condition. To check the accuracy of the asymptotic local power approximations, we simulate the powers of the LR and Q tests in finite samples.

Toda (1994, 1995) suggests a VAR(1) data generating process (DGP) which is a canonical form for investigating the properties of tests for the

Figure 1: Local power of the LR test with no deterministic terms.

Figure 2: Local power of the Q test with no deterministic terms.

Figure 3: Local power of the LR test with restricted constant.

Figure 4: Local power of the LR test with unrestricted constant.

Figure 5: Local power of the Q test with unrestricted constant.

Figure 6: Local power of the LR test with unrestricted constant and restricted trend.

Figure 7: Local power of the LR test with unrestricted constant and trend.

Figure 8: Local power of the Q test with unrestricted constant and trend.

cointegration rank, since other VAR(1) processes can be obtained from it by linear transformations. In the simulations we use a three-dimensional version of Toda's DGP from Hubrich et al. (2001) of the form

$$\mathbf{X}_t = \begin{pmatrix} \phi_1 & 0 & 0 \\ 0 & \phi_2 & 0 \\ 0 & 0 & 1 \end{pmatrix} \mathbf{X}_{t-1} + \begin{pmatrix} 0 \\ 0 \\ \mu \end{pmatrix} + \boldsymbol{\varepsilon}_t, \quad \boldsymbol{\varepsilon}_t \sim \text{NID} \left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 & \theta_1 & \theta_2 \\ \theta_1 & 1 & 0 \\ \theta_2 & 0 & 1 \end{pmatrix} \right). \quad (28)$$

Analogous with the asymptotic analysis in Section 4, we only consider the cases when the cointegration rank is $r = 0$ or $r = 1$. Then $\phi_2 = 1$, and we can set $\theta_2 = 0$. If $\phi_1 = 1$, the cointegration rank is zero, and if $|\phi_1| < 1$, the cointegration rank is one. The case $f = 0$ in Section 4 corresponds to $\phi_1 = 1$ and $f < 0$ corresponds to $\phi_1 < 1$. The quantity θ_1 is the correlation between the first component of the process and the second component. The case $g = 0$ corresponds to $\theta_1 = 0$, and then the two components are uncorrelated. In line with the asymptotic analysis in Section 4, we impose $\theta_1 = 0$ (the results for different values of θ_1 are available upon request). The initial value X_{10} is defined as $X_{10} = \gamma/(1 - \phi_1^2)$. A value of $\gamma = 1$ corresponds to the initial condition being equal to one standard deviation of the unconditional distribution of X_{1t} when $|\phi_1| < 1$. The series X_{2t} and X_{3t} have initial values 0. If $\mu \neq 0$ there is a linear trend in the random walk component and $\mu = 0$ corresponds to no linear trend. In the simulations we set $\mu = 0$.

We estimate a VAR(1) model with an unrestricted constant:

$$\Delta \mathbf{X}_t = \Pi \mathbf{X}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\varepsilon}_t, \quad (29)$$

and test the null hypothesis of cointegration rank $r = 0$. We report in Table 1 the rejection probabilities of the LR and Q tests for different values of ϕ_1 and a range of initial conditions γ . We let ϕ_1 take the values 1, 0.99, 0.975, 0.95 and 0.90, which are the ones recommended by Busetti et al. (2006) for simulating tests of convergence. For quarterly data and $T = 100$, $\phi_1 = 0.95$ is a plausible value. A value of $\phi_1 = 0.90$ represents fast convergence and a value of $\phi_1 = 0.975$ corresponds to slow convergence. Note that when $\phi_1 = 1$, the sizes of the tests do not depend on X_{10} , so the simulations for $\phi_1 = 1$ are from a single experiment with $X_{10} = 0$. The values of $|\phi_1| < 1$ give the powers of the tests. We let γ vary among 0, 0.5, 1, 2, 4. The nominal significance level is 5% and the number of replications is 100000. The computations and simulations are performed as described in Section 4.

The rejection probabilities in Table 1 mirror the local powers in Section 4. We find that when γ is large, the LR test has high power whereas the Q test has low power. For example, if $\phi_1 = 0.95$, the power of the LR test is 0.080 when $\gamma = 0$, 0.291 when $\gamma = 2$, and 0.955 when $\gamma = 4$. The corresponding powers of the Q test are 0.046 when $\gamma = 0$, 0.019 when $\gamma = 2$, and 0.019 when $\gamma = 4$.

Table 1: Rejection probabilities of the LR and Q tests in model (29) with an unrestricted constant. The DGP is given by (28) with $\phi_2 = 0$, $\mu = 0$ and $\theta_1, \theta_2 = 0$. The series lengths are $T = 100$ and the nominal significance level is 5%.

γ / ϕ_1	1	0.99	0.975	0.95	0.90	1	0.99	0.95	0.975	0.90
	LR					Q				
0	0.067	0.069	0.071	0.080	0.117	0.033	0.039	0.046	0.061	0.118
0.5	0.067	0.063	0.077	0.088	0.124	0.033	0.026	0.036	0.054	0.112
1	0.067	0.073	0.104	0.114	0.145	0.033	0.019	0.024	0.041	0.096
2	0.067	0.141	0.277	0.291	0.276	0.033	0.020	0.019	0.025	0.067
4	0.067	0.548	0.929	0.955	0.876	0.033	0.018	0.019	0.020	0.042

6 Price Convergence

In this section we illustrate the impact of the initial condition on the powers of the LR and Q tests by investigating price convergence of transition economies in Central Europe and the Baltics to the Euro Area. For the former, the initial observation on the real exchange rate is off its equilibrium level. We study the effect of the dimension of the VAR model on the powers of the tests by including UK and Danish real exchange rates in an extended VAR system. The UK and Denmark are currently outside the Euro Area.

We begin by discussing a time series definition of convergence. Let p_{it} , $i = 0, 1, \dots, p$, denote a log price index for country i , and let e_{ijt} denote the log nominal exchange rate between countries i and j . Then the real exchange

rate, or the real price difference, z_{ijt} , is defined as

$$z_{ijt} = p_{it} + e_{ijt} - p_{jt}. \quad (30)$$

Busetti et al. (2006) adapt a definition of output convergence in Bernard and Durlauf (1996) to price convergence. According to this definition, the price levels between countries i and j are said to converge if z_{ijt} satisfies the condition

$$\lim_{\tau \rightarrow \infty} E(z_{ij,t+\tau} \mid \mathcal{F}_t) = 0, \quad (31)$$

where $\mathcal{F}_t$ is the information set at time t . The definition entails that absolute purchasing power parity is expected to hold, as τ goes to infinity, since the choice of base year for price indices is arbitrary. Busetti et al. (2006) distinguish between price level convergence and inflation convergence, since prices are often found to contain an $I(2)$ component. They further distinguish between countries in the process of converging and countries that have converged, by employing stationarity tests to z_{ijt} . Since this distinction is not a major concern here, we do not discuss it further, and refer to Busetti et al. (2006) for details. Moreover, none of the z_{ijt} in our data was found to contain an $I(2)$ component. Thus we may exclusively focus on price level convergence.

An important implication of condition (31) is that the real exchange rate is asymptotically stationary under convergence, since the conditional expectation would not be defined otherwise. Nevertheless, it has typically been

difficult to reject a unit root in real exchange rates. Such persistence may reflect nonlinear threshold dynamics rather than 'true' unit root behaviour, as shown by Taylor et al. (2001), among others. In this case, the real exchange rate will behave like a unit root process within some small interval around zero, and long time series are required to detect mean reversion. However, the increase in power of the LR test of cointegration rank due to a large initial condition in the real exchange rate of transition economies may allow rejection of the unit root hypothesis in shorter time series.

The data consist of monthly observations on the consumer price index and nominal exchange rate for the Euro Area (EA), the Czech Republic (CZ), Estonia (EE), Poland (PL), the UK and Denmark (DK), obtained from the EuroStat database. The sample period is 1996:1–2007:12 and the number of monthly observations is 144. We index the Euro Area as $j = 0$, and take it as the reference economy, i.e. we fix $j = 0$ in (30). Following Buseti et al. (2006), we take the last period to be the base year. This normalisation yields the maximum power gain in the model with no deterministic terms, but has no consequences for the other specifications of the deterministic terms.

Figure 9 plots the real exchange rates. As can be seen from the figure, the initial condition is most pronounced in the real exchange rate of the Czech Republic and Estonia, and also to some extent Poland, which show clear convergence trends. In contrast, UK and Danish real exchange rates are more consistent with threshold dynamics. For instance, both series show persistent deviations on both sides of the zero line, although the Danish

Figure 9: Real exchange rates of Czech republic (CZ), Estonia (EE), Poland (PL), UK and Denmark (DK) against the Euro Area (EA).

deviations are of a smaller magnitude than the UK. Thus, at the outset, and in concurrence with previous evidence on real exchange rates, we would expect to find unit roots in the UK and Danish series.

We extend the univariate approach of Buseti et al. (2006) to a multivariate framework by estimating the VAR model (2) on a vector of real exchanges rates and using the LR and Q tests to infer the rank, r , of $\Pi = \alpha\beta'$. The null hypothesis that country i converge to the Euro Area or, equivalently that z_{i0t} is stationary, can be formulated as $\beta = (e_i, \beta_{-i})$, where e_i is a unit vector and β_{-i} is a $p \times (r - 1)$ dimensional matrix of unrestricted coefficients. The test of this hypothesis is described in Johansen (1996), and the corre-

sponding LR statistic is asymptotically distributed as χ^2 with $p - r$ degrees of freedom under the null hypothesis.

An advantage of the multivariate framework is that it permits convergence to other economies than the Euro Area. Although such z_{ijt} are unlikely to have a large initial condition, and are therefore not of interest from the perspective of the present paper, they can nevertheless easily be tested as restrictions on β . To see this, note that convergence between i and j can be formulated as the hypothesis $z_{ijt} = z_{i0t} - z_{j0t} \sim I(0)$.

We first focus on the transition economies by setting $\mathbf{X}_{1t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t})'$, where the initial conditions are likely to be large, and later extend this vector to $\mathbf{X}_{2t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t}, z_{UK0t}, z_{DK0t})'$. This setup allows us to analyse the impact of the initial condition on the powers of the LR and Q tests for different dimensions of the random walk component, since the UK and Danish real exchange rates are likely to display independent unit roots behaviour, as discussed above. We consider models with no deterministic terms, a constant restricted to the cointegration space, and an unrestricted constant. A model with no constant is consistent with (31), provided that the last observation, which is set to zero, is in equilibrium. Including a constant in the model allows for uncertainty with respect to this normalisation. If the constant is unrestricted, a linear trend is permitted in z_{ijt} under the alternative hypothesis of no convergence, whereas no trend is permitted when the constant is restricted to the cointegration space.

Initial modelling of $\mathbf{X}_{1t}$ and $\mathbf{X}_{2t}$ suggests two lags ($k = 2$) in both cases,

Table 2: Dummy variables included in the estimated models. The intervention dummies take the value 1 in the quarter yymm and 0 otherwise. The transitory dummies take the value 1 in the quarter yymm, -1 in the consecutive quarter, and 0 otherwise.

Country	Intervention	Transitory
CZ	$D9705, D9807, D9901$	–
EE	$D9609, D9701, D9711, D9802$	$D9703, D9707$
PL	$D0107$	–
UK	–	$D0004$
DK	$D9810$	–

based on information criteria. The models are estimated using CATS in RATS 2.0. We included dummy variables, listed in Table 2, to account for outliers. However, the results are only marginally different if the dummies are removed from the analysis. Table 3 reports misspecification statistics for the estimated models. There are no serious misspecifications in the models, apart from rejection of normality due to some large outliers in the beginning of the sample for the transition economies.

6.1 The Transition Economies

Table 4 reports the LR and Q test for models with $\mathbf{X}_{1t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t})'$, and different deterministic terms. As can be seen from the table, $r = 0$ and $r = 1$ are rejected by the LR test when there is no constant in the model (Model 1), whereas $r = 0$ is not rejected by the Q test. Thus, the LR test suggests $r = 2$, and the Q test suggest $r = 0$. Table 5 reports the results

Table 3: Misspecification statistics for the estimated models in Tables 4 and 6. Models 2 and 3, as well as 5 and 6 are identical. LM(1) and LM(12) are Lagrange multiplier tests for residual autocorrelation. ARCH(1) is the test for autoregressive conditional heteroskedastic residuals and JB is the Jarque–Bera test for normality of the residuals. All statistics are χ^2 . * denotes statistically significant at the 5 per cent level. ** denotes statistically significant at the 1 per cent level.

	LM(1)	LM(12)	ARCH(1)	JB
Model 1	3.682	5.184	46.506	17.042**
Model 2	4.129	5.156	50.465	16.021*
Model 4	28.574	24.089	240.387	58.897**
Model 5	29.019	25.549	256.589	57.097**

from tests of unit vectors in β given $r = 2$, corresponding to stationary z_{i0t} s. As can be seen from the table, given $r = 2$, both Czech and Estonian real exchange rates have unit vectors in β , implying convergence, or remarkably, that purchasing power parity holds. These findings are in line with the results in Sections 4 and 5 for the model with no constant, in which the initial condition produced the highest power gain for the LR test, but no such gain for the Q test. However, when a constant is added, either restricted (Model 2) or unrestricted (Model 3), the power gain of the LR test gradually disappears, yielding $r = 1$ and $r = 0$, respectively. The Q test, on the other hand, suggests $r = 0$ irrespective of the deterministic terms. We note that the finding of $r = 1$ in Model 2 is incompatible with unit vectors in both Czech and Estonian real exchange rates, as is suggested by Table 5. However, if we impose $r = 1$ in Model 2, only the Estonian real exchange rate has a

Table 4: LR and Q tests for the transition economies with $\mathbf{X}_{1t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t})'$.

r	$\hat{\lambda}_{r+1}$	LR	95%	p -value	$\hat{\nu}_{r+1}$	Q	95%	p -value
Model 1: No deterministic terms								
0	0.231	53.94	24.21	0.00	0.983	-16.41	-32.60	0.494
1	0.085	16.59	12.28	0.01	0.950	-9.41	-18.47	0.304
2	0.027	3.94	4.07	0.05	0.950	-2.41	-8.00	0.286
Model 2: Restricted constant								
0	0.237	55.65	35.07	0.00	0.984	-15.64	-44.75	0.914
1	0.088	17.31	20.16	0.12	0.953	-8.94	-27.72	0.775
2	0.029	4.23	9.14	0.39	0.953	-2.23	-14.03	0.753
Model 3: Unrestricted constant								
0	0.094	27.05	29.80	0.10	0.984	-15.64	-44.75	0.914
1	0.070	12.97	15.41	0.12	0.953	-8.94	-27.72	0.775
2	0.019	2.68	3.84	0.10	0.953	-2.23	-14.03	0.753

reasonably large β coefficient, indicating that only this series converges.

6.2 The extended set of economies

Table 6 reports the LR and Q test for models with $\mathbf{X}_{2t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t}, z_{UK0t}, z_{DK0t})'$, and different deterministic terms. As before, in the model with no deterministic terms, $r = 0$ and $r = 1$ are rejected by the LR, but not by the Q test. However, the hypothesis $r = 1$ is now close to not being rejected with a p -value of 0.04. Moreover, the results in Table 5 (Model 4)

Table 5: Tests of unit vectors in β .

Model	r	z_{CZ0t}	z_{EE0t}	z_{PL0t}	z_{UK0t}	z_{DK0t}
1	2	0.07	0.06	0.00		
2	1	0.07	0.13	0.00		
4	1	0.00	0.00	0.00	0.00	0.00
4	2	0.00	0.00	0.00	0.00	0.00
5	1	0.00	0.00	0.00	0.00	0.00
6	1	0.00	0.00	0.00	0.00	0.00

suggest that there are no unit vectors in β regardless of the choices $r = 1$ or $r = 2$. This outcome excludes price convergence to the Euro Area level, but does not exclude the possibility of price convergence between other country pairs. However, we do not find any homogeneously cointegrated price pairs z_{ijt} (not reported), and consequently no evidence of such convergence. For the models with a constant (Models 5 and 6), the results of the LR test and Q tests unambiguously suggest $r = 1$ and $r = 0$ respectively. Also for these model, there are neither unit vectors in β , nor other homogeneously cointegrated price pairs. Hence, compared to the VAR system for the transition economies, the increase in dimension seems to have reduced the power gained by the LR test due to the initial condition.

7 Conclusions

We have derived the local powers of the LR and Q tests for cointegration rank when the initial condition is nonzero and does not vanish asymptotically.

Table 6: LR and Q tests for the extended set of economies $\mathbf{X}_{2t} = (z_{CZ0t}, z_{EE0t}, z_{PL0t}, z_{UK0t}, z_{DK0t})'$.

r	$\hat{\lambda}_{r+1}$	LR	95%	p -value	$\hat{\nu}_{r+1}$	Q	95%	p -value
Model 4: No deterministic terms								
0	0.351	102.80	97.78	0.00	0.990	-37.53	-72.45	0.816
1	0.169	41.35	31.18	0.04	0.969	-19.81	-50.96	0.877
2	0.063	15.13	7.39	0.45	0.950	-12.81	-32.60	0.688
3	0.029	5.88	3.43	0.45	0.950	-5.81	-18.47	0.554
4	0.012	1.70	1.26	0.22	0.874	-1.49	-8.00	0.397
Model 5: Restricted constant								
0	0.354	108.41	76.81	0.00	0.967	-41.39	-90.56	0.968
1	0.171	46.42	53.95	0.20	0.967	-29.42	-65.76	0.911
2	0.089	19.87	35.07	0.73	0.943	-17.45	-44.75	0.863
3	0.033	6.63	20.16	0.91	0.915	-9.38	-27.72	0.747
4	0.013	1.82	9.14	0.81	0.915	-4.69	-14.03	0.465
Model 6: Unrestricted constant								
0	0.272	81.58	69.61	0.00	0.967	-41.39	-90.56	0.968
1	0.124	36.47	47.71	0.38	0.967	-29.42	-65.76	0.911
2	0.089	17.68	29.80	0.60	0.943	-17.45	-44.75	0.863
3	0.026	4.44	15.41	0.86	0.915	-9.38	-27.72	0.747
4	0.005	0.73	3.84	0.39	0.915	-4.69	-14.03	0.465

Our main results are that the local power of the LR test is increasing in the magnitude of the initial condition, whereas the local power of the Q test is decreasing. If the dimension of the process is large, or if deterministic terms are fitted, the impact of the initial condition is mitigated. We investigate the behaviour of the tests in an application to price convergence. Our results help explain the outcomes of tests for price convergence of transition economies, where the initial condition is expected to be large. In particular, in models that exploit the power gain due to the initial condition, we establish price convergence, or that purchasing power parity holds.

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