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Hiding Loan Losses: How to Do It? How to Eliminate It?

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Abstract

This paper introduces three methods to hide loan losses and analyzes how they affect bank’s loan interest income, payments on deposits, liquidity and moral hazard. The analysis reveals that two hiding methods represent a Ponzi scheme. Contrary to classic theory, e.g. Diamond (1984), moral hazard may arise even though bank’s loan portfolio is diversified. Alternative instruments to eliminate hiding are investigated. Under specific circumstances, a Ponzi scheme may provide a socially optimal method to create liquidity and prevent the failure of a solvent but illiquid bank.

JEL Classification: G21, G28.

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1. Introduction

A bank’s opportunity to hide loan losses has played an important part in recent banking crises, most of all in emerging economies. This can be confirmed by abundant evidence. To begin, De Juan (1996, p. 91) describes the methods of hiding as follows:

“..when a loan – particularly a large loan – becomes questionable or bad because of the borrower’s lack of repayment capacity, the bank rolls over the loan so that it does become past due. Alternatively, the borrower may be given a new loan to repay the previous loan. The rolled-over loan does not become past due in the books and the new loan is not in arrears, but the actual debt is”

The hiding methods – a bank rolls over the defaulted loans or refinances them with subsequent loans - are so effective that a bank may seem to be greatly profitable even when it possesses a large burden of hidden problem loans and is de facto insolvent. These types of occurrences are documented by numerous researchers.

“Yet, during the previous banking crises, many of Latin America’s banking systems have reported positive net income to assets, whereas banking systems in industrial countries have reported significant negative net income to assets. Rojas-Suarez & Weissbrod (1996, p. 13).”

Improvements in accounting and transparency, a bank run, a banking crisis or an accurate bank audit finally reveals the bank’s true financial condition. In Baltic States, for instance, this is reported by Hansson & Tombak (1999, p.217)

“The ability of banks to roll over problem loans concealed their true solvency and created a false picture of health. Bank profits and thus net worth were overstated. When the hidden problems finally emerged, especially through improved accounting and auditing, the resulting erosion of profits and
capital was unexpected. When large, these changes could transform a seemingly solvent bank into an
apparently insolvent one.”

Since a bank is often capable of hiding loan losses for a long time, they accumulate with time and
the magnitude of the surfacing loan losses may eventually be massive. In Argentina, for example,
the ratio of non-performing loans to total loans was over 30% in 1986 and in Uruguay it was almost
46%. Even more dramatic, in Bulgaria, the ratio was over 60%. The ratios are much larger than in
the industrialized countries where transparency is relatively good and bank regulators close down or
recapitalize insolvent banks. In sharp contrast to emerging economies, during the savings and loan
crisis in the U.S.A, ratio of non-performing loans to total loans reached a peak value, 4.1%, in 1987
(Sheng, 1996a). For more evidence on hidden loan losses and banking crises see Kanya & Woo

Given the high frequency of recent banking crises, the crucial role of their hidden loan
losses and the complexity of the hiding methods, it is important to investigate hiding in detail. Few
researchers have investigated banking under hidden loan losses: e.g. Aghion & Bolton & Fries
With full agreement with the significance of their contributions, the hiding process is not explored
in detail, since they focus on optimal bailout policies. The paper aims to fulfil this gap in the
literature by investigating alternative methods to hide loan losses. Do banks’ loan interest income
and payments on deposits vary under alternative methods? What is the most profitable hiding
method? Does an opportunity to hide loan losses worsen the moral hazard problem? How can a
regulator eliminate this type of moral hazard? Do alternative methods of hiding require different
instruments from the regulator? The paper explores those two methods of hiding which are
documented in the literature: the rolling over method (a bank extends the maturity of a problem loan
and capitalizes unpaid interest in the loan) and the refinancing method (a bank grants a subsequent
loan to a borrower who cannot repay his original loan. The loan capital of the subsequent loan is
used to repay the original loan). In addition, the paper introduces the third method to hide loan losses: a *compensating balance method*. A bank grants an oversize loan to a borrower, who must maintain a part of it in his bank account (= a compensating balance) but the borrower can invest the rest of the loan capital in a project. Since the loan repayments are paid at the beginning from the compensating balance account, each borrower can then service his loan whether or not his investment project is successful. Therefore, at the beginning the bank bears no loan losses and it generates handsome profits. Finally, the true condition of the financed projects and of the bank surfaces.

The results indicate that the rolling over method is the least profitable alternative to hide loan losses. A defaulted loan incurs no losses to a bank (the lost loan capital is not deducted from the bank’s income and bank capital) but it yields no repayments either. The refinancing method proves to be as profitable as the compensating balance method; a defaulted loan incurs no losses to the bank and it yields repayments. Both methods represent a Ponzi scheme. If the share of defaulted loans is large, the bank may be illiquid under the rolling over method, since the defaulted loans yield no loan interest income. The illiquidity reveals the hidden loan losses to outsiders. The regulator may be able to mitigate the profitability of the rolling over method by forcing banks to diversify their lending or by excluding unpaid loan interest from retained earnings and thereby from bank capital. Yet, the regulator’s main instrument against hiding proves to be auditing.

Since the paper explores moral hazard under an opportunity to hide loan losses, it is related to rich research on moral hazard in banking: e.g. Merton (1977), Matutes & Vives (1996, 2000), Blum (1999, 2002), Chiesa (2001), Decamps & Rochet & Roger (2004), Freixas & Rochet & Parigi (2004) and Repullo (2004a). The paper differs from these articles because it investigates how hiding affects moral hazard. In addition, the paper extends fresh research on Ponzi schemes, e.g. Bhattacharya (2003) and Araujo & Pasoa & Torres-Martinez (2002).

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1 For an extensive survey regarding this literature, see Freixas & Rochet (1997).
The paper is organized as follows. Section 2 presents a model, whereas Section 3 is devoted to banking with monitoring. Banking without monitoring is examined in Section 4, while Section 5 characterizes diversification and Ponzi schemes. The compensating balance method is studied in Section 6, Section 7 analyses bank supervision and Section 8 concludes.

2. Economy

The model includes entrepreneurs (=borrowers), banks and a bank regulator. Everyone is risk neutral and the banking sector is fully competitive. The model has two periods: period-1 and period-2. Period-1 begins at time point 0 and ends at time point 1. Period-2 begins at time point 1 and ends at time point 2.

2.1 Project types

The total amount of entrepreneurs is 1 in both periods. At the beginning of period-1, each entrepreneur can undertake an investment project. When the project is started, its upcoming type is uncertain. The realized project type is learned during period-1 after the investment. Three alternative project types exist.

A fast project lasts for a period. If successful, it produces $Y$ units of output at the end of the period-1.

A slow project lasts for two periods. It produces no interim output at the end of period-1. If successful, it produces $Y_2$ at the end of period-2, $Y_2 > Y^2$. The liquidation value of the slow project is zero at the end of period-1.

A failed project has no value and the failure is irreversible.
If an entrepreneur exerts effort to his project, the project represents a good project variety and it succeeds with certainty. A good project becomes later either the fast project or the slow project. Without effort, a bad project comes true and the project succeeds with probability \( p \), \( 0 < p < 1 \), in each period and fails with probability \( 1 - p \). A bad project becomes later the fast project, the slow project or a failed project.

An investment project requires a unit of capital input. An entrepreneur now has capital of his own and he needs to seek for a bank loan. Since the upcoming project type is unknown when the bank grants a loan, the bank lends the funds for a period at the gross loan interest rate \( 1 + R_i \), \( R_i \in \{R_m, R_{nm}\} \). \( R_m = r + m \) denotes the breakeven loan interest rate with bank monitoring while \( R_{nm} = r \) represents the loan interest rate without monitoring. Here \( r \) is the interest rate of the economy, which is the cost of bank deposits and bank capital, whereas \( m \) denotes the non-monetary costs of monitoring. If the financed project proves to belong to the slow type, it yields no output at the end of period-1. Since the slow project has a small liquidation value, \( 0 \), but very large long-term output at the end of period-2, \( Y_2 \), it is optimal to reschedule the loan repayments. At the beginning of period-1, the bank commits to reschedule the original loan at the fixed loan interest rate, if the financed project proves to be slow. Two alternative methods for rescheduling exist.

**Rolling over**: The original loan is rolled over and its extent is \( 1 + R_i \) during period-2 because unpaid interest is capitalized in the loan. During period-2, the loan interest rate is again \( R_i \). If the project succeeds, the bank receives \( (1 + R_i)^2 \) at the end of period-2.

**Refinance**: At the end of period-1, the bank grants a new short-term loan (= a subsequent loan) to a borrower with a slow project. The extent of the subsequent loan is \( 1 + R_i \) and its interest rate is \( R_i \).
The borrower repays the original loan, $(1 + R_1)$, at the end of period-1 using the funds of the subsequent loan. The repayment of the subsequent loan, $(1 + R_1)^2$, takes place at the end of period-2.

More precisely, during period-1 an entrepreneur and his bank recognize the realized project type. The realized type is private information and unobservable to outsiders (even if the project output is assumed to be publicly observable). The bank reschedules the loan if the financed project proves to be slow. In this way, the interruption of the productive long-term project can be prevented.

A standard effort aversion problem is now constructed. A project has positive NPV only with effort. Yet, given the limited liability of debt finance and the non-monetary costs of effort, $e$, an entrepreneur will shirk effort exertion without bank monitoring. Effort aversion can be eliminated only through monitoring, which incurs non-monetary costs, $m$, to a bank. The effort aversion problem is detailed as follows.

**Assumption 1.** With effort, the NPV of the each upcoming project type is clearly positive.

\[ Y > 1 + r + e + m + \bar{A}, \quad ii.) Y_2 > Y^2. \]

The first inequality states that the NPV of the fast project is clearly positive with effort; the output covers the repayment of the principal and interest, $1 + r$, the costs of effort exertion, $e$, the costs of monitoring, $m$ and the cost of bank auditing. $\bar{A}$ denotes the highest auditing cost of the bank regulator and is defined later. The second inequality displays that the slow project is even more productive than the fast project and thus its NPV is positive.

**Assumption 2.** Without effort, the NPV of each project type is negative.
i.) \( pY < 1 + r \),  
ii.) \( pY_2 < 1 + r \).

According to the first inequality, the NPV of the fast project is negative without effort. The second equality expresses the same results for the slow project, when the project has matured for a period. This ensures that the NPV of the slow project is negative, when the project is started.

**Assumption 3. In the absence of monitoring, an entrepreneur shirks effort.**

i.) \( p[Y - (1 + r)] > Y - (1 + r) - e \),

ii.) \( p[Y_2 - (1 + r)] > Y_2 - (1 + r) - e \),

iii.) \( \alpha p[Y - (1 + r)] + (1 - \alpha)\bar{\delta}p^2[Y_2 - (1 + r)] > \alpha[Y - (1 + r)] - e + (1 - \alpha)\delta[Y_2 - (1 + r) - e] \).

The first inequality states that an entrepreneur shirks effort if he faces the fast project. The entrepreneur shirks effort also during the second period of the slow project. This is shown by the second inequality. As to the third inequality, it implies that the entrepreneur shirks effort at the beginning of the period-1, when the upcoming type of the project is still unknown, but it will be fast with probability \( \alpha \) and slow with probability \( 1 - \alpha \).

Consequently, the effort aversion problem appears and it can be eliminated only by monitoring borrowers. The task of monitoring is delegated to banks, which charge breakeven interest, \( R_m = r + m \), on loans. This does not, however, ensure that the banks monitor their borrowers. Given the limited liability of banks, they may neglect costly monitoring. A bank
monitors only if monitoring is at least as profitable as the non-monitoring strategy. This moral hazard problem is investigated in later sections.\(^2\)

2.2 Bank’s balance sheet

In period-1 the volume of new projects is 1. The bank finances the projects and funds its operations with the fixed amount of equity capital, \(E\), and deposits. As mentioned before, the interest rate of the economy, \(r\), represents the cost of capital and deposits.

With monitoring, a stochastic share \(s\) of financed projects proves to be slow. Here \(s\) has a support \([S, \tilde{S}]\), \(0 \leq S < \tilde{S} < 1\), continuous density \(g\), and distribution \(G\). The rest of the financed projects, \(1 - s\), are fast. Without monitoring, a stochastic share \(l\) of financed projects fails. Here \(l\) has a support \([L, \tilde{L}]\), \(0 < L < \tilde{L} < 1\), continuous density \(f\), and distribution \(F\). Regarding the rest of the assets, \(1 - l\), a stochastic share \(s\) of those will be slow and the rest \(1 - s\) will be fast. Thus, the volumes of financed projects are: \(l\) failed projects, \((1 - l)s\) slow projects and \((1 - l)(1 - s)\) fast projects.

Since a few projects are fast and mature at the end of period-1, the bank’s loan portfolio has room for fresh loans at the beginning of period-2. These funds are invested in fresh projects which are known to be fast and which mature at the end of period-2.

To clarify connections between symbols, it is useful to note that under shirking a project’s expected probability to success meets

\[
p = \int_{L}^{\tilde{L}} (1-l) f(l) \, dl .
\]  

\(^2\)The following values, for example, satisfy Assumptions 1-3: \(r = 0.02\), \(e = 0.08\), \(m = 0.03\), \(p = 0.75\), \(Y = 1.15\), \(Y_2 = 1.33\), \(\bar{A} = 0.01\), \(\alpha = 0.8\). Hence, it is known that \(R_m = 0.05\) .
Since an average project is unprofitable in the absence of monitoring, loans are on average also unprofitable without monitoring

$$\int\limits_{\bar{l}}^{\bar{l}} (1 - l)(1 + R_m) f(l) \, dl < 1 + r .$$

Recall that slow projects as well as failed projects yield no output at the end of period-1. In addition, the shares of both project types are stochastic. Furthermore, the loans that are granted for these projects can be rescheduled. Since the realized project type is unobservable to outsiders, they cannot know whether a bank reschedules a loan in order to delay the repayment of a slow project (which is socially valuable) or to hide a loan loss and thereby applying the non-monitoring strategy (which is socially harmful). In the bank’s loan portfolio, the share of rescheduling loans is smaller with monitoring, \(s\), than without monitoring, \(s + l(1 - s) > s\).

The bank regulator (= she) insures deposits and audits banks. She pre-commits to close down banks that neglect monitoring. A closed bank is liquidated and the liquidation proceeds are first and foremost utilized for payments on deposits. The remainder of the proceeds, if any, is paid to the banker. The regulatory instruments are used by the regulator in such a way that banks prefer the monitoring strategy to non-monitoring. Therefore, in equilibrium banks monitor.

The regulator cannot directly observe whether a bank monitors or not. Furthermore, at the end of period-1 loans cannot be liquidated, since it would interrupt slow projects. This is known by the banks that attempt to hide their loan losses by rescheduling the defaulted loans. The regulator aims to reveal hidden loan losses by auditing banks at the end of period-1. With probability \(h\), she succeeds in revealing a hiding attempt and closes down the bank. The quality of the auditing system can be chosen by the regulator. When a bank manages to hide its loan losses with probability \(h\),
this incurs costs $A(1-h)$ to the regulator.\(^3\) It is assumed that: $A(0) = 0,$
\[
A'(1-h) \geq 0, \quad A'(0) = 0, \quad A''(1-h) > 0, \quad A(1) = \bar{A}.
\]
Given Assumption 1, banking is profitable even under the highest quality of auditing, $\bar{A}$. The regulator finances the auditing system by taxing successful entrepreneurs but aims to minimize the costs of auditing subject to the bank’s incentive constraint.

Even when the regulator cannot uncover a hiding attempt, loan losses surface if their realized share is so large that the bank is illiquid. The bank is then closed down. In sum, the time line is the following.

1.1 The regulator imposes an equity capital requirement, $E$, to banks. Banks are established. Each bank maintains the same amount, $E$, of capital and attracts the amount $1-E$ of deposits.
1.2 The regulator chooses the quality of the auditing system. The choice is public information.
1.3 Banks grant loans and decide whether or not to monitor.
1.4 Without monitoring, some projects fail.
1.5 The end of period-1: fast projects mature and these loans are repaid. Banks attract deposits for period-2 and reschedule the loans that are allocated for slow projects. If a bank has neglected monitoring, it reschedules the defaulted loans. Banks fulfil the rest of their loan portfolio with fresh short-term loans for fast projects.
1.6 The regulator audits banks. If she observes loan losses, she closes down the bank.
1.7 Banks repay the deposits of period-1. If a bank is illiquid due to a large burden of loan losses, it cannot repay deposits, the hiding attempt surfaces and the regulator closes down the bank.

Liquid banks repay deposits and the dividends of period-1.

2.0 The entrepreneurs invest the loan capital in the fresh fast projects for period-2.
2.1 The bank decides whether or not to monitor during period-2.

\(^3\) Here $h < 1$ either because the regulator does not audit each bank of the auditing system is imperfect.
2.2 At the end of period-2, all loans mature and banks are closed down. They repay deposits and the banker receives the remaining returns.

Finally, few simplifying assumptions are made.

**Assumption 4.** If a bank chooses the non-monitoring strategy in period-1, it follows the non-monitoring strategy also during period-2.

Under some parameter values, the following strategy is possible. A bank neglects monitoring during period-1 and thereafter observes the realized share of loan losses. If the share of losses is small, the bank may optimally turn to the monitoring strategy during period-2. This kind of strategy is, however, unrealistic and thus we have rejected it by making Assumption 4.

**Assumption 5.** The fixed amount of equity capital satisfies: \(0 < E < L\).

Assumption 5 simplifies the analysis. Furthermore, under some parameter values, it is possible that the bank neglects monitoring in period-1 and checks the realized share of loan losses. Thereafter, the banker may optimally reveal loan losses to the regulator although the banker knows that the regulator closes down the bank. In practise, this strategy is highly unlikely, since liquidation rarely yields any returns to the bank’s owners. This unrealistic strategy is rejected with Assumption 5.\(^4\)

\(^4\) Liquidation at the end of period-1 can be made unprofitable in several ways. For example, it is possible to assume that the share of slow projects is always so large that liquidation is unprofitable. We have chosen Assumption 5.
**Assumption 6.** Under the non-monitoring strategy, the maximum share of rolled over loans or refinanced loans satisfies
\[
\left( L + (1 - L)S \right) (1 + R_m) \leq 1 \quad \text{and} \quad \left( 1 - \left( L + (1 - L)S \right) \right) R_m < (1 - E) r .
\]

Assumption 6 ensures that the loan portfolio has enough room to hide loan losses even when the bank does not grow. That is, the bank’s inability to grow does not reveal the hiding strategy. The second inequality makes it possible to explore the illiquidity of the bank. This option enriches the analysis.

**Assumption 7.** Under the rolling over method,
\[
\left( 1 - \left( L + (1 - L)S \right) \right) R_m > (1 - E) r .
\]

Assumption 7 states that the minimum share of rolled over loans – that is, the total share of loans that are channelled either for slow projects or for defaulted projects – is so small that a bank can hide its loan losses by rolling over these loans (the bank is not illiquid with certainty under the rolling over method). If Assumption 7 is not satisfied, the rolling over method cannot be used to hide loan losses but the refinance method can be used. This alternative is, of course, possible but not as interesting.

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5 By denying the growth of the bank, we deny a complex problem whether or not the banker will inject fresh capital in the bank with hidden loan losses at the beginning of period-2. This problem is already explored in Niinimaki (2007).

6 In addition to footnote 2, suppose that \( L = 0.05, \overline{L} = 0.45, S = 0, \overline{S} = 0.4, E = 0.01 \). Then, Assumptions 5-7 are also satisfied.
3. Monitoring bank

This section sheds light on the operations of a monitoring bank under both rescheduling methods. In Subsection 3.1 a bank reschedules the loans of slow projects by rolling over these loans. In Subsection 3.2 the bank refinances these loans. Since the bank has no hidden loan losses, audits by the regulator have no effect on banking.

3.1 Rolling over

Recall that with monitoring, a stochastic share $s$ of projects is slow, whereas the rest are fast. At the end of period-1, a monitoring bank rolls over the loans that are channelled for slow projects. Those $1 - s$ loans, which are allocated for fast projects, mature yielding loan repayments $(1 - s)(1 + R_m)$. The bank attracts fresh deposits, $1 - E$, for period-2 and pays back the deposits of period-1, $(1 + r)(1 - E)$. The banker’s earnings during period-1 – that is, the profit of the bank from which the cost of monitoring and the cost of injected bank capital are deducted – amounts to

$$\int_{1 - s}^{1} (1 - s)R_m - (1 - E)r \cdot g(s)ds - m - E(1 + r).$$

(3.1)

If $\hat{s}$ denotes the realized share of the rolled over loans, the banker’s earnings are $(1 - \hat{s})R_m - r - m - E$ or $\hat{s}R_m - E$. Here $\hat{s}R_m$ represents bank’s interest receivables from the rolled over loans. The interest receivables belong to the returns from period-1 although they are paid out from the bank at the end of period-2. Because the receivables are not paid out at the end of period-1, they
increase the retained earnings of the bank and thus raise its capital. For the same reason, the need for deposits for period-2 is \(1 - E - \bar{s}R_m\).

During period-2, the loan portfolio includes the rolled over loans for slow projects, \(s(1 + R_m)\). The rest of the loan portfolio, \(1 - s(1 + R_m)\), is reinvested at the beginning of period-2, since a share \(1 - s\) of loans matured at the end of period-1. Thus, the bank can grant \(1 - s(1 + R_m)\) loans for fresh, fast projects.

At the end of period-2, the loans mature. The loans for slow projects yield \(s(1 + R_m)^2\), while the loans for fresh fast projects yield \([1 - s(1 + R_m)](1 + R_m)\). The loan repayments total \(1 + R_m\). After payments on deposits, \((1 - E - sR_m)(1 + r)\), the banker’s earnings during period-2 amount to

\[
\int_{\frac{1}{2}}^{\frac{1}{2}} (1 + R_m - (1 - E - sR_m)(1 + r)g(s)ds - m.\tag{3.2}
\]

Given \(s = \bar{s}\), this simplifies to \(\bar{s}R_m(1 + r) + E(1 + r)\). The earnings are positive during period-2, but equal to the present value of the losses from period-1. The life-time earnings from the monitoring strategy are zero.

### 3.2 Refinance

Again, at the end of period-1 those \(1 - s\) borrowers who face a fast project can repay their loans in total. The borrowers, who encounter a slow project, have no funds for repayment and the bank grants a subsequent loan, \(1 + R_m\), to each of them. Immediately, the borrowers use the subsequent loans to repay their original loans. Since each borrower can repay his original loan, the bank obtains

\footnote{The amount of deposits is positive, thanks to Assumptions 5 and 6.}
income $1 + R_m$. It attracts deposits, $1 - E$, for period-2 and pays back the deposits of period-1, $(1 + r)(1 - E)$. Hence, the bank profits in period-1 amount to $R_m - (1 - E)r = m + Er$. If we deduct the cost of monitoring and the cost of bank capital, $(1 + r)E$, from $m + Er$, we obtain the banker’s returns $-E$.

During period-2 the loan portfolio consists of the subsequent loans that are allocated for slow projects, $s(1 + R_m)$, and of fresh loans, $1 - s(1 + R_m)$, that are channelled at the beginning of period-2 to fresh fast projects.

At the end of period-2, the fast projects mature, yielding loan repayments $\left[1 - s(1 + R_m)\right](1 + R_m)$, whereas the subsequent loans yield $s(1 + R_m)^2$. Therefore, the loan repayments total $1 + R_m$, which is spent to repay interest on deposits, $(1 + r)(1 - E)$. The bank enjoys profit $1 + R_m - (1 + r)(1 - E)$ or $m + (1 + r)E$. When the non-monetary costs of monitoring are subtracted from this, we obtain the banker’s profits during period-2, $E + (1 + r)E$. Under the lifetime of the bank, the NPV of the banker’s returns is $-E + \delta(1 + r)E = 0$.

### 3.3 Discussion I

It is interesting to note that even though the two methods of rescheduling loan repayments provide the same returns, 0, to the banker during the life-time of the bank, the banker’s returns differ between period-1 and period-2. Under the rolling over method the returns are $sR_m$ units smaller than under the refinance method during period-1, but $sR_m(1 + r)$ units larger during period-2.

Importantly, in subsection 3.1 it is implicitly assumed that under the rolling over method the bank can always pay interest on deposits after period-1, $(1 - \bar{S})R_m - (1 - E)r \geq 0$. Yet, it is possible that $(1 - \bar{S})R_m - (1 - E)r < 0$. Then, the bank is illiquid; its loan interest income does not
cover interest payments on deposits. This is true if the maximum share of slow projects (and thereby the maximum share of the rolled over loans), $\bar{S}$, is sufficiently high. The problem is avoided under the refinancing method. Then, each loan is repaid and the bank is liquid.

Under the rolling over method, the illiquidity effect depends crucially on whether or not the interest receivables can be incorporated into the regulator’s capital requirement, $E$. To see this, recall that the unpaid loan interest of the rolled over loans, $sR_m$, in all, represents bank’s interest receivables. Because the receivables cannot be paid out from the bank at the end of period-1, they boost the retained earnings of the bank and thus raise its capital. Recall the capital requirement, $E$. Given the retained earnings, the total amount of capital, $E + sR_m$, exceeds the requirement, $E$. If the retained earnings can be incorporated into the regulator’s capital requirement, the bank can release excessive capital, $\min[E, sR_m]$, at the end of period-1. If $\min[E, sR_m] = sR_m$, the bank’s funds consist of loan interest income, $(1-s)R_m$, and released capital, $sR_m$, $R_m$ in all, which cover interest payments on deposits, $(1-E)r$. During period-2 the capital amounts to $E$ which consists of retained earnings, $sR_m$, and remaining initially injected capital, $E - sR_m$. Hence, illiquidity is avoided and the bank obtains in both periods the very same returns as under the refinance method! If $\min[E, sR_m] = E$, the bank’s income totals $(1-s)R_m + E$, which can be smaller than $(1-E)r$. It is possible that the bank is illiquid and fails at the end of period-1. To avoid this, the bank can optimally follow the refinance method that avoids the problem of illiquidity. We will see later that the refinance method represents a Ponzi scheme. Consequently, it may be socially optimal to obey a Ponzi scheme and thus avoid the failure of the solvent bank due to temporary illiquidity. A summary follows.
**Proposition 1.** When a bank monitors borrowers and reschedules the loans that are channelled for the slow projects, both rescheduling methods yield equal returns to the bank during its life-time. If the retained earnings cannot be incorporated into the regulator’s capital requirement, the refinance method is more profitable during period-1 but the rolling over method is more profitable during period-2. If the retained earnings can be incorporated into the regulator’s capital requirement and if the amount of the rolled over loans is sufficiently small \( E > sR_m \), the methods yield equal returns in both periods.
4. In the absence of monitoring

This section investigates bank returns in the absence of monitoring when a bank hides its loan losses either by rolling over defaulted loans (subsection 4.1) or by refinancing failed projects (subsection 4.2). Thereafter the profitability of the methods is compared in subsection 4.3.

4.1 Rolling over

Consider a representative bank that neglects monitoring during period-1 and a stochastic share $l_1$ of loans default (subscript 1 stresses that the realized loan losses stem from period-1). The expected bank returns can be found out by aggregating returns under four situations.

With probability $h$ the bank manages to hide its loan losses and generates during period-1 expected returns

\[
\pi_{11}^{\text{re}} = \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2}}^{\frac{1}{2}} (1 - l_1)(1 - s)R_m - (1 - E)r f(l_1)dl_1 g(s)ds ,
\]

(4.1)

which can be paid out as dividends to the banker at the end of period-1. Here $(1 - l_1)(1 - s)R_m$ marks the total loan interest income from the fast projects. The bank rolls over $l_1$ defaulted loans and $(1 - l_1)s$ loans, which are allocated for slow projects. The outsiders do not observe whether the loans have been rolled over in order to finance slow projects or to hide loan losses. The second term $(1 - E)r$ indicates interest payments on deposits. Only if the loan interest income is adequate to pay interest on deposits, hiding is possible. In (4.1) $\bar{l}_1$ represents the highest possible share of loan losses which satisfies $(1 - \bar{l}_1)(1 - S)R_m \geq (1 - E)r$, where $\bar{l}_1 \in \left[L, \bar{L}\right]$ (recall that $E$ is fixed). Thus, $\bar{l}_1$ is the highest possible share of loan losses such so the bank is still liquid when the realized share
of slow projects is at the minimal level, $\underline{S}$. We may have $\hat{\bar{l}}_i = \overline{L}$. In addition, $\hat{S}_1(\hat{l}_i)$ denotes the highest realized share of slow projects so that the bank is liquid, when the realized share of defaulted loans is given, $\hat{l}_i$, 

$$
(1 - \hat{l}_i) \left[ 1 - \hat{\bar{s}}(\hat{l}_i) \right] R_m - (1 - E)r \geq 0 \ , \ \hat{s}_i \in [\underline{S}, \overline{S}].
$$

(4.2)

It may be possible that $\hat{s}_i = \underline{S}$, if $\hat{l}_i$ is sufficiently small. On the other hand, if the constraint (4.2) is not satisfied even when $\hat{s}_i = \underline{S}$ then $\hat{s}_i = \overline{S}$. The probability that the realized share of slow projects is so low that the bank is liquid, $\hat{s}_i \leq \hat{\bar{s}}_1$, is

$$
\Omega = \int_{\underline{S}}^{\hat{\bar{s}}_1} \int_{\underline{S}}^{\hat{s}_i} f(l) g(s) \, dl \, ds.
$$

(4.3)

The optimality of hiding compared with the revelation of loan losses is shown in Appendix A.

With probability $1 - h$ the audit reveals hidden loan losses, the regulator closes down the bank, liquidates it and repays the deposits. The banker receives the rest of the liquidation proceeds

$$
\pi_{12}^{sw} = \int_{\underline{S}}^{\hat{l}_{12}} \int_{\underline{S}}^{\hat{s}_i} (1 - l_1)(1 - s)(1 + R_m) - (1 - E)(1 + r) \ f(l_i) \, dl_i \ g(s) \, ds,
$$

(4.4)

where $\hat{l}_{12}$ is the highest share of loan losses which satisfies $(1 - \hat{l}_{12})(1 - \overline{S})(1 + R_m) \geq (1 - E)(1 + r)$, $\hat{l}_{12} \in [L, \overline{L}]$. If the constraint is not satisfied even when the realized share of loan losses is at the minimum level, $\hat{l}_i = L$, it is known that $\hat{l}_{12} = \overline{L}$. If the constraint is satisfied even when the realized
share of loan losses exceeds $\bar{L}$, that is $\hat{L}_i > \bar{L}$, then $\tilde{L}_{12} = \bar{L}$. In addition, $\bar{S}_{12}(\hat{L}_{12})$ is the highest share of slow projects that satisfies $(1 - \tilde{L}_{12})(1 - \bar{S}(\hat{L}_{12}))(1 + R_w) \geq (1 - E)(1 + r)$ when $\bar{S}(\hat{L}_{12}) \in [\underline{S}, \bar{S}]$ and $\hat{L}_{12}$ is given.

With probability $h[1 - \Omega]$ the audit does not reveal loan losses, but the true financial condition of the bank surfaces due to illiquidity. The burden of rolled over loans, which yield no loan interest payments at the end of period-1, is so large that the bank is illiquid; the realized loan interest income, $(1 - \hat{L}_i)(1 - \bar{s})R_w$, is insufficient for the interest on deposits, $(1 - E)r$. This reveals the hiding attempt and the bank is closed down. The banker receives the remainder of the liquidation proceeds

$$\text{Max}\left\{0, (1 - \hat{L}_i)(1 - \bar{s})(1 + R_w) - (1 - E)(1 + r)\right\},$$

where $\hat{L}_i$ and $\bar{s}$ are the realized shares of loan losses and slow projects. Some manipulation gives

$$\text{Max}\left\{0, (1 - \hat{L}_i)(1 - \bar{s})R_w - (1 - E)r\right\} + \left\[(1 - \hat{L}_i)(1 - \bar{s}) - (1 - E)\right\}.$$

Here the term in the first brackets is negative due to the illiquidity and the term in the second brackets is negative because the term in the first brackets is negative and $R_w > r$. Thus, it is known that $\text{Max}\{\ldots\} = 0$. When the loan losses surface due to illiquidity and the bank is closed down, the banker receives no returns.\(^8\)

---

\(^8\) De Juan (1996, p. 93) highlights the important signalling effect of illiquidity: “In the mid 1980s, Argentina suffered a very serious banking crisis that affected mostly new banks and banks run by new bankers. Some two to three hundred banks experienced interventions and/or were liquidated. Practically all were insolvent, but intervention was triggered by illiquidity. Only through illiquidity was the insolvency discovered.”
With probability $h\Omega$ the bank manages to hide loan losses during period-1 and reaches period-2. Then, the loan portfolio includes the rolled over loans. In addition, since a part of the loans is allocated for fast projects during period-1, these loans mature at the end of period-1 and the funds can be used to finance fresh, fast projects during period-2. At the end of period-2 the loans mature. The bank’s expected returns from period-2 are (the ex ante point of view, when the share of period-1 loan losses has not been realized)

$$
\pi^\text{re}_2 = \int \int \int_L \left[1 - (1 + R_n)l_2 \right] (1 + R_m)(1 - l_2) - (1 - E - \Delta R_m)(1 + r) \ f(l_1) dl_1 \ g(s) ds \ f(l_2) dl_2, \quad (4.7)
$$

where $\Delta = l_1 + (1 - l_1)s$ represents the total amount of the rolled over loans. Given Assumptions 5 and 6, it known that the amount of deposits is positive, $1 - E - \Delta R_m > 0$. In addition, $\hat{I}_2^\text{re}(\hat{l}_1)$ is the highest realized share of loan losses during period-2 so that the bank can repay deposits

$$
\left[1 - (1 + R_m)l_1 \right] (1 + R_m)(1 - \hat{I}_2^\text{re}) - (1 - E - \Delta R_m)(1 + r) \geq 0, \quad \hat{I}_2^\text{re} \in \{L, \bar{L}\}. \quad (4.8)
$$

If the bank makes a profit even when the realized share of loan losses is at the upper limit, $\hat{I}_2^\text{re} = \bar{L}$, we have $\hat{I}_2^\text{re} = \bar{L}$. On the other hand, if the bank cannot repay deposits even when the realized share of loan losses is at the lower limit, $\hat{I}_2^\text{re} = L$, we have $\hat{I}_2^\text{re} = L$ and $\pi^\text{re}_2 = 0$. In (4.8) the loan repayments, $\left[1 - (1 + R_m)l_1 \right] (1 + R_m)(1 - l_2)$, consist of two parts. The first part includes the loan repayments from the successful loans that are granted by the bank at the beginning of period-2 for
fresh fast projects, \([1 - \Delta (1 + R_m) ] (1 + R_m) (1 - l_z)\). The second part represents loan repayments from the loans that were channelled for slow projects, \((1 - l_z) s (1 + R_m)^2 (1 - l_z)\).

The banker’s total earnings from the non-monitoring strategy consist of expected bank returns from which the costs of injected bank equity are subtracted

\[
\Pi_{ro} (E; h) = h \pi_{11}^{ro} + (1 - h) \pi_{12}^{ro} + h \delta \pi_{2}^{ro} - (1 + r) E. 
\]  

(4.9)

We are implicitly assuming that \(\Pi_{ro} (E; 0) > 0\). The amount of bank capital is so small that the rolling over strategy yields a profit to the banker if the quality of the auditing system is zero.\(^{10}\)

---

\(^9\) Sheng (1996b, p. 151) documents how the rolling over method was used during the Chilean banking crisis at the beginning of 1980s. “Auditors for Banco Espanol qualified their report for 1979 by stating that 37\% of loans could not be evaluated because of lack of information on the debtors’ ability to pay – even through the loans had been rolled over repeatedly.”

\(^{10}\) Recall the numeric example in Footnotes 2 and 5. Under these values, it is known that \(\pi_{12}^{ro} = 0\). From (4.2) it is possible to observe that the maximum share of rolled over loans, \(\Delta\), such that a non-monitoring bank is liquid, is 0.604. Inserting this to (4.8) indicates that the payments on deposits are at least 0.978. Since the loan repayments are at most 0.95, the bank fails with certainty at the end of period-2, \(\pi_{2}^{ro} = 0\). The banker’s expected earnings from the non-monitoring strategy, (4.9), are \(\Pi_{ro} = h \pi_{11}^{ro} - (1 + r) E\). Let us extend the numeric example by assuming that both the share of defaulted loans and the share of slow projects have a continuous uniform distribution. It is possible to find out that \(\pi_{11}^{ro} = 0.012\). Hence, it is known that \(\Pi_{ro} = h \ast 0.012 - 0.0102\). If \(h = 1\), the non-monitoring strategy yields profits 0.0018 and it is the optimal to neglect monitoring. To make the non-monitoring strategy unprofitable, the regulator needs to invest in auditing so that \(h = 0.85\) or smaller.
4.3 Refinance

The bank neglects monitoring, learns the realized share of loan losses and the realized share of slow projects. Each fast project yields repayment, $1 + R_m$ units. The bank refinances the failed projects and slow projects by granting subsequent loans to these borrowers. The subsequent loans are used to repay the original ones. Consequently, the bank receives loan interest income $1 + R_m$. It attracts deposits for period-2, $1 - E$, and repays the deposits of period-1, $(1 - E)(1 + r)$. In period-1, the bank enjoys profit

$$\pi_{r1} = R_m - (1 - E)r,$$  \hspace{1cm} (4.10)

which can be paid out to the banker. This represents returns in the case that the hiding attempt is successful. Since the loan interest income exceeds the payments on deposits, $R_m > (1 - E)r$, loan losses never surface due to the illiquidity, as with the rolling over method.

With probability $1 - h$, the audit reveals hidden loan losses and the bank is closed. The returns are the same as with the rolling over method, $\pi_{ro}^r$.

With probability $h$, the bank manages to hide and keeps on operating during period-2. Then, the loan portfolio includes $l_f(1 + R_m)$ subsequent loans, which are used to hide loan losses and $(1 - l_f)s(1 + R_m)$ subsequent loans that are allocated for slow projects. The rest of the loan portfolio, $1 - l_f(1 + R_m) - (1 - l_f)s(1 + R_m)$, consists of short-term loans to finance fresh, fast projects.

At the end of period-2, each project and loan matures. The slow projects yield loan interest payments $s(1 - l_s)(1 + R_m)^2(1 - l_z)$. The loans for fresh, fast projects yield repayments,

$$[1 - l_f(1 + R_m) - s(1 - l_f)(1 + R_m)](1 - l_z)(1 + R_m).$$

Given these, the loan interest income
totals \[1 - l_1 (1 + R_m) \big( 1 - l_2 (1 + R_m) \big) \big( 1 + R_m \big) \], whereas payments on deposits amount to \((1 - E)(1 + r)\). The bank generates expected profits

\[
\pi^{Re}_2 = \int \int \int \left[ 1 - (1 + R_m) l_1 \right] \left[ 1 + R_m \right] \left[ 1 - (1 - E)(1 + r) \right] f(l_1) dl_1 f(l_2) dl_2 ,
\]

(4.11)

where \( \bar{I}^{Re}_2 \), \( \bar{I}^{Re}_2 \in [L, \bar{L}] \), is the highest share of loan losses during period-2 which satisfies

\[
\left[ 1 - (1 + R_m) \hat{l}_1 \right] \left[ 1 + R_m \right] \left[ 1 - \bar{l}^{Re}_2 \right] - (1 - E)(1 + r) \geq 0 .
\]

(4.12)

Here \( \bar{l}^{Re}_2 = \bar{L} \) if the bank can repay deposits with each realized share of loan losses during period-2. In addition, \( \bar{l}^{Re}_2 = L \), if the realized share of loan losses is always so high that the bank cannot pay back deposits. The banker’s expected returns during its life time total\(^{11}\)

\[
\Pi^{Re} = h \pi^{Re}_1 + (1 - h) \pi^{Ro}_{12} + \delta h \pi^{Re}_2 - (1 + r) E .
\]

(4.13)

\(^{11}\) Recall the numeric example in Footnotes 2, 6 and 10. Under these values, it is known that \( \pi^{Ro}_{12} = 0 \). During period-2, the payments on deposits exceed 1, whereas the loan repayments are less than 0.95. Hence, the bank fails with certainty and \( \pi^{Ro}_2 = 0 \). Under the refinancing method, the banker’s expected earnings from the non-monitoring strategy simplify to

\[
\Pi^{Re} = h \pi^{Re}_1 - (1 + r) E \quad \text{or} \quad h \cdot 0.03 - 0.01 .
\]

Suppose that \( h = 1 \). It is easy to observe that the refinancing method is much more profitable than the rolling over method, 0.02 > 0.0018. To make the refinancing method unprofitable, the regulator needs to invest in auditing so that \( h \) is at most 1/3.
4.3 Discussion II

This subsection explores which method, the rolling over method or the refinance method, is more profitable to a non-monitoring bank. Two cases are examined depending on whether or not interest receivables are allowed to be incorporated in the required amount of bank capital. In the first scenario, this is not possible, but in the second, it is.

By deducting the bank returns under the rolling over method from the bank returns with refinance, we obtain difference, $\Pi^{\text{re}} - \Pi^{\text{ro}}$, or

$$\begin{align*}
&\int h_{\frac{7}{7}} \left( R_m - (1-E)r f(l_s) dl_s g(s)ds + \int h_{\frac{7}{7}} \Delta R_m f(l_s) dl_s g(s)ds + h \delta \left[ \pi^{\text{re}}_2 - \pi^{\text{ro}}_2 \right] \right).
\end{align*}$$

(4.14)

Here $\Delta = l_s + (1-l_s)s$ represents the total amount of rolled over loans. The difference consists of three terms in such a way that the first term and the second term describe returns in period-1. The first, positive term indicates returns from refinancing when the realized share of rolled over loans is so large that the rolling over method is unprofitable due to the illiquidity. Since the bank is always liquid under the refinance method, this method yields profits. The second, positive term expresses the difference in returns when both methods yield profits. The realized share of the rolled over loans is so small that the bank is liquid also under the rolling over method. Yet, under the refinance method each loan yields the loan interest income, $R_m$, whereas under the rolling over method only a share $(1-l_s)(1-s)$ of loans yield interest income. Obviously, the refinancing method is more profitable. The third term in (4.14) shows the difference in expected returns during period-2 and can be detailed as (recall (4.7) and (4.11))
Both methods yield an equal loan interest income but the payments on deposits are smaller under the rolling over method, since unpaid loan interest is capitalized in the loan size. This raises bank capital and thereby reduces the need for deposits. The effect makes the rolling over method more profitable than the refinance method during period-2. Yet, the probability that the bank achieves period-2 is smaller under the rolling over method because illiquidity may reveal hidden loan losses at the end of period-1. As a result, we do not know which method yields higher expected returns from period-2. Appendix B shows the difference in expected returns during period-2,

\[ h \delta \left[ \pi_{2}^{Re} - \pi_{2}^{Ro} \right] , \text{is larger than} \]

\[ -h \int_{\frac{1}{2}}^{1} \int_{\frac{1}{2}}^{1} \Delta R_{m} f(l_1)g(s) \, ds \, dl_2 . \]  

(4.16)

Hence, it is known that (4.14) is positive; the refinance method is more profitable than the rolling over method. More precisely, the refinance method is more profitable than the rolling over method during period-1 while during period-2 the reverse is true. The effect of period-1, however, dominates for two reasons. To begin, under the rolling over method the bank achieves period-2 only if it is liquid during period-1. Thus, the bank does not achieve “the relatively high returns of period-2” with certainty. In addition, during period-2 the hidden loan losses surface. The burden of loan losses is likely to be so large that the bank fails whether it has rolled over the defaulted loans or refinanced them. The fact that the bank returns are larger under the rolling over method is
meaningless when both methods yield negative returns and the banker is protected by limited liability. A conclusion can be made.

**Proposition 2.** When interest receivables cannot be incorporated into the regulator’s capital requirement, the refinancing method is more profitable for a non-monitoring bank than the rolling over method.

Consider now that the regulator imposes a capital requirement for banks and that interest receivables, which belong to retained earnings, can be incorporated into the required capital. Thus, after period-1, bank capital totals \( E + \Delta R_m \). The bank can lower the amount of capital by releasing excessive capital by \( \text{Min}[E, \Delta R_m] \). This creates two cases.

If \( \text{Min}[E, \Delta R_m] = \Delta R_m \), it is possible to lower capital back to the required level, \( E \).

During period-2 bank capital, \( E \), consists of retained earnings, \( \Delta R_m \), and the remaining, initially injected capital, \( E - \Delta R_m \). Thus, at the end of period-1 the bank can release \( \Delta R_m \) units of initially injected capital and spend these funds to cover interest payments on deposits. These funds, \( \Delta R_m \), and the loan interest income, \((1 - s)(1 - \ell)R_m \) or \( R - \Delta R_m \), together amount to \( R_m \), which covers interest payments on deposits, \((1 - E)r \). In period-1, the bank generates returns \( R_m - (1 - E)r \) and in period-2 it makes returns \( \pi^{\text{Re}}_2 \). In both periods the returns are the same as under the refinance method!

If \( \text{Min}[E, \Delta R_m] = E \), the bank can drop capital to \( \Delta R_m > E \) for period-2. Then, bank capital consists entirely of the retained earnings, \( \Delta R_m \), and it exceeds \( E \). The bank can release the initially injected capital in total and spend these funds to cover interest payments on deposits at the end of period-1. These released funds, \( E \), and the loan interest income, \( R_m - \Delta R_m \), together amount
to $R_n - (\Delta R_n - E)$. Since $\Delta R_n > E$, the income is lower during period-1 than under the refinance method. Hence, during period-1, the refinance method yields larger returns. During period-2, the rolling over method is more profitable thanks to the larger amount of bank capital, $\Delta R_n > E$, and thus smaller payments on deposits. Appendix C shows that even when $\Delta R_n - E > 0$ is minimal, the refinance method is more profitable than the rolling over method during the lifetime of the bank. The intuition is obvious. Since the bank goes into bankruptcy with a positive probability during period-2, the expected, relatively high returns from the rolling over method during period-2 are insufficient to cover its relative losses during period-1.

**Proposition 3.** When interest receivables can be incorporated into the regulator’s capital requirement, the profitability of the methods depends on the realized amount of rolled over loans, $\Delta$. If $\Delta R_n < E$, the refinance method and the rolling over method yield equal returns for a non-monitoring bank. If $\Delta R_n > E$, it is more profitable to refinance than to roll over loans.

Given Proposition 2 and Proposition 3, the refinancing method is always at least as profitable as the rolling over method.

Obviously, the regulator should not allow banks to incorporate interest receivables into the required bank capital, since this option increases the returns from hiding under the rolling over method. The regulator cannot be sure whether or not the interest receivables are based on performing loans (slow projects) or on defaulted loans. Suppose that the bank neglects monitoring, rolls over the defaulted loans and thus obtains interest receivables. If the bank can incorporate the interest receivables into the required bank capital, the receivables raise the capital and the bank can pay out excessive capital at the end of period-1. This increases the expected returns from the non-monitoring strategy. At the end of period-2, the true financial condition of the bank surfaces; a large share of interest receivables proves to be worthless and the bank is likely to be insolvent.
5. Diversification and Ponzi

In his seminal article, Diamond (1984) utilizes the weak law of large numbers as well as an assumption on independent and identically distributed project returns to demonstrate how perfect diversification within the bank eliminates moral hazard. As the number of financed projects multiplies without bound, the risk of project returns is eliminated through diversification. Thus, the bank’s income is fixed and it cannot gamble with deposits.\(^\text{12}\)

In our model, suppose first that a bank has no equity capital and it cannot hide loan losses. In addition, the bank neglects monitoring and the realized share of loan losses is at the expected level

\[
p = \int_{\frac{1}{2}}^{1} (1-l) f(l) \, dl . \tag{5.1}
\]

At the end of period-1, the regulator observes loan losses, closes down the bank and liquidates it. Given (2.2), the bank cannot repay deposits even if the liquidation value of the slow projects was one

\[
p(1 + R_m) - (1 + r) = \int_{\frac{1}{2}}^{1} (1-l)(1+R_m) f(l) \, dl - (1 + r) < 0. \tag{5.2}
\]

Since the liquidation value of slow projects is, however, zero, the non-monitoring strategy is even more unprofitable (here the realized share of slow projects is at the minimum level)

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\(^\text{12}\) Diamond (1984) analyzes ex post monitoring, whereas this paper investigates interim monitoring.
Therefore, the moral hazard problem is eliminated when loan losses are observable. Let us again assume that loan losses can be hidden. The bank rolls over the defaulted loans and manages to hide the loan losses. At the end of period-1, the successful fast projects yield loan interest income, $p(1 - \bar{s})R_m$, to the bank that pays interest $r$ on deposits and achieves returns

$$p(1 - \bar{s})R_m - r,$$  \hspace{1cm} (5.4)

which can be rewritten as

$$\left[ p(1 - \bar{s})(1 + R_m) - (1 + r) \right] + \left[ 1 - p + p\bar{s} \right].$$  \hspace{1cm} (5.5)

Given (5.3), the term in the first brackets, which expresses bank returns without hiding, is negative. The term in the second brackets is positive. It indicates the extra returns that a bank can achieve by hiding its loan losses. If the second term is small, bank returns (5.5), are negative and the non-monitoring strategy is unprofitable even with hiding. More precisely, we have $p(1 - \bar{s})R_m < r$; the bank is illiquid. When the second term in (5.5) is sufficiently large, the bank returns (5.5), are positive. The bank’s chance to hide its loan losses by rolling over the defaulted loans makes the non-monitoring strategy profitable although the loan portfolio is strongly diversified; that is, the realized share of loan losses is at the average level. Consequently, the chance to hide loan losses extends the magnitude of the moral hazard problem.

The positive incentive effect of diversification is based on the principle that loan losses are deducted from the repayments of successful loans and bank capital. The subtracted volume is so large under perfect diversification that the non-monitoring strategy is unprofitable.
This effect eliminates moral hazard also in our setting when loan losses are observable, (5.3). The effect fades when the bank can hide its loan losses by rolling over defaulted loans. Since no loan losses officially exist, no losses are subtracted. The extra benefit is represented by $1 - p$ in the second brackets of (5.5). The existence of slow projects also mitigates the problem of moral hazard when loan losses are observable (see (5.3)). Since these loans have low liquidation value, their existence decreases the bank returns when the auditor observes hidden loan losses at the end of period-1 and closes down the bank. This effect is thus avoided when the bank can hide its loan losses ($pS$ in the second brackets of (5.5)).

Although the bank can dampen the effects of diversification by rolling over defaulted loans, diversification still influences the bank returns, because the defaulted loans yield no interest income. The larger the share of defaulted loans, the smaller the bank returns are. As a result, improved diversification may make the rolling over method unprofitable. Consider two distributions. The support of the first distribution is $[L_1, \bar{L}_1]$ and the second support is $[L_2, \bar{L}_2]$ so that $L_1 < L_2 < \bar{L}_2 < \bar{L}_1$. It is possible that

$$(1 - L_1)(1 - S)R_w > r, \quad (1 - L_2)(1 - S)R_w < r.$$  (5.6)

The bank can hide loan losses only if its loan interest income is based on distribution $[L_1, \bar{L}_1]$, which is relatively less diversified than $[L_2, \bar{L}_2]$. Under distribution $[L_2, \bar{L}_2]$, a hiding attempt surfaces with certainty due to illiquidity. Thus, the regulator may optimally force banks to diversify their lending in order to eliminate moral hazard. This positive effect of diversification exists, however, only when the bank uses the rolling over method in hiding. If the bank adopts the refinancing method, each loan is repaid at the end of period-1 and diversification has no effect.
More precisely, the chance to hide loan losses increases bank returns in two ways compared with the returns without hiding.

i.) Bank returns increase because the lost principals of the defaulted loans are not deducted from the bank’s loan interest income and bank capital at the end of period-1.

ii.) Bank returns increase because the defaulted loans also yield loan interest income, $R_m$.

Under the rolling over method, only the first effect increases bank returns while under the refinance method, both effects increase them. Interestingly, the refinancing method represents a Ponzi scheme. Kane (1989, p. 17) gives the following definition to a Ponzi scheme:

“In a Ponzi scheme, a fund-raising enterprise operates with little or none of the earning assets that a sound enterprise requires to generate a projected stream of cash flows with which to service lenders and investors. Instead, the enterprise relies on expanding its liabilities faster than its interest and dividend payments expand. The enterprise pays interest or dividends each period to its old clients— not from earnings but from funds that are provided by new lenders and investors. As long as new funds can be attracted into the scheme fast enough, the enterprise’s managers can get corporate obligations as they come due and pay themselves handsomely at the same time.”

Consider the refinance method. A borrower with a failed project cannot repay his original loan. The bank grants a subsequent loan to him and he uses the loan capital to repay the original loan. Hence, the bank’s loan interest income from the original loan, $R_m$, is paid using the capital of the subsequent loan, which is funded with deposits. Hence, the bank uses, de facto, deposits to pay interest on its original loans. The loan repayments are not based on project output but on the funds that are provided by depositors. Since each loan is repaid, the bank generates high loan interest income, can pay interest on deposits and pay out dividends. This can be summarized as follows.
**Proposition 4.** Even when diversification eliminates moral hazard under symmetric information, it does not eliminate moral hazard with certainty when banks can hide their loan losses.

Diversification can make hiding unprofitable, if the bank hides the loan losses by rolling over these loans, but diversification has no effect on bank returns if the bank hides the loan losses by refinancing these loans. Under the refinance method, a bank is running a Ponzi scheme.

Recall from Section 3 that refinancing – and thus a Ponzi scheme – may be socially optimal. Even with monitoring, the share of slow projects may be so large that the loan interest income does not cover interest payments on deposits. The bank is illiquid in the short run (during period-1) but solvent in the long run (during period-2) and it goes into a bankruptcy due to illiquidity. If the bank is allowed to refinance the slow projects of its borrowers, it can pay interest on deposit and it avoids bankruptcy. As the matter of fact, the bank is creating liquidity through refinancing because it can issue liquid demand deposits and pay interest on them even though a major share of its funds is tied in loans that yield no loan interest income during period-1.
6. Compensating balances

Until now we have investigated two methods to hide loan losses: the refinance method and the rolling over method. This section introduces the third method of hiding: compensating balances. Mishkin (2004, p. 219) defines compensating balances as follows:

“A firm receiving a loan must keep a required minimum amount of funds in a checking account at the bank. For example, a business getting a $10 million loan may be required to keep compensating balances of at least $1 million in its checking account at the bank. This $1 million in compensating balances can then be taken by the bank to make up some of the losses on the loan if the borrower defaults. Besides serving as collateral, compensating balances help increase the likelihood that a loan will be paid off.”

A bank constructs the following arrangement. It grants loans at the beginning of period-1. Each loan consists of two parts. The size of the primary loan is 1 unit, and the bank charges interest $R_m$ on it. The size of the secondary loan is $R_m$. For as long as an entrepreneur keeps the secondary loan in his checking account at the bank, the loan interest rate is zero, since the bank can reinvest these funds in government bonds at the risk free interest rate of the economy.\(^{13}\) When the entrepreneur withdraws the secondary loan from the bank account, the interest rate is $R_m$. At the end of period-1, each borrower uses his compensating balance to repay the interest of the primary loan, $R_m$. Hence, borrowers have no funds in their compensating balance accounts after period-1.

An entrepreneur, who encounters a slow project, cannot repay his loan in all at the end of period-1. Thus, these loans are rolled over. Since each borrower can now pay loan interest, $R_m$, at the end of period-1, the rolling over method is different than in Section 3.

\(^{13}\) Since the funds are invested in risk-free government bonds, which have 0% risk weighting in Basel II, the secondary loans create no capital requirement.
Rolling over 2. The bank can roll over the primary loans and the secondary loans at the end of period-1. Since the loan interest of period-1 is now paid by each borrower, it is not capitalized in the loan size. During period-2 borrowers pay interest $R_m$ on their loans. That is, at the end of period-2, the repayment of the primary loan is $1 + R_m$ and the repayment of the additional loan is $R_m(1 + R_m)$, $(1 + R_m)^2$ in all.

In the next subsections we first explore banking with monitoring. Thereafter, banking without monitoring is examined.

6.1 A monitoring bank

During period-1 an entrepreneur learns the realized type of his project: slow or fast. In both cases, he uses his compensating balance to pay the interest of the primary loan, $R_m$, at the end of period-1. Since the secondary loan is kept in the checking account during period-1, its loan interest rate is zero. An entrepreneur, who has faced a fast project, spends the project output, $Y$, to repay the principal of the primary loan and the secondary loan, $1 + R_m$ in all. Thereafter, he has got out of his debts. An entrepreneur, who has faced a slow project, pays the loan interest of period-1, $R_m$, using his compensating balances. Thereafter, the loans are rolled over. Thus, the entrepreneur still owes the principal of the primary loan and the secondary loan. Since each loan yields the loan interest income to the bank and since the interest payments on deposits amount $(1 - E)r$, the bank generates in period-1 returns

$$R_m - (1 - E)r.$$ (6.1)
During period-2, the loan portfolio includes \( s(1 + R_m) \) rolled over loans (the primary loans and the secondary loans). The rest of the loan portfolio, \( 1 - s(1 + R_m) \), is used for fresh, fast projects.

At the end of period-2, the slow projects mature. A borrower repays the primary loan, \( 1 + R_m \) units (the principal and the loan interest of period-2), and the secondary loan, \( R_m(1 + R_m) \) units (the principal and the loan interest of period-2), \( (1 + R_m)^2 \) in all. The repayments from slow projects add up to \( s(1 + R_m)^2 \). Fresh loans, which were granted at the beginning of period-2, also mature yielding \( [1 - s(1 + R_m)](1 + R_m) \). The loan interest income totals \( 1 + R_m \), while payments on deposits amount to \( (1 - E)(1 + r) \). Hence, in period-2 the bank generates returns

\[
R_m - (1 - E)r + E .
\]  

(6.2)

Given the cost of monitoring and the costs of injected equity capital, the banker’s profit is zero during the life-time of the bank.

### 6.2 Non-monitoring bank

During period-1 an entrepreneur learns the type of his project: slow, fast or defaulted. Independent of the realized project type, he uses his compensating balance to pay interest on the loan. An entrepreneur who has faced a fast project also repays his primary loan and the secondary loan in all. An entrepreneur who has encountered a slow project or a defaulted project lets the bank to roll over both loans. Since each borrower is able to repay the loan interest at the end of period-1, the loan interest income amounts to \( R_m \). Given interest on deposits, \( (1 - E)r \), bank returns during period-1 are \( R_m - (1 - E)r \).
During period-2, the loan portfolio includes both fresh short-term loans and rolled over loans. The amount of rolled over primary loans is \( l_1 + (1 - l_1)s \) while the rolled over secondary loans amount to \( \left[ l_1 + (1 - l_1)s \right] R_m \), \( \left[ l_1 + (1 - l_1)s \right] (1 + R_m) \) in all. The rest of the loan portfolio, \( 1 - \left[ l_1 + (1 - l_1)s \right] (1 + R_m) \), which is positive, is filled with fresh, short-term loans for fast projects.

After period-2, the loans mature. The rolled over primary and secondary loans yield \((1 - l_1)s(1 + R_m)^2(1 - l_2)\). The fast projects yield \([1 - \left[ l_1 + (1 - l_1)s \right] (1 + R_m)](1 - l_2)(1 + R_m)\). Hence, the loan repayments total \([1 - l_1(1 + R_m)](1 - l_2)(1 + R_m)\), while payments on deposits amount to \((1 - E)(1 + r)\). Therefore, the bank generates during period-2 expected returns

\[
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[ 1 - (1 + R_m)l_1 \right] (1 + R_m)(1 - l_2) - (1 - E)(1 + r) \int l_1 dL_1 \int l_2 dL_2 . \tag{6.3}
\]

A conclusion follows.

**Proposition 5.** Under the compensating balance method, the bank generates in both periods the very same returns than under the refinancing method independent of its decision to monitor or neglect monitoring.

Consequently, the magnitude of the moral hazard problem is the same under both methods. The bank is again running a Ponzi scheme. The loan interest payments of period-1 are paid using the funds of the compensating balance accounts. These accounts are funded with the secondary loans, which are funded with deposits. Thus, the loan interest payments of the failed projects and slow projects are funded with deposits. The bank is, de facto, using its own deposits to pay loan interest.
to itself. In addition, since the bank rolls over the defaulted loans, the lost principal is not subtracted from the bank’s income or its capital.

Why is the compensating balance method worth studying? It may be argued that a hiding attempt can be easily observed by auditing the growth of borrowing; loans to an insolvent borrower accumulate period after period. This is true under the rolling over method and under the refinance method. Yet, it is false under the compensating balance method. During period-1, the bank lends \( 1 + R_w \) units to each borrower. During period-2 a borrower with a failed project has the same amount of loans. Consequently, the regulator cannot reveal hidden loan losses by focusing on borrowers with accumulating amounts of debt.

7. Incentive compatible quality of auditing

The regulator earns zero returns. Since the non-monitoring strategy is unproductive, the regulator operates so that banks optimally monitor.

Given the fixed bank capital, the regulator chooses such a quality of auditing that the banks are indifferent between the monitoring and non-monitoring strategies. Since the monitoring strategy yields zero returns, the bank optimally monitors if the expected returns from the non-monitoring strategy are zero or negative. The expected returns from the non-monitoring strategy are highest under the refinance method and under the compensating balance method. It is necessary to choose such a quality of auditing that the expected bank returns are zero under these methods

\[
h \pi_1^{Re} + (1 - h) \pi_1^{Re} + \partial h \pi_2^{Re} - (1 + r)E \leq 0,
\]

or
The term in the first brackets is larger than the term in the latter brackets. The amount of capital must be so large that the term in the second brackets is negative. Thereafter, it is easy to see that when $h$ is small enough, the bank optimally monitors. Since the good projects are profitable under the maximum quality (and maximum cost) of auditing, $\bar{A}$, there exist always such a quality of auditing, $A^* < \bar{A}$, that makes the non-monitoring strategy unprofitable. Thus, it is possible to finance the auditing system by taxing firms.

8. Conclusions

This paper challenges the classic arguments, which are initially based on Diamond (1984), that perfect diversification of loan returns eliminates the moral hazard problem in banking. The paper shows that when a loan is repaid gradually over several periods and when a bank can hide its loan losses, perfect diversification does not eliminate moral hazard. Consequently, the paper is very sceptical as to the depositors’ ability to control banks. Even when a bank seems to enjoy handsome profits and its balance sheet seems to be in an excellent financial condition, the bank may be de facto insolvent and operating under a large burden of hidden loan losses.

This paper illustrates three methods of hiding. A bank can hide its loan losses by rolling over these loans, by refinancing them with subsequent loans that are spent to repay the defaulted original ones, or by utilizing compensating balances. The profitability of the rolling over method is smaller (or equal) than the profitability of the other methods. Its profitability increases if loan interest receivables can be in incorporated in capital requirement, and decreases, if the regulator can force banks to diversify their lending in such a way that a non-monitoring bank is
always illiquid. The compensating balance method is as profitable as the refinancing method. Both methods represent a Ponzi scheme. The paper fails to explain why banks so commonly adopt the rolling over method even though it is not the most profitable one.

How can bank regulators eliminate hiding? A key problem is that it may sometimes be socially optimal to reschedule loan repayments when a financed project is profitable in a long run, but cannot yield loan repayments in the short run. Section 3 indicates that a bank may sometimes optimally run a Ponzi scheme. Without it the bank may fail due to illiquidity although it is solvent in the long run. It is difficult to evaluate how common this type of problem is in reality.

In the model framework, hiding can be mitigated simply by reducing the dividend payouts of a bank after period-1 if the bank possesses numerous of rescheduled loans. This instrument is, however, a bit unrealistic. In practise, banks have long-term lending relationships, which include sequential loans that are based on the stochastic needs of borrowers. It is not possible to reduce the dividend payouts of each bank with long-term lending relationships.

Careful bank auditing may provide the one and only effective instrument against hidden loan losses. The regulators need to audit not only the financial condition of a bank but also the financial condition of its borrowers. This is a demanding task.
Appendix A

Appendix A determines that a bank rather intends to hide its loan losses by rolling over these loans than reveal them to the regulator. When the bank makes this decision, it already knows the realized shares of loan losses and slow projects in period-1. By hiding loan losses the bank makes profits

\[(1 - \hat{l}_t)(1 - \hat{s})R_m - (1 - E)r, \]  

(A.1)
in period-1 and expected profit

\[\int_{\hat{l}_t}^{1} \left[1 - (1 + R_m)^{\hat{l}_t}(1 - l_z) - \left[1 - E - \hat{l}_tR_m - (1 - \hat{l}_t)\hat{s}R_m \right](1 + r) \right] f(l) \, dl, \]  

(A.2)
in period-2. If the bank reveals its loan losses to the regulator, it earns

\[Max \left[0, (1 - \hat{l}_t)(1 - \hat{s})(1 + R_m) - (1 - E)(1 + r) \right]. \]  

(A.3)

Next we subtract (A.1) from (A.3). The difference is at most

\[(1 - \hat{l}_t)(1 - \hat{s}) - (1 - E). \]  

(A.4)

If this is negative, we know with certainty that hiding is profitable. This is true if

\[L \text{ or } S \text{ is large enough (since } L \leq \hat{l}_t, S \leq \hat{s}). \]  

In this model the profitability of hiding is based on Assumption 5, which ensures that (A.4) is negative. Q.E.D.
Appendix B

Appendix B shows the difference in expected returns in period-2, \( h \delta \left[ \pi^R_{2} - \pi^{Ro}_{2} \right] \), is larger than

\[
- h \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2}}^{\frac{1}{2}} \Delta R_m \ f(l_1) dl_1 \ g(s) ds . \tag{B.1}
\]

Now \( \bar{T}^R_{2} \leq \bar{T}^{Ro}_{2} \) since the rolling over method is more profitable in period-2. We can rewrite (4.15) as

\[
\begin{align*}
&\int_{\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2}}^{\frac{1}{2}} \left[ 1 - (1 + R_m)l_1 \right] \left[ 1 + R_m \right] \left( 1 - l_2 \right) - \left( 1 - E \right) \left( 1 + r \right) f(l_1) dl_1 \ g(s) ds f(l_2) dl_2 \tag{B.2} \\
&- \delta h \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2}}^{\frac{1}{2}} \Delta(1 + r) R_m \ f(l_1) dl_1 \ g(s) ds f(l_2) dl_2 \\
&- \delta h \int_{\frac{1}{2}}^{\frac{1}{2}} \int_{\frac{1}{2}}^{\frac{1}{2}} \left[ 1 - (1 + R_m)l_1 \right] \left[ 1 + R_m \right] \left( 1 - l_2 \right) - \left( 1 - E - \Delta R_m \right) \left( 1 + r \right) f(l_1) dl_1 \ g(s) ds f(l_2) dl_2 .
\end{align*}
\]

The top line is positive if \( \bar{T}^R_{2} > L \) and zero if \( \bar{T}^R_{2} = L \). It expresses expected returns when banking is profitable under the refinance method in period-2, but the bank cannot achieve period-2 by rolling over its defaulted loans due to illiquidity. The median line indicates returns when the bank is profitable during period-2 under both methods. It is negative if \( \bar{T}^R_{2} > L \) and zero if \( \bar{T}^{Ro}_{2} = \bar{L} \). The bottom line shows returns when banking is profitable under the rolling over method but unprofitable under the refinance method. It is zero if \( \bar{T}^{Ro}_{2} = \bar{L} \) or \( \bar{T}^R_{2} = \bar{L} \) and negative in other cases.

Note that

\[
\left\{ \left[ 1 - (1 + R_m)l_1 \right] \left[ 1 + R_m \right] \left( 1 - l_2 \right) - \left( 1 - E \right) \left( 1 + r \right) \right\} + \Delta(1 + r) R_m . \tag{B.3}
\]

\[42\]
When \( \tilde{t}^\text{Re}_2 < \tilde{t}_2^- < \tilde{t}^\text{Re}_2 \) the term in parenthesis is negative but (B.3) is positive. Thus, (B.3) is smaller than \( \Delta(1 + r)R_m \).

We will now show that (B.2) is non-positive but larger than (B.1). The proof consists of four cases. First, suppose that \( \tilde{t}^\text{Re}_2 = \tilde{L} \). Then \( \tilde{t}^\text{Re}_2 = \tilde{L} = \tilde{L} \), the bottom line of (B.2) is zero and (B.2) equals

\[
\delta h \int_{\tilde{L}}^{\tilde{t}_2} \int_{l_1}^{l_2} \left[ 1 - (1 + R_m) l_1 \right] (1 + R_m) (1 - l_2) - (1 - E)(1 + r) f(l_1) dl_1 \ g(s) ds \ f(l_2) dl_2 
\]

\[
- \delta h \int_{\tilde{L}}^{\tilde{t}_2} \int_{l_1}^{l_2} \Delta(1 + r) R_m f(l_1) dl_1 \ g(s) ds \ f(l_2) dl_2 .
\]

Now the term on the lower line is equal to (B.1), but the term on the upper line is positive. Hence, (B.2) is larger than (B.1). Suppose now \( \tilde{L} > \tilde{t}^\text{Re}_2 > \tilde{L} \). Given (B.3), (B.2) is now larger than

\[
\delta h \int_{\tilde{L}}^{\tilde{t}_2} \int_{l_1}^{l_2} \left[ 1 - (1 + R_m) l_1 \right] (1 + R_m) (1 - l_2) - (1 - E)(1 + r) f(l_1) dl_1 \ g(s) ds \ f(l_2) dl_2 
\]

\[
- \delta h \int_{\tilde{L}}^{\tilde{t}_2} \int_{l_1}^{l_2} \Delta(1 + r) R_m f(l_1) dl_1 \ g(s) ds \ f(l_2) dl_2 ,
\]

in which the lower line cannot be smaller than (B.1) and the upper line is positive. Hence, (B.5) and (B.2) are larger than (B.1). Third, suppose that \( \tilde{t}^\text{Re}_2 = \tilde{L} \). Now the top line and the median line of (B.2) are zero and (B.2) simplifies to
\[-\delta h \int_0^{t^*} \int_{[\frac{1}{2}, \frac{1}{2}]} \left[ 1 - (1 + R_m^*) l_t \right] (1 + R_m)(1 - l_t) - \left( 1 - E - \Delta R_m \right)(1 + r) \cdot f(l_t) dl_t \cdot g(s) ds \cdot f(l_t) dl_t. \quad (B.6)\]

Given (B.3) this is larger than (B.1). Finally, when \( \bar{I}_2^{Re} = L \), then \( \bar{I}_2^{Ro} = \bar{I}_2^{Re} = L \) and (B.2) is zero.

Consequently, the difference in expected returns in period-2, \( h\delta \left[ \pi_2^{Re} - \pi_2^{Ro} \right] \) or (B.2), is always larger than (B.1). Q.E.D

**Appendix C**

This Appendix shows that when \( \text{Min} \left[ E, \Delta R_m \right] = E \), where \( \Delta R_m = l_t R_m + (1 - l_t) s R_m \), the bank returns are lower under the rolling over method than under the refinance method even through the interest receivables can be incorporated in the capital requirement. To begin, let \( \varepsilon \) denote \( \varepsilon = \Delta R_m - E > 0 \). Since \( \text{Min} \left[ E, \Delta R_m \right] = E \), the bank can release \( E \) units of initially injected capital at the end of period-1. During period-2 the amount of capital is the sum of the initially injected capital, \( E \), and retained earnings, \( \Delta R_m = l_t R_m + (1 - l_t) s R_m \), from which the released capital, \( E \), is subtracted; that is, the amount of capital is \( E + \varepsilon \). Given the released capital, \( E \), and the loan interest income, \( (1 - l_t)(1 - s) R_m \), the funds of the bank total \( (1 - l_t)(1 - s) R_m + E \) or \( R_m + (E - \Delta R_m) \) or \( R_m - \varepsilon \) during period-1. The payments on deposits amount to \( (1 - E)r \).

Appendix B has already explored the case where \( \varepsilon \) is so large that the bank is illiquid during period-1, \( R_m - \varepsilon < (1 - E)r \). This is easy to see by reviewing Appendix B when \( \Delta R_m = \varepsilon \).

Thereafter, \( \varepsilon \) is assumed to be so small that the bank is liquid during period-1; \( R_m - \varepsilon > (1 - E)r \). When the life-time returns from the rolling over method are subtracted from the life-time returns of the refinance method, we obtain
The first term indicates returns during period-1 and the second term shows returns during period-2. Note that since the bank is liquid at the end of period-1, (C.1) differs from (4.14). It is again necessary to explore expected returns from period-2, $h\delta \left[ \pi^{re}_2 - \pi^{re}_1 \right]$, in detail. Recall the difference from (B.2) when $\bar{l}_1 = \bar{L}$, and $\bar{s}_i = \bar{S}$ (the bank is never illiquid under the rolling over method)

\begin{equation}
- \partial h \int_{\bar{l}}^{\bar{L}} \int_{\bar{s}}^{\bar{S}} \epsilon (1 + r) f(l_1) dl_1 g(s) ds f(l_2) dl_2
\end{equation}

\begin{equation}
- \partial h \int_{\bar{l}}^{\bar{L}} \int_{\bar{s}}^{\bar{S}} \left[ 1 - (1 + R_n)l_1 (1 + R_n) (1 - l_2) - (1 - E - \epsilon)(1 + r) f(l_1) dl_1 g(s) ds f(l_2) dl_2. \right]
\end{equation}

We will show that (C.2) is larger than

\begin{equation}
- h \int_{\bar{l}}^{\bar{L}} \int_{\bar{s}}^{\bar{S}} \epsilon f(l_1) dl_1 g(s) ds f(l_2) dl_2,
\end{equation}

which ensures that (C.1) is positive. From (C.2) it is easy to see that (C.2) is always larger than (C.3) when $\bar{l}_2^{re} < \bar{L}$ (recall (B.3)). It is enough to examine the case $\bar{l}_2^{re} = \bar{L}$. When $\bar{l}_2^{re} = \bar{L}$, (C.2) is equal to (C.3) and thus (C.1) is zero. But when $\bar{l}_2^{re} = \bar{L}$, the bank always succeeds during period-2. We will show that this cannot be possible, since the non-monitoring strategy can be profitable only if the bank fails with a positive probability and thus benefits from limited liability. When $\bar{l}_2^{re} = \bar{L}$ the banker’s expected returns under the rolling over method are
where the terms in the big brackets and parenthesis are negative. Hence, banking can be profitable
(or yield zero returns) under the rolling over method only if \( I_{z}^{Re} < L \). But when \( I_{z}^{Re} < L \), (C.2)
exceeds (C.3) and thus (C.1) is positive. The refinance method is more profitable than the rolling
over method. Q.E.D
References


